GRAVITATIONAL LENSING

LECTURE 3

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CONTENTS

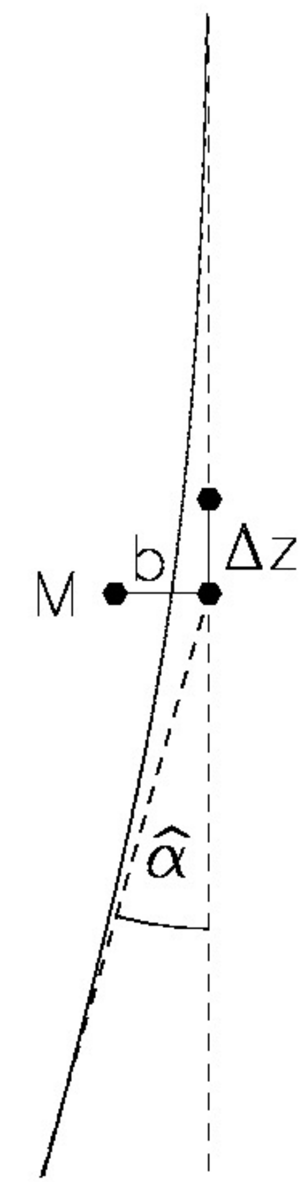
- Deflection by an ensemble of point masses
- Deflection by an extended mass distribution

DEFLECTION ANGLE OF A POINT MASS

$$\hat{\vec{\alpha}} = \frac{2}{c^2} \int_{-\infty}^{+\infty} \vec{\nabla}_\perp \Phi dz$$

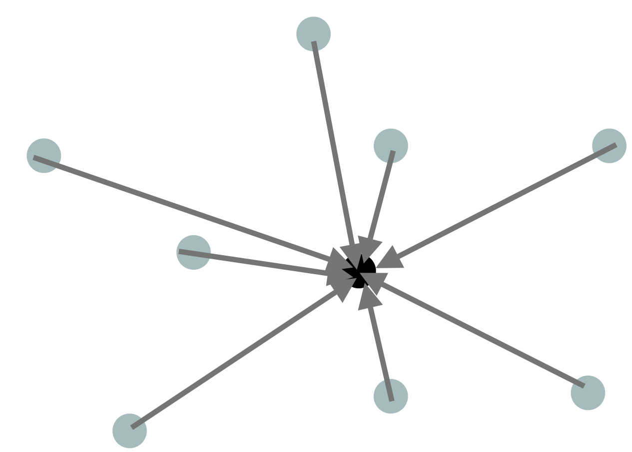
$$\Phi = -\frac{GM}{r}$$

$$\begin{aligned} \hat{\vec{\alpha}}(b) &= \frac{2GM}{c^2} \begin{pmatrix} x \\ y \end{pmatrix} \int_{-\infty}^{+\infty} \frac{dz}{(b^2+z^2)^{3/2}} \\ &= \frac{4GM}{c^2} \begin{pmatrix} x \\ y \end{pmatrix} \left[\frac{z}{b^2(b^2+z^2)^{1/2}} \right]_0^{\infty} = \frac{4GM}{c^2 b} \begin{pmatrix} \cos\phi \\ \sin\phi \end{pmatrix} \end{aligned}$$



DEFLECTION BY AN ENSEMBLE OF POINT MASSES

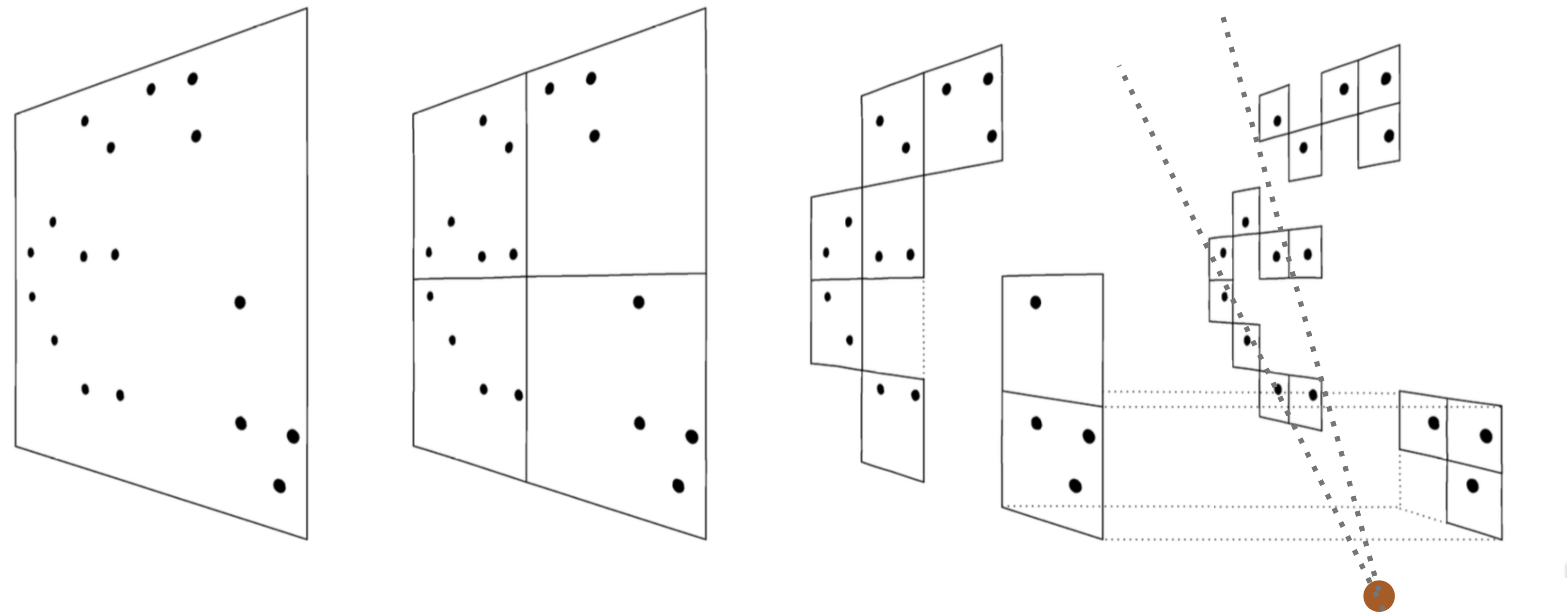
- Remaining in the weak field limit, one can use the superposition principle
- The deflection angle by a system of point masses is the vectorial sum of the deflection angles of the single lenses
- The calculation of the deflection angle by direct summation of all contributions from each point less has a computational cost $O(N^2)$



$$\hat{\vec{\alpha}}(\vec{\xi}) = \sum_i \hat{\vec{\alpha}}_i(\vec{\xi} - \vec{\xi}_i) = \frac{4G}{c^2} \sum_i M_i \frac{\vec{\xi} - \vec{\xi}_i}{|\vec{\xi} - \vec{\xi}_i|^2}$$

POSSIBLE SOLUTION: TREE ALGORITHM (BARNES & HUT, 1986)

Barnes & Hut
"oct-tree"



Short-range contributions (direct summation): particles in cells subtending large angles

Long-range contributions (grouped, Taylor expansion of the deflection potential,…): particles in cells subtending angles smaller than a chosen threshold

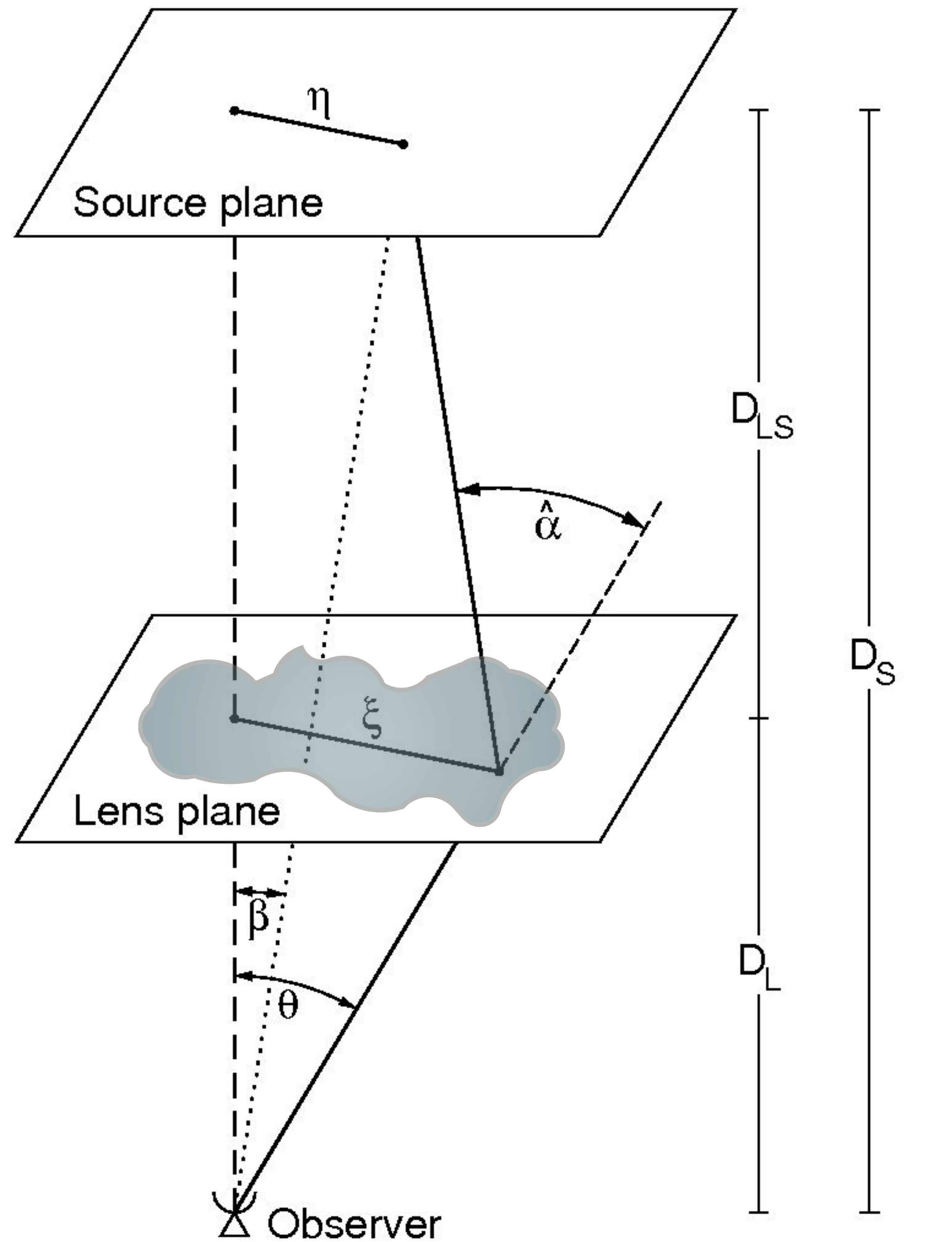
Cost of calculations scales as O(NLog(N))

DEFLECTION BY AN EXTENDED MASS DISTRIBUTION

- This can be easily generalized to the case of a continuum distribution of mass
- Assumption: thin screen approximation

$$\Sigma(\vec{\xi}) = \int \rho(\vec{\xi}, z)\ \mathrm{d}z$$

$$\vec{\hat{\alpha}}(\vec{\xi}) = \frac{4G}{c^2} \int \frac{(\vec{\xi} - \vec{\xi}')\Sigma(\vec{\xi}')}{|\vec{\xi} - \vec{\xi}'|^2}\ \mathrm{d}^2\xi'$$



HOW TO COMPUTE THIS DEFLECTION ANGLE?

$$\vec{\hat{\alpha}}(\vec{\xi}) = \frac{4G}{c^2} \int \frac{(\vec{\xi} - \vec{\xi}')\Sigma(\vec{\xi}')}{|\vec{\xi} - \vec{\xi}'|^2} \, \mathrm{d}^2\xi'$$

This is a convolution!

Kernel function:

$$\vec{K}(\vec{\xi}) \propto \frac{\vec{\xi}}{|\vec{\xi}|^2}$$

$$\tilde{\hat{\alpha}}_i(\vec{k}) \propto \tilde{\Sigma}(\vec{k})\tilde{K}_i(\vec{k})$$

This is the typical problem to be solved using FFT (Cooley and Tukey, 1965)