GRAVITATIONAL LENSING

LECTURE 4

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CONTENTS

- distances in cosmology

HUBBLE DISTANCE

suggested reading: http://arxiv.org/pdf/astro-ph/9905116v4.pdf

- The Hubble constant is the proportionality constant between the recession velocity and the distance in an expanding universe:

$$v = H_0\, d$$

- As you can see the dimensionality of the Hubble constant is the inverse time:

$$t_{\mathrm{H}} \equiv \frac{1}{H_0} = 9.78 \times 10^9\, h^{-1}\ \mathrm{yr} = 3.09 \times 10^{17}\, h^{-1}\ \mathrm{s}$$

- In this time the light travels the *Hubble* distance:

$$D_{\mathrm{H}} \equiv \frac{c}{H_0} = 3000\, h^{-1}\ \mathrm{Mpc} = 9.26 \times 10^{25}\, h^{-1}\ \mathrm{m}$$

SCALE FACTOR AND EXPANSION OF THE UNIVERSE

- Starting from the cosmological principle and from the Einstein equations, we can derive the Friedmann equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2(t) = H_0^2 E(z)^2$$

- Assuming that the universe is only made of matter and vacuum energy in the form of a cosmological constant:

$$E(z) \equiv \sqrt{\Omega_{\rm M}\,(1+z)^3 + \Omega_k\,(1+z)^2 + \Omega_\Lambda}$$

$$\Omega_{\rm M} \equiv \frac{8\pi\, G\, \rho_0}{3\, H_0^2}$$

$$\Omega_\Lambda \equiv \frac{\Lambda\, c^2}{3\, H_0^2}$$

$$\Omega_{\rm M} + \Omega_\Lambda + \Omega_k = 1$$

- The expansion of the universe is given by the scale factor $a(t)$ which is related to the redshift by

$$1 + z = \frac{1}{a} \qquad |dz| = \frac{|da|}{a^2}$$

COMOVING DISTANCE (ALONG THE LINE OF SIGHT)

- From the Friedmann equation we obtain

$$\frac{da}{dt}\frac{1}{a} = \frac{dz}{dt}a = H_0 E(z) \Rightarrow \frac{cdt}{a} = \frac{c}{H_0}\frac{dz}{E(z)} = D_H \frac{dz}{E(z)}$$

- Integrating:

$$(1+z)ct = D_c = D_H \int_0^z \frac{dz'}{E(z')}$$

- This distance is called "Comoving distance (along the line of sight)": This is **the distance between two points which remains constant over time if the two points move with Hubble flow.**

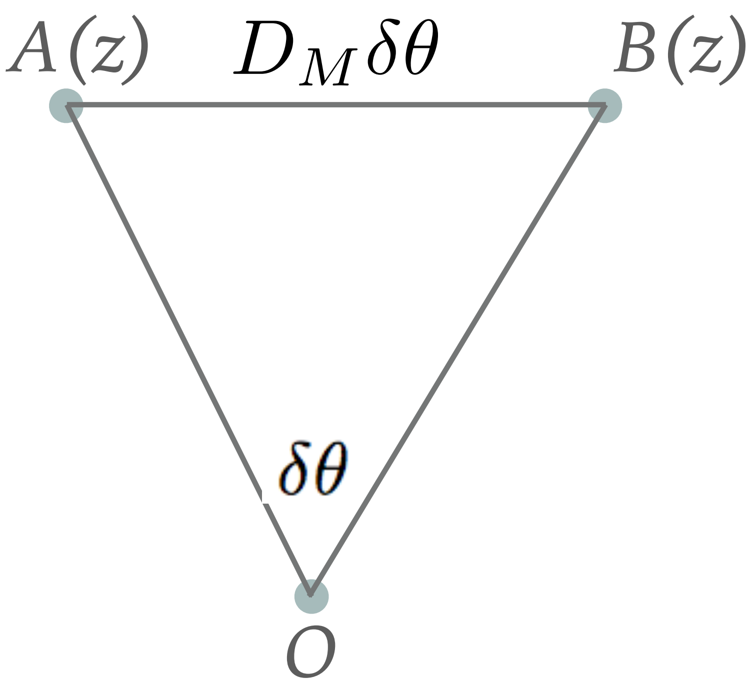
PROPER DISTANCE

- We can turn this distance into a ***proper*** distance by means of

$$D_{pr}(1+z) = D_c$$

- This is the **distance between the two points measured by rulers at the time they are being observed**

ANGULAR DIAMETER DISTANCE



$D_M\delta\theta$ = *comoving transversal distance*

$$D_{\rm M} = \begin{cases} D_{\rm H}\frac{1}{\sqrt{\Omega_k}}\sinh\left[\sqrt{\Omega_k}\,D_{\rm C}/D_{\rm H}\right] & \text{for } \Omega_k > 0 \\ D_{\rm C} & \text{for } \Omega_k = 0 \\ D_{\rm H}\frac{1}{\sqrt{|\Omega_k|}}\sin\left[\sqrt{|\Omega_k|}\,D_{\rm C}/D_{\rm H}\right] & \text{for } \Omega_k < 0 \end{cases}$$

$$D_{\rm A} = \frac{D_{\rm M}}{1+z}$$

= *angular diameter distance*

= *ratio of the physical (proper) transverse size to its angular size*