5 Lensing by galaxies and galaxy clusters

In this chapter we will review the most important applications of lensing by galaxies and galaxy clusters. So far, we have learned:

- what are the properties of lenses with a given shape, either circular or elliptical;
- how to model external perturbations, i.e. mass sheets and external shear.

We can now employ this knowledge to interpret the lensing features around galaxies and clusters and learn about:

(1) how mass is distributed as a function of radius;

(2) if mass is distributed smoothly or in substructures;

(3) what is the best cosmological model which explains the amplitude and the redshift dependence of the lensing effects;

(4) what are the properties of the sources being lensed.

Applications 1 and 2 aim at understanding the internal structure of the lenses. Therefore, they are *lens oriented* applications. Interesting questions are: what is the density profile of galaxies and clusters? what is the interplay between baryons and dark-dark matter? Is the currently favored model of cold-dark-matter (CDM) consistent with how matter is distributed in the cosmic structures?

Application 3 aims at using gravitational lensing by bound structures such as galaxies and clusters as a probe for cosmology. The strength and the abundance of the lenses depends on the cosmic distances and on the growth of the cosmic structures. Thus, the idea is using lensing for cosmography and constraining the structure formation process.

Application 4 is *source oriented*. Once the lens properties are known, i.e. once we know how a certain galaxy or a cluster bend light, then we can use this knowledge to de-lens the sources and profit of the lensing magnification to resolve features in the sources which would be unobservable otherwise.

5.1 Galaxies and clusters as gravitational lenses

We may list here few key properties of the lenses we will discuss in this chapter.

Elliptical galaxies For the purposes of this discussion, we will consider two very broad classes of galaxies: ellipticals and spirals. Elliptical galaxies have ellipsoidal/spheroidal shapes, thus we will attempt to model them using the simple lens models studied earlier in this book.

Ellipticals typically have steep surface brightness profiles, well described by the Sérsic law

$$S(R) = S_e \exp\left\{-b_n\left[\left(\frac{R}{R_e}\right)^{1/n} - 1\right]\right\}, \tag{5.1}$$

with index $n \gtrsim 4$. In the formula above, S is the surface brightness, R is the radius, R_e and S_e are the effective radius and the surface brightness at R_e, and b_n is a positive parameter that, for a given n, can be determined from the definition of r_e and S_e.

Their masses cover a wide range, from $10^7\ M_\odot$ in the case of dwarf ellipticals, to $10^{12}\ M_\odot$ in the case of the brightest central galaxies of clusters. Nevertheless, ellipticals are the principal contributors to the high mass end of the galaxy mass function. Together with the fact that they are abundant at low redshift, rather than at high redshift, these galaxies are the most probable strong gravitational lenses we may encounter in the sky.

Ellipticals are mainly constituted by old stellar populations and contain little gas. Therefore, they are typically red in color. Moreover, their abundance is higher in dense environments than in the field. For example, they make up to $\sim 70-80$ of the galaxies in clusters.

Spiral galaxies Spiral galaxies have a more complex internal structure for the purpose of lens modeling. We should at least account for a bulge and a disk, where stars have different radial distributions, in addition to their dark matter halo. Being sustained by rotation, disks are extended and flat, with surface brightness profiles well described by Sérsic models with $n \sim 1$. The bulges have surface brightness distributions more similar to that of ellipticals.

Spiral galaxy masses are in the range $10^9 - 10^{11}\ M_\odot$. The disks of these galaxies contain large amounts of gas and dust, from which young stars have birth. Thus, spirals are bluer than elliptical galaxies. As we can imagine, the color contrast is particularly important to disentangle lenses and sources. Spiral galaxies (and we include also irregular galaxies in this broad class of galaxies) are abundant in the field and at higher redshift, thus being the typical sources in a galaxy-galaxy lens system.

Galaxy clusters Galaxy clusters are ensembles of thousands of galaxies. In the framework of a hierarchical model of structure formation, these cosmic structures form late in cosmic time via mergers of smaller structures. Because of this and of the fact that they are still young and evolving systems, they often have irregular shapes.

Their content reflects the overall composition of the universe. Their major constituent is dark matter, accounting for about 85% of their content. The remaining 15% is made by the hot (1-10 keV) intra-cluster plasma, and by stars, mainly locked in the cluster galaxies.

Clusters are the largest bound structures in the universe and the populate the exponential tail of the structure mass function. Their masses are in the range $10^{13}\ , M_\odot$ (groups) $10^{15}\ , M_\odot$ (clusters). Because of their huge masses, clusters are the most efficient gravitational lenses in the universe. At the same time they are rarer objects compared to galaxies.

The cluster galaxy population is dominated by red elliptical galaxies, which lay on a red sequence in the color-magnitude diagram. About $20-30\%$ of the galaxies are bluer spirals. The fraction of spirals and elliptical is a function of cluster centric distance, with the spirals becoming more abundant towards the cluster outskirts.

5.2 Strong lensing

5.2.1 General considerations

Strong lensing occurs in the central regions of galaxies and galaxy clusters when the lens is "critical". This happens when it develops extended critical lines. As seen earlier, these form where

$$\lambda_t = 1 - \kappa - \gamma = 0 \quad (5.2)$$
$$\lambda_r = 1 - \kappa + \gamma = 0 \; . \quad (5.3)$$

The first equation defines the *tangential* critical line, the second the *radial* critical line. In the case of an axially symmetric lens, the tangential critical line is the Einstein radius.

It is obvious that extended systems are much more complex lenses than point masses. For what said above, the bulk of the mass in galaxies and clusters is dark and believed to be diffusely distributed in an halo. Observations of galaxies in clusters obviously show that clusters contain mass clustered at smaller scale than their halo, i.e. in galaxies. Observations of the local group show that even the galaxies contain smaller substructures, on the scale of *dwarf satellites*. On an even smaller scale, we finally see mass locked in stars. All these components concur to determine the intensity of the overall lensing effects of galaxies and clusters, but, taken individually, all these components produce lensing signals at different scales .

To quantify these scales, we may use the SIS model with typical values of the velocity dispersion σ_v. This scales with mass as $\sigma_v \propto M^{1/2}$. Therefore, if a galaxy cluster has σ_v is ~ 1000 km/s, a massive elliptical may have velocity dispersion $\sigma_v \sim 300$ km/s and a dwarf satellite $\sigma_v \sim 10$ km/s. The sizes of the Einstein radii of such SIS lenses are shown in Fig. 5.1 as a function of the lens redshift and for different source redshifts (a concordance ΛCDM cosmological model with $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$ was used to draw the plot. The figure tells us that strong lensing effects of the galaxy and of the cluster as a whole should be detectable on scale of arcsecs and tens of arcsecs, respectively. Substructures in galaxies produce effects on scales of milli-arcsecs, instead. As we have seen, the typical size of the stellar Einstein radius is few micro-arcsec. Thus, we should account for macro-, milli-, and micro-lensing effects when investigating lenses on the scales of galaxies and galaxy clusters. When modeling such lenses, this is a complication of course. However, depending on the circumstances (quality of the observations, type of the source, amount of lensing features) accurate lens modeling may be possible allowing us to investigate the properties of these lenses down to the smallest scales.

5.2.2 Parametric modeling

We can now outline a modeling strategy. We will re-think the lenses using the following paradigma:

- galaxies and clusters are made of dark matter and baryons (stars, gas). These components may have different spatial distributions, but it is often convenient to be guided by the stars to figure out what is the shape of the overall mass distribution;
- we may think of extended lenses as composed of a **main clump of matter** and many **substructures** orbiting around it. As such, the lens mass distribution may be written as

$$\kappa(\vec{x}) = \kappa_{sm}(\vec{x}) + \sum_i^{n_{sub}} \kappa_{sub,i}(\vec{x}) \quad (5.4)$$