4 Microlensing

In this chapter, we introduce *microlensing*. With this term, we intend a vast class of lensing phenomena by lenses that can be safely treated as point masses. We refer in particular to events occurring within our galaxy (*galactic microlensing*), produced by objects as small as planets. Typical lenses in microlensing events are stars in the disk of our galaxy, lensing other stars in the galactic bulge. Alternatively, microlensing events are searched towards other regions of the sky, where many potential lensed sources are observable, i.e. towards the Magellanic Clouds, or even the Andromeda galaxy.

In this chapter, we provide the theoretical basis for understanding the microlensing phenomena and briefly review their observational signatures. Suggested readings include the book "Gravitational lensing and microlensing" by Mollerach & Roulet (2002), and several reviews like e.g. Paczynski (1996); Gaudi (2003, 2012); Mao (2012); Rahvar (2015).

The chapter is structured as follows. We start by discussing the lensing properties of a single, isolated point mass. Then, we consider ensembles of point masses as lenses, focussing in particular on binary lenses. As a particular case of binary lens, we consider the system composed by a star and an orbiting planet.

4.1 Point masses

In this section, we review the lensing properties of a point mass. Its deflection angle was already found to be

$$\hat{\vec{\alpha}} = -\frac{4GM}{c^2 b}\vec{e}_r \; , \tag{4.1}$$

where $\vec{e}_r$ is the unit vector in radial direction. No direction is prefered in an axisymmetric situation like that, so we can identify $\vec{e}_r$ with one coordinate axis and thus reduce the problem to one dimension. Then

$$\hat{\alpha} = \frac{4GM}{c^2 b} = \frac{4GM}{c^2 D_\mathsf{L}\theta} \; , \tag{4.2}$$

where we have expressed the impact parameter by the angle θ, $b = D_\mathsf{L}\theta$.

The lensing potential is given by

$$\hat{\Psi} = \frac{4GM}{c^2}\frac{D_\mathsf{LS}}{D_\mathsf{L}D_\mathsf{S}}\ln|\vec{\theta}| \; , \tag{4.3}$$

as one can show using

$$\nabla \ln|\vec{x}| = \frac{\vec{x}}{|\vec{x}|^2} \; . \tag{4.4}$$

The lens equation reads

$$\beta = \theta - \frac{4GM}{c^2 D_{\mathsf{L}} \theta} \frac{D_{\mathsf{LS}}}{D_{\mathsf{S}}} \ . \tag{4.5}$$

With the definition of the Einstein radius,

$$\theta_E \equiv \sqrt{\frac{4GM}{c^2} \frac{D_{\mathsf{LS}}}{D_{\mathsf{L}} D_{\mathsf{S}}}} \ , \tag{4.6}$$

we have

$$\beta = \theta - \frac{\theta_E^2}{\theta} \ . \tag{4.7}$$

Dividing by θ_E and setting $y = \beta/\theta_E$ and $x = \theta/\theta_E$, the lens equation in its adimensional form is written as

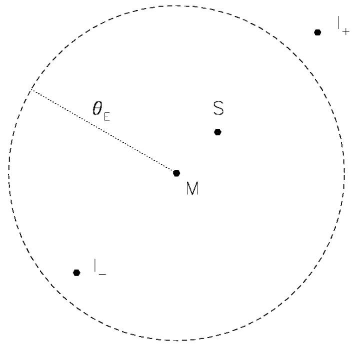
$$y = x - \frac{1}{x} \tag{4.8}$$

Multiplication with x leads to

$$x^2 - xy - 1 = 0 \ , \tag{4.9}$$

which has two solutions:

$$x_\pm = \frac{1}{2} \left[y \pm \sqrt{y^2 + 4} \right] \ . \tag{4.10}$$



Thus, a point-mass lens has two images for any source, irrespective of its distance y from the lens. Why not three? Because its mass is singular and thus the time-delay surface is not continously deformed.

If $y = 0$, $x_\pm = \pm 1$; that is, a source directly behind the point lens has a ring-shaped image with radius θ_E. For order-of-magnitude estimates:

$$\begin{aligned} \theta_E &\approx (10^{-3})'' \left(\frac{M}{M_\odot} \right)^{1/2} \left(\frac{D}{10\mathrm{kpc}} \right)^{-1/2} , \\ &\approx 1'' \left(\frac{M}{10^{12} M_\odot} \right)^{1/2} \left(\frac{D}{\mathrm{Gpc}} \right)^{-1/2} , \end{aligned} \tag{4.11}$$

where

$$D \equiv \frac{D_{\mathsf{L}} D_{\mathsf{S}}}{D_{\mathsf{LS}}} \tag{4.12}$$

is called *effective lensing distance*.

As $\beta \to \infty$, we see that $\theta_- = x_- \theta_E \to 0$, while obviously $\theta_+ = x_+ \theta_E \to \beta$: when the angular separation between the lens and the source becomes large, the source is unlensed. Formally, there is still an image at $\theta_- = 0$.

The magnifications follow from from the Jacobian. For any axially-symmetric lens,

$$\begin{aligned}\det A &= \frac{y}{x}\frac{\partial y}{\partial x} = \left(1-\frac{\alpha}{x}\right)\left(1-\frac{\partial \alpha}{\partial x}\right) \\ &= \left(1-\frac{1}{x^2}\right)\left(1+\frac{1}{x^2}\right) = 1-\left(\frac{1}{x}\right)^4 \\ &\Rightarrow \ \mu = \left[1-\left(\frac{1}{x}\right)^4\right]^{-1} ,\end{aligned} \tag{4.13}$$

i.e. a source at $y = 1$ has two images at

$$x_{\pm} = \frac{1 \pm \sqrt{5}}{2} , \tag{4.14}$$

and their magnifications are

$$\mu_{\pm} = \left[1-\left(\frac{2}{1 \pm \sqrt{5}}\right)^4\right]^{-1} . \tag{4.15}$$

For general source positions,

$$\begin{aligned}\mu_{\pm} &= \left[1-\left(\frac{1}{x_{\pm}}\right)^4\right]^{-1} \\ &= \frac{x_{\pm}^4}{x_{\pm}^4 - 1} = \frac{1}{2} \pm \frac{y^2+2}{2y\sqrt{y^2+4}} .\end{aligned} \tag{4.16}$$

Note that $\lim_{y\to\infty} \mu_- = 0$ and that $\lim_{y\to\infty} \mu_+ = 1$: even if the lens equation has always two solutions, for large angular separations between the source and the lens, one image desappears because it is demagnified. The other is completely undistiguishable from the source because it has the same flux and the same position.

The total magnification of a point source by a point mass is thus

$$\mu = |\mu_+| + |\mu_-| = \frac{y^2+2}{y\sqrt{y^2+4}} , \tag{4.17}$$

and the magnification ratio of the two images is

$$|\frac{\mu_-}{\mu_+}| = \left(\frac{y-\sqrt{y^2+4}}{y+\sqrt{y^2+4}}\right)^2 = \left(\frac{x_-}{x_+}\right) . \tag{4.18}$$

If $\beta = \theta_E$, $y = 1$ and the total magnification is $\mu = 1.17 + 0.17 = 1.34$. In terms of magnitudes, this correspond to $\Delta m = -2.5 \log \mu \sim 0.3$. The image forming at x_+ contributes for $\sim 87\%$ of the total magnification.