GRAVITATIONAL LENSING

LECTURE 1

Docente: Massimo Meneghetti

AA 2015-2016

DOCENTE

Massimo Meneghetti

Ricercatore

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ricevimento: da concordare via e-mail o telefono

google group: GravLens_2016

MATERIALE DIDATTICO

dispense del corso:

Introduction to Gravitational Lensing - lecture scripts, M. Meneghetti

scaricabile da: **http://pico.bo.astro.it/~massimo/pico/Teaching.html**

altro materiale:

Gravitational Lenses, P. Schneider, J. Ehlers, E.E. Falco, Springer-Verlag, 1992

Proceedings of the 33rd Saas Fee Advanced Course on Gravitational Lensing, scaricabile da http://www.astro.uni-bonn.de/~peter/SaasFee.html

eventuale ulteriore materiale (presentazioni, articoli, esercizi, ecc.) verrà fornito durante le lezioni

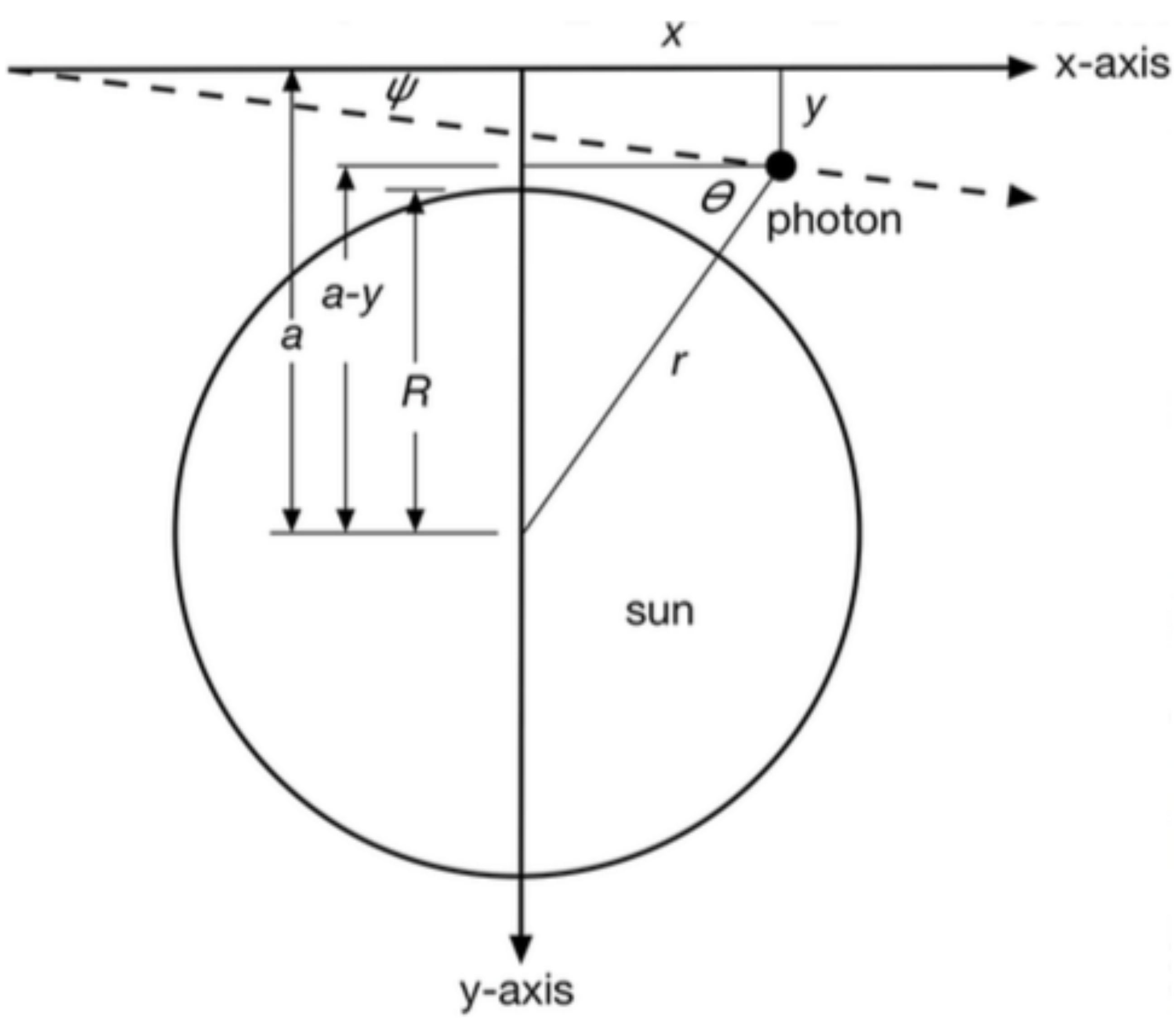
THE COURSE

- Basics of Gravitational Lensing Theory
- Applications of Gravitational Lensing:
  - microlensing in the MW
  - lensing by galaxies
  - lensing by galaxy clusters
  - lensing by the LSS
- Exercises and practical activities
- Final exam

CONTENTS

- Gravitational lensing in the Newtonian limit: what if photons had mass?
- Gravitational lensing in the context of general relativity
- The deflection angle

DEFLECTION OF A LIGHT CORPUSCLE



- Assumptions:
  - photons have an inertial gravitational mass
  - photons propagate at speed of light
  - Newton's law of gravity
  - Newton's principle of equivalence

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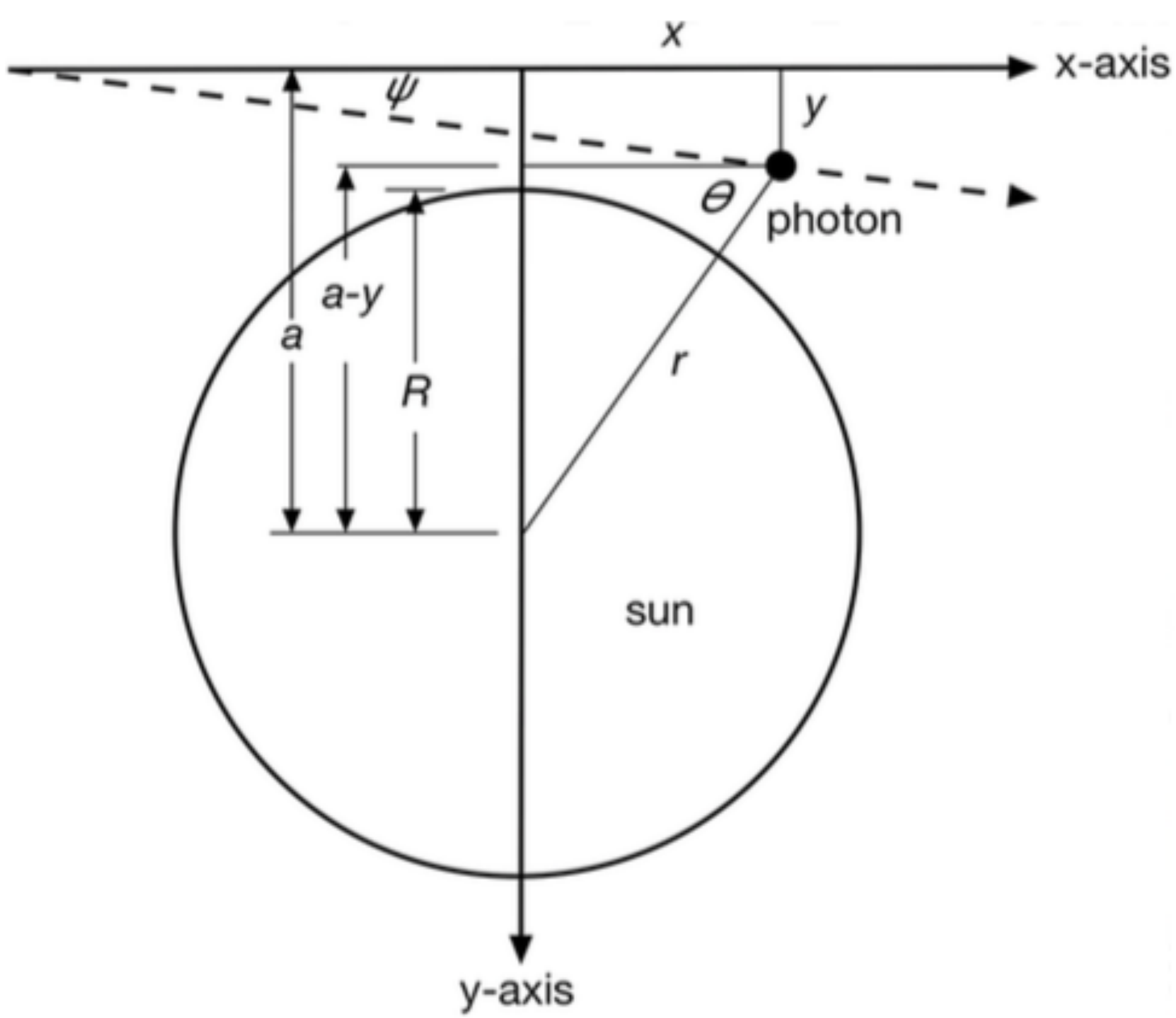

$$m = \frac{p}{c}$$

$$x = ct$$

$$\begin{aligned} \vec{F} &= \frac{d\vec{p}}{dt} \\ &= |F|(\cos\theta, \sin\theta) \\ &= \frac{GMm}{r^2}(\cos\theta, \sin\theta) \end{aligned}$$

$$r^2 = x^2 + (a - y)^2$$

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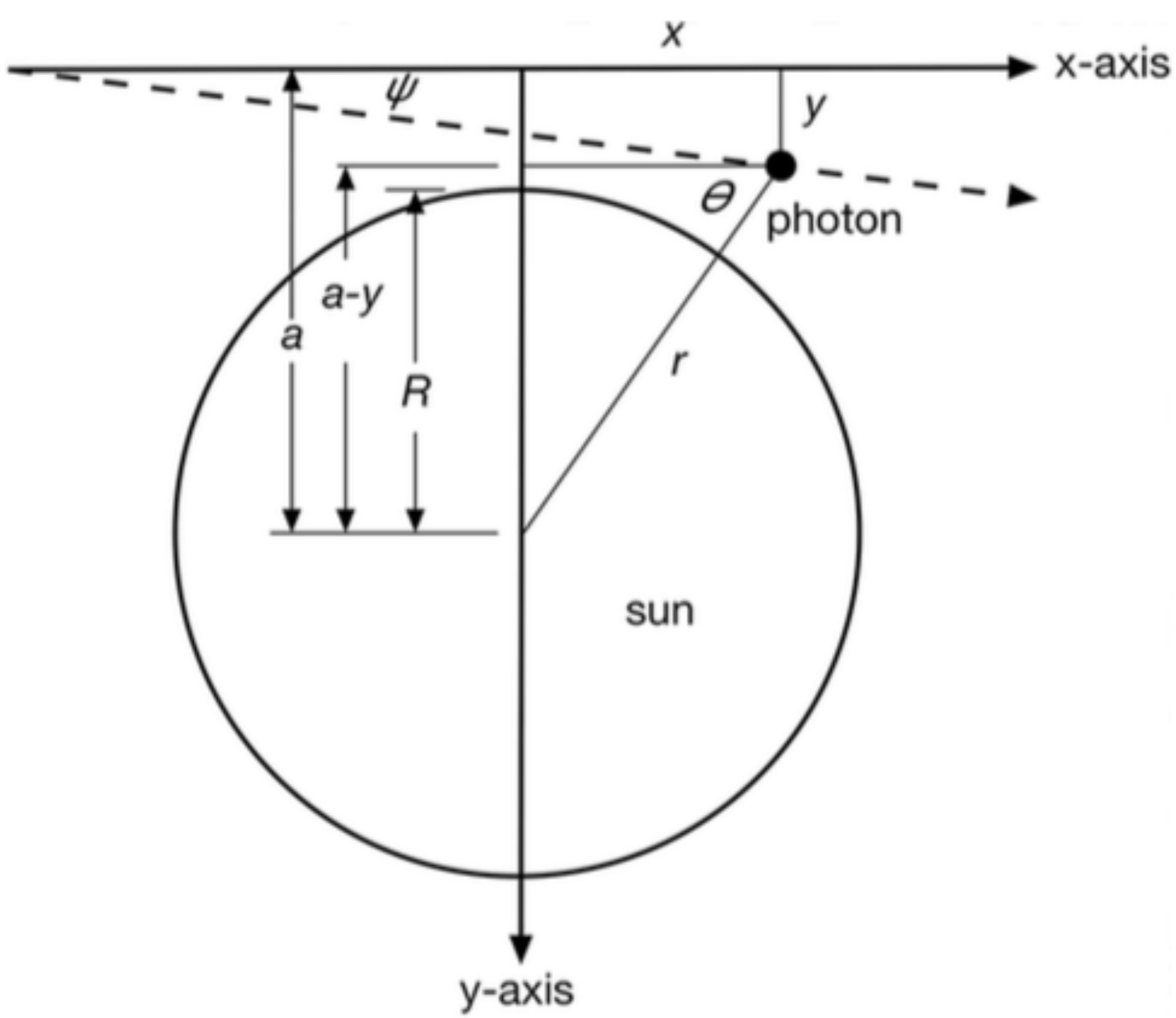

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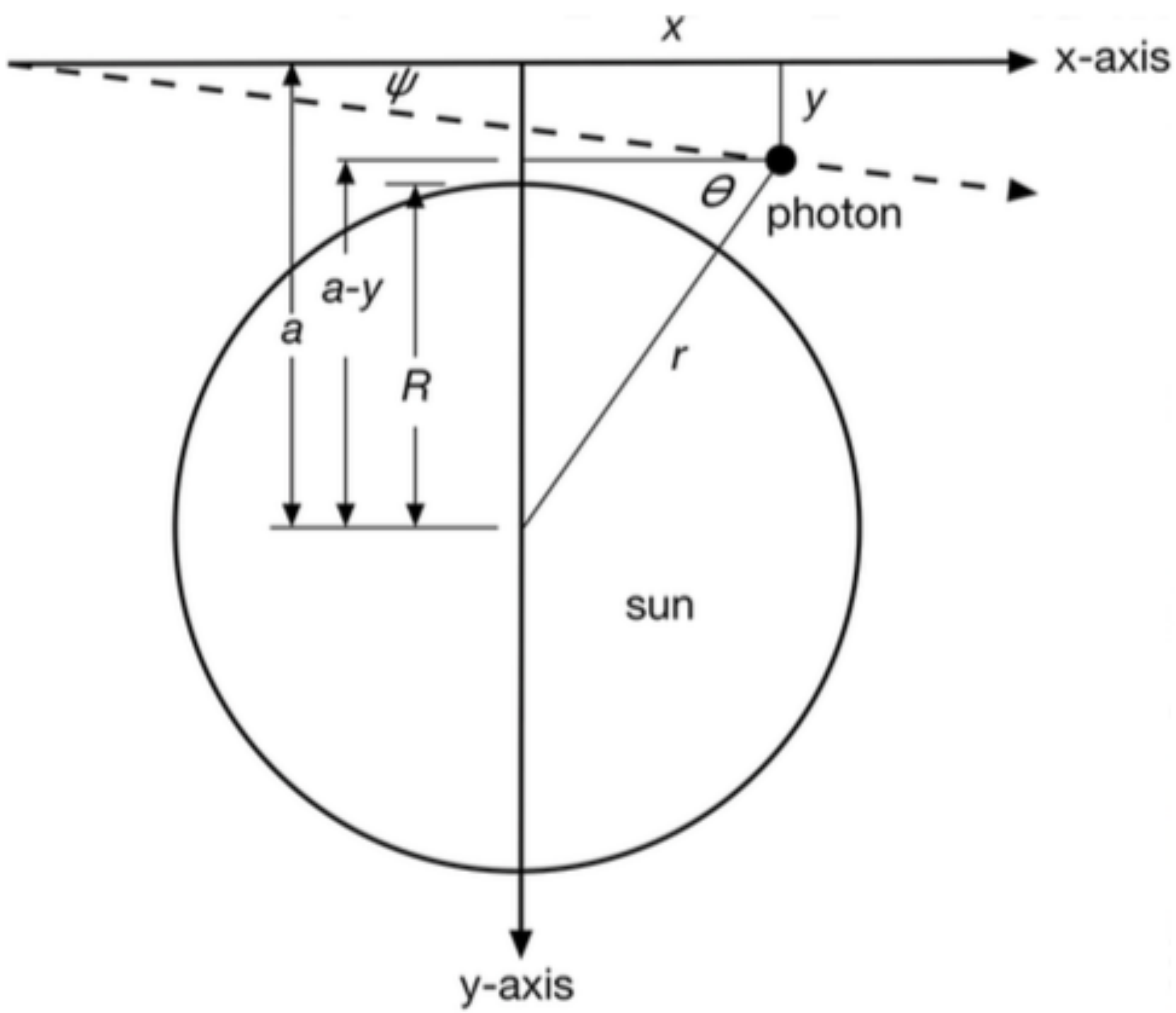

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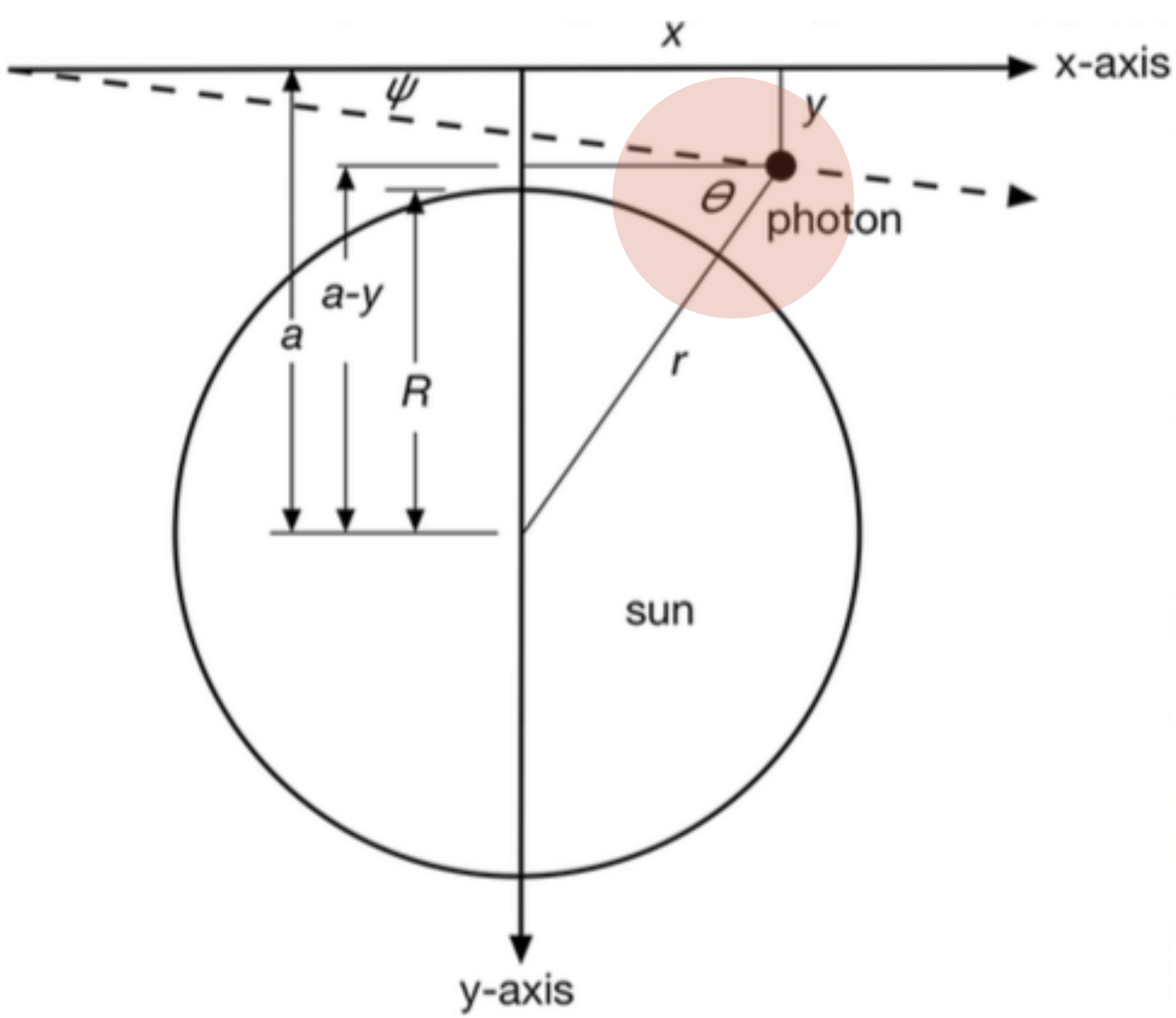

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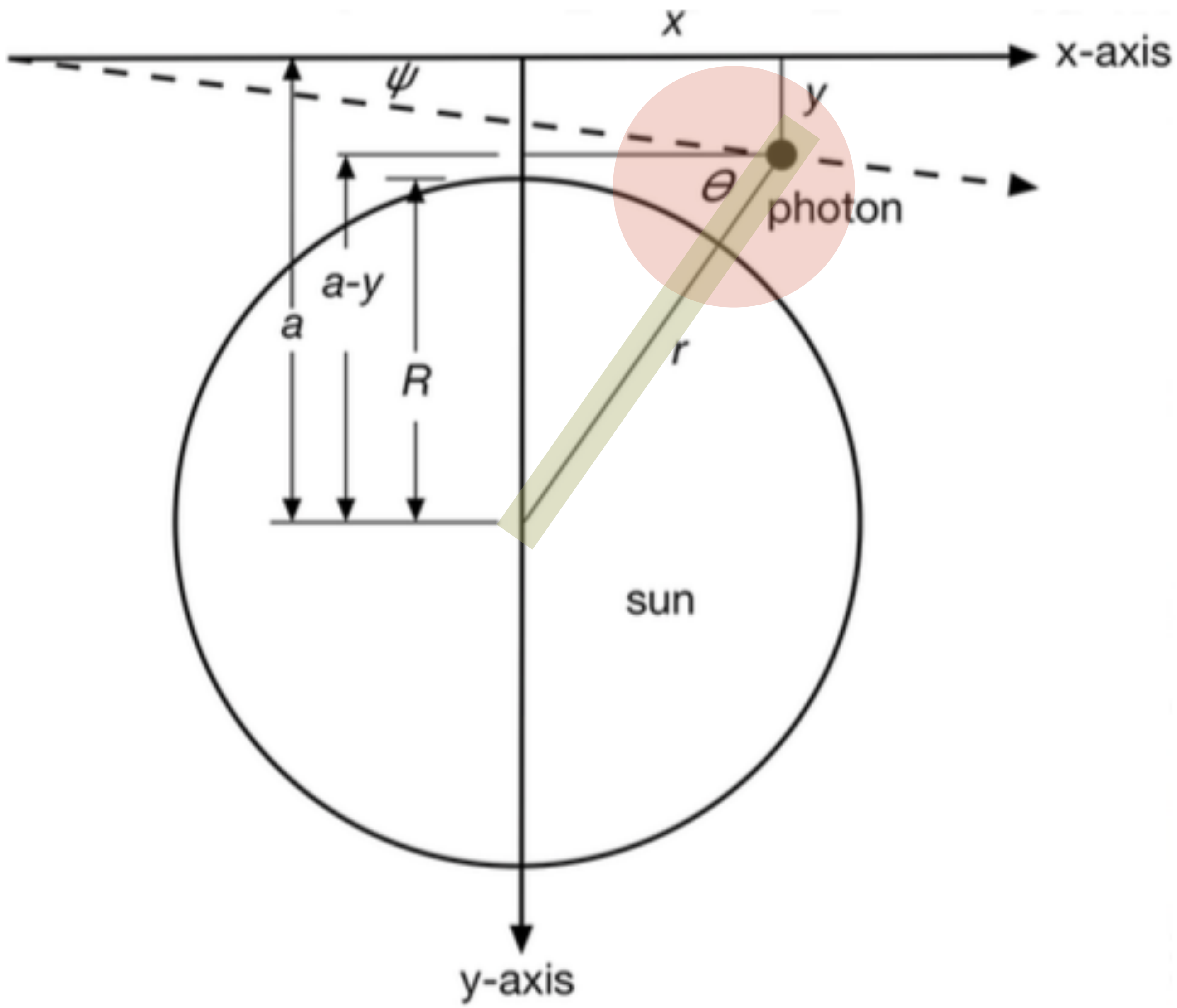

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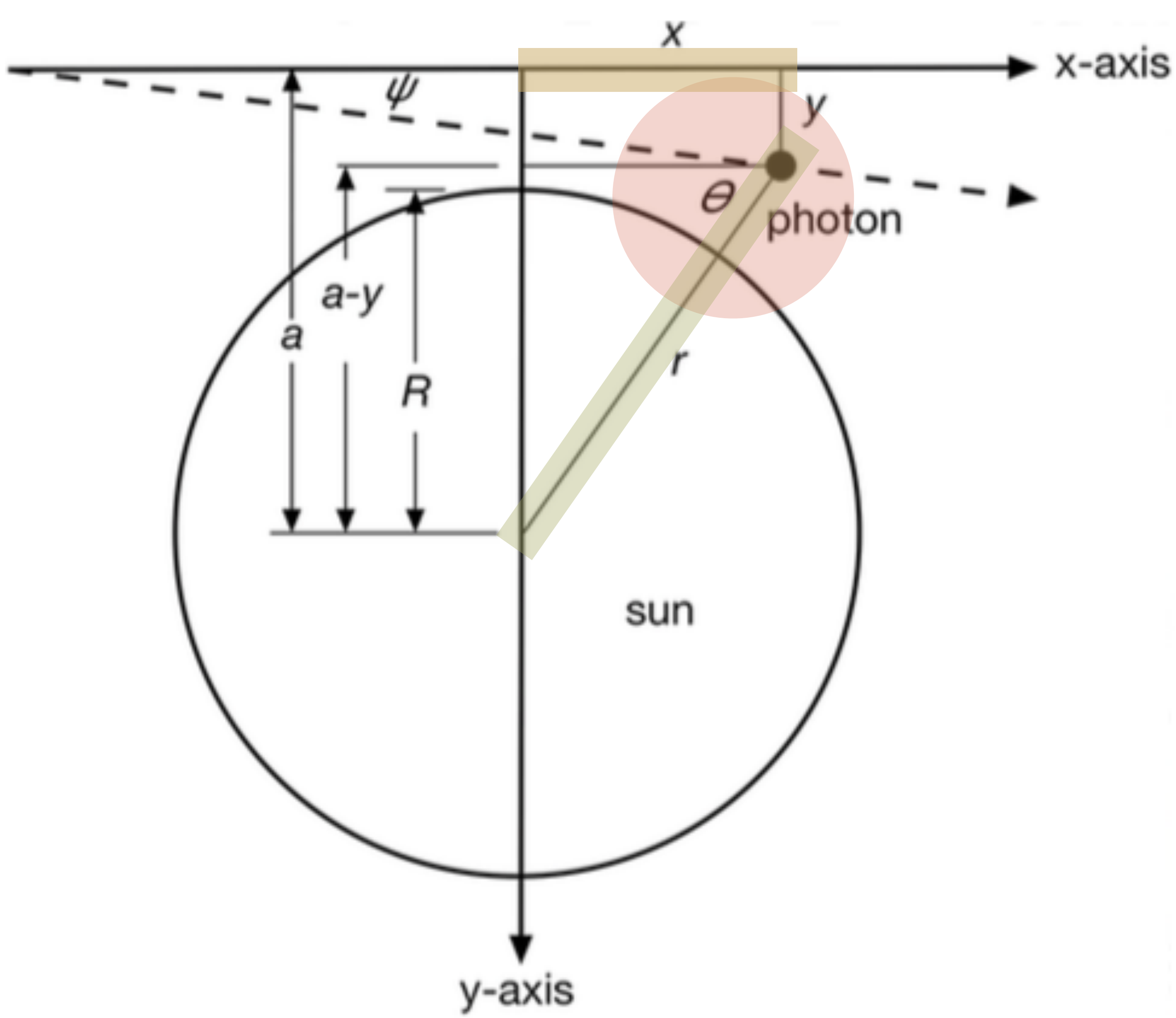

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DEFLECTION OF A LIGHT CORPUSCLE

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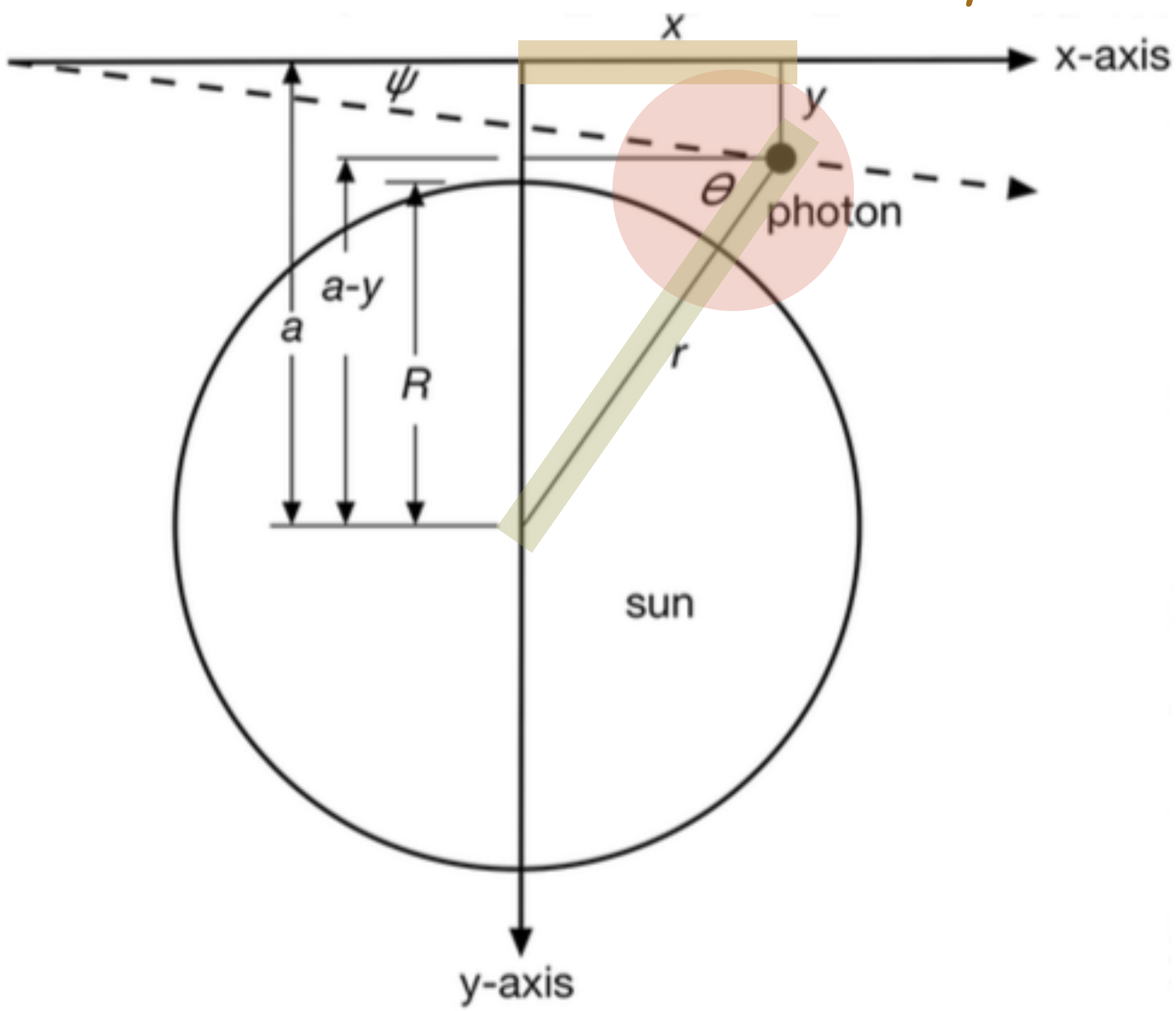

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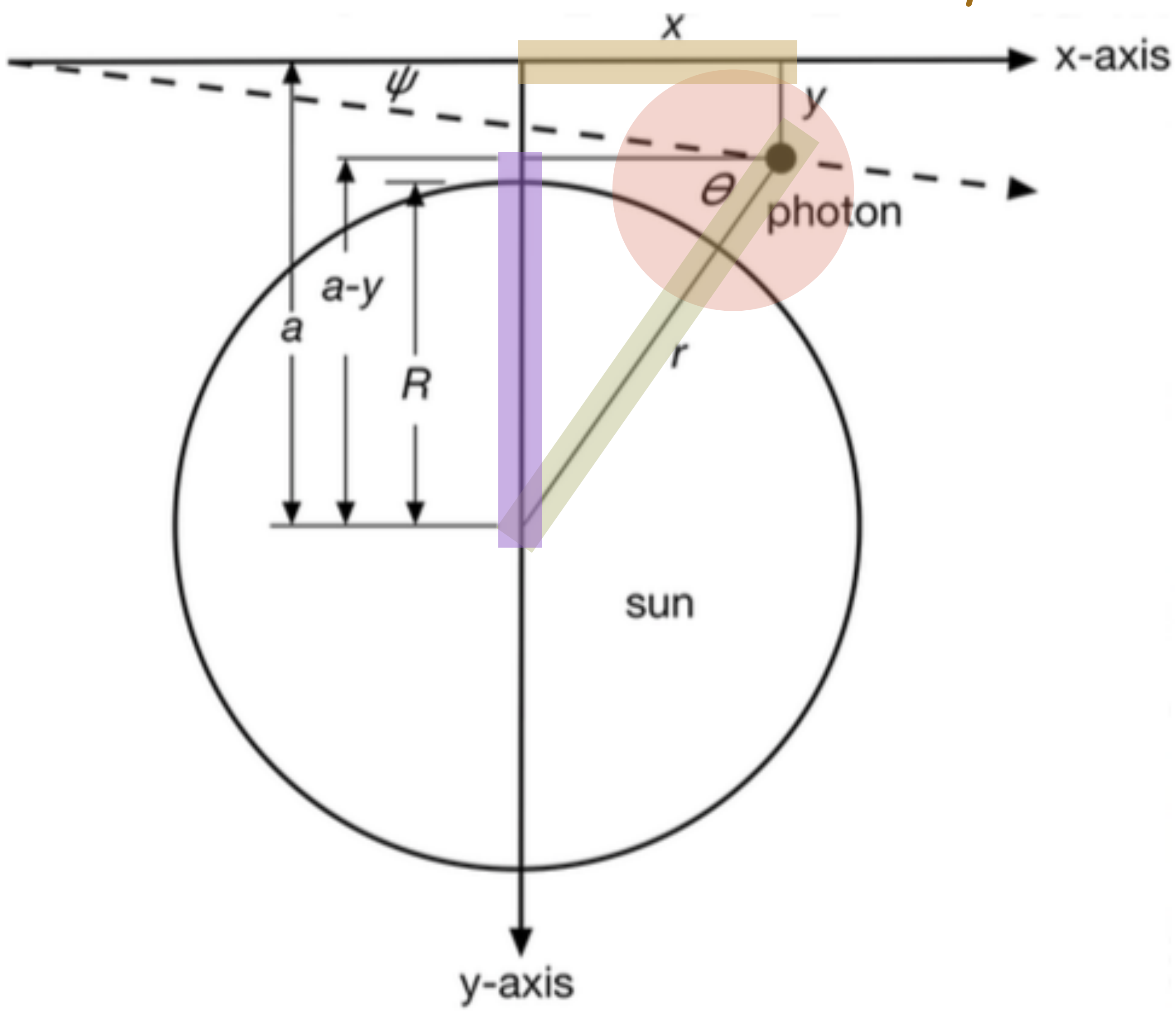

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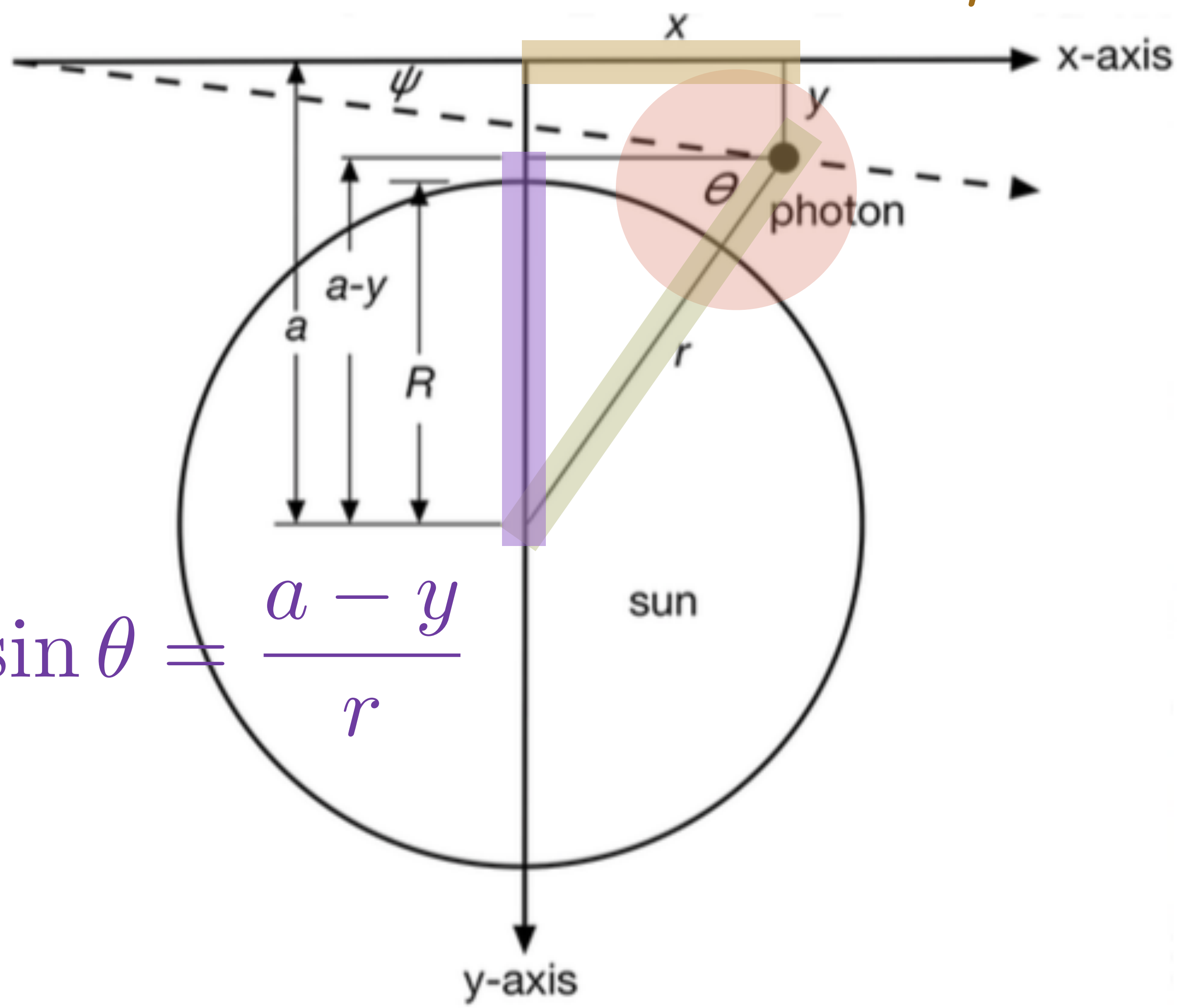

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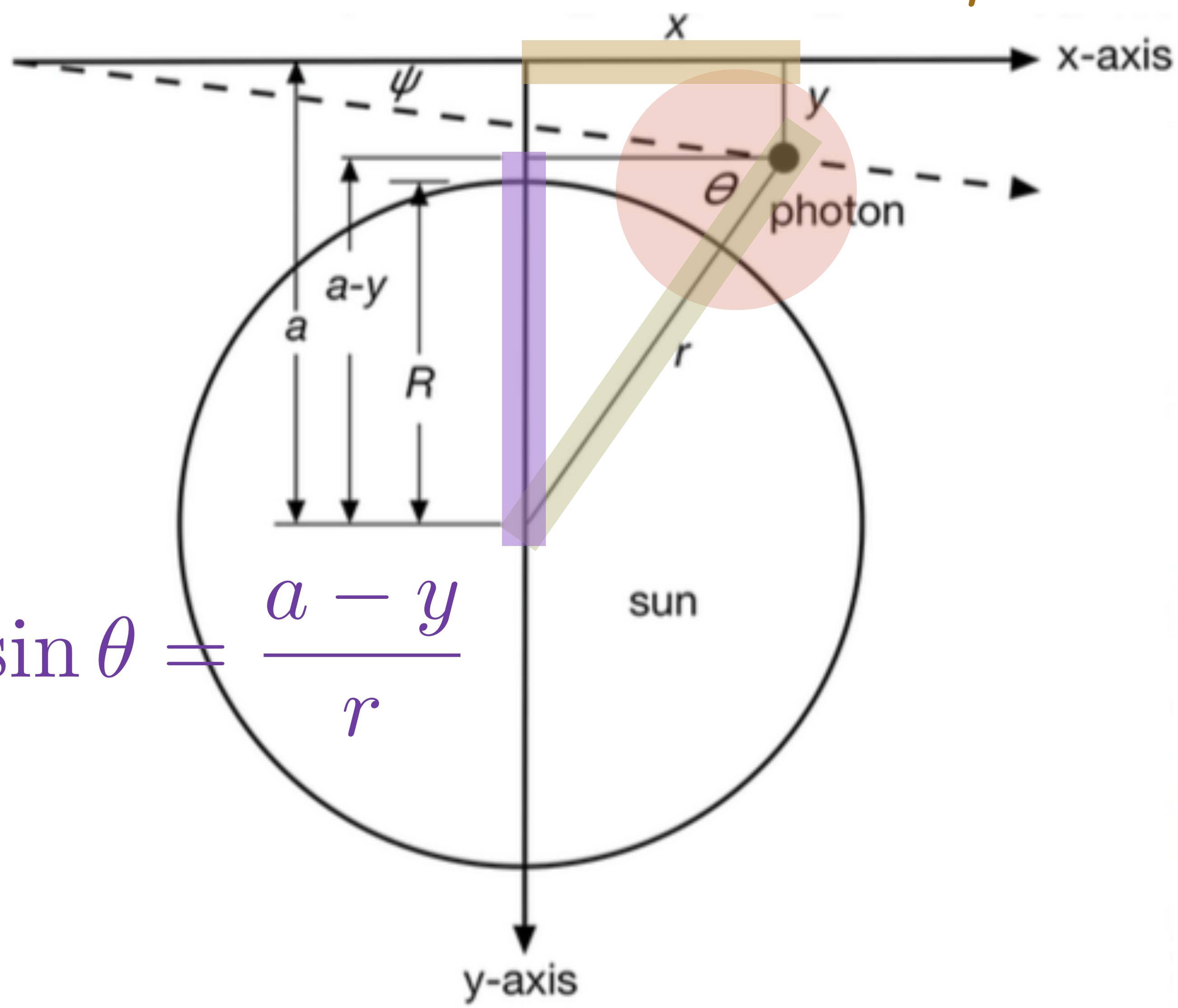

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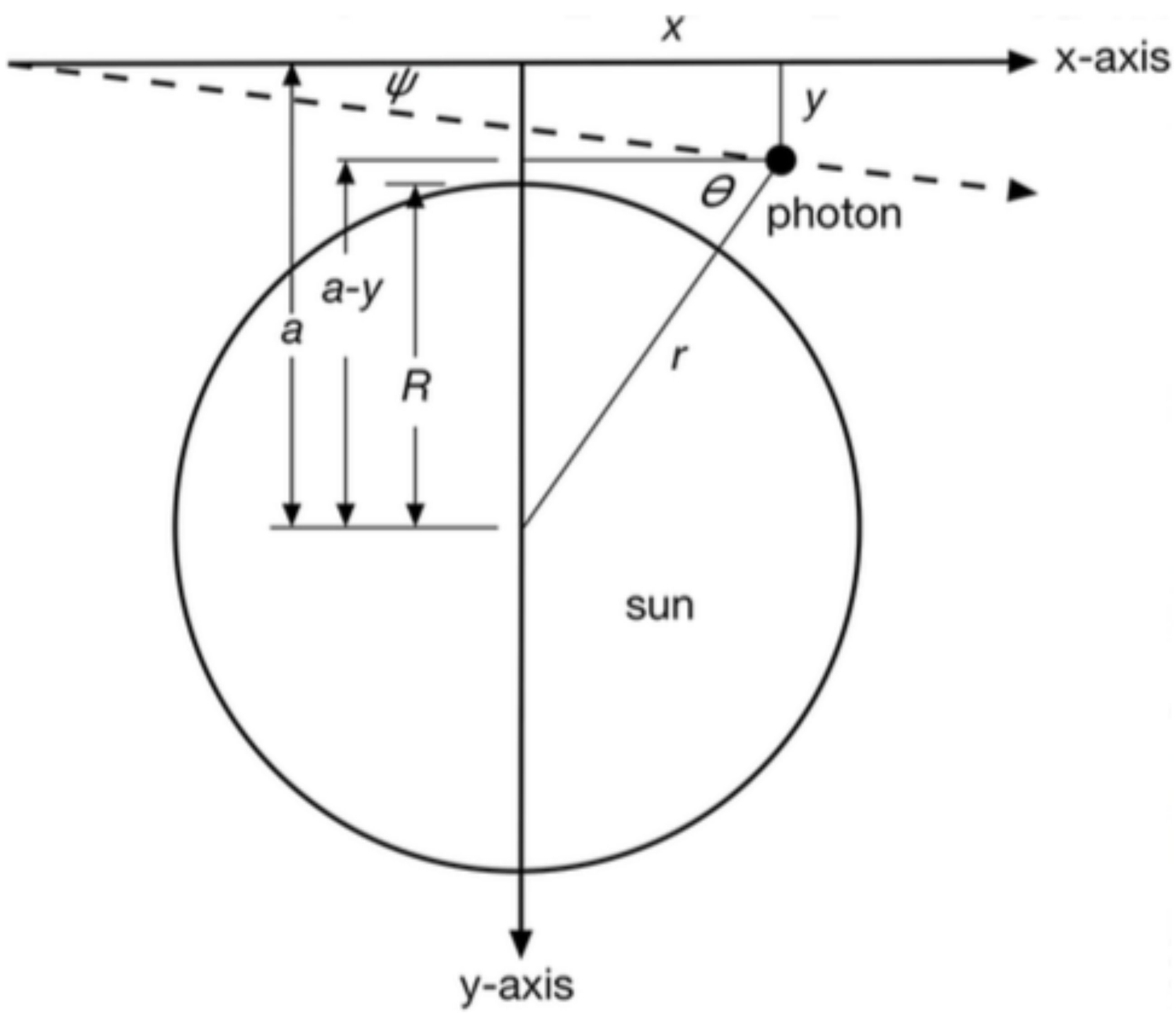

$$x = ct$$

$$dx = cdt$$

$$\frac{dp_i}{dt} = \frac{dp_i}{dx}\frac{dx}{dt} = c\frac{dp_i}{dx}$$

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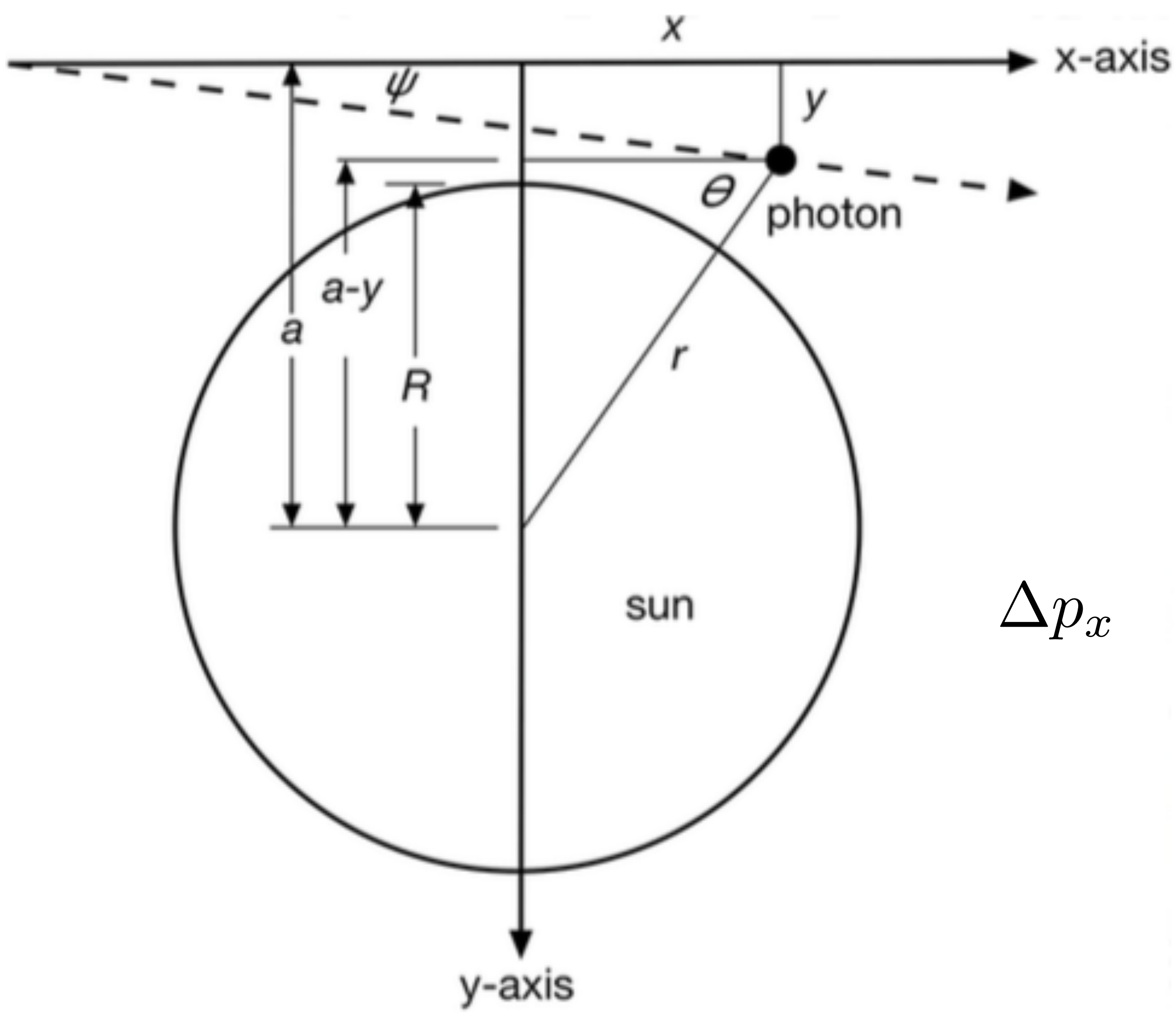

$$\frac{dp_x}{dx} = \frac{GMp}{c^2} \frac{x}{(x^2 + (a-y)^2)^{3/2}}$$

$$\begin{aligned} \Delta p_x &= \frac{GMp}{c^2} \int_{-\infty}^{+\infty} \frac{x}{(x + (a-y)^2)^{3/2}} dx \\ &= \frac{GMp}{c^2} \left[\log[(a-y)^2 + x^2] \right]_{-\infty}^{+\infty} \\ &= 0 \end{aligned}$$

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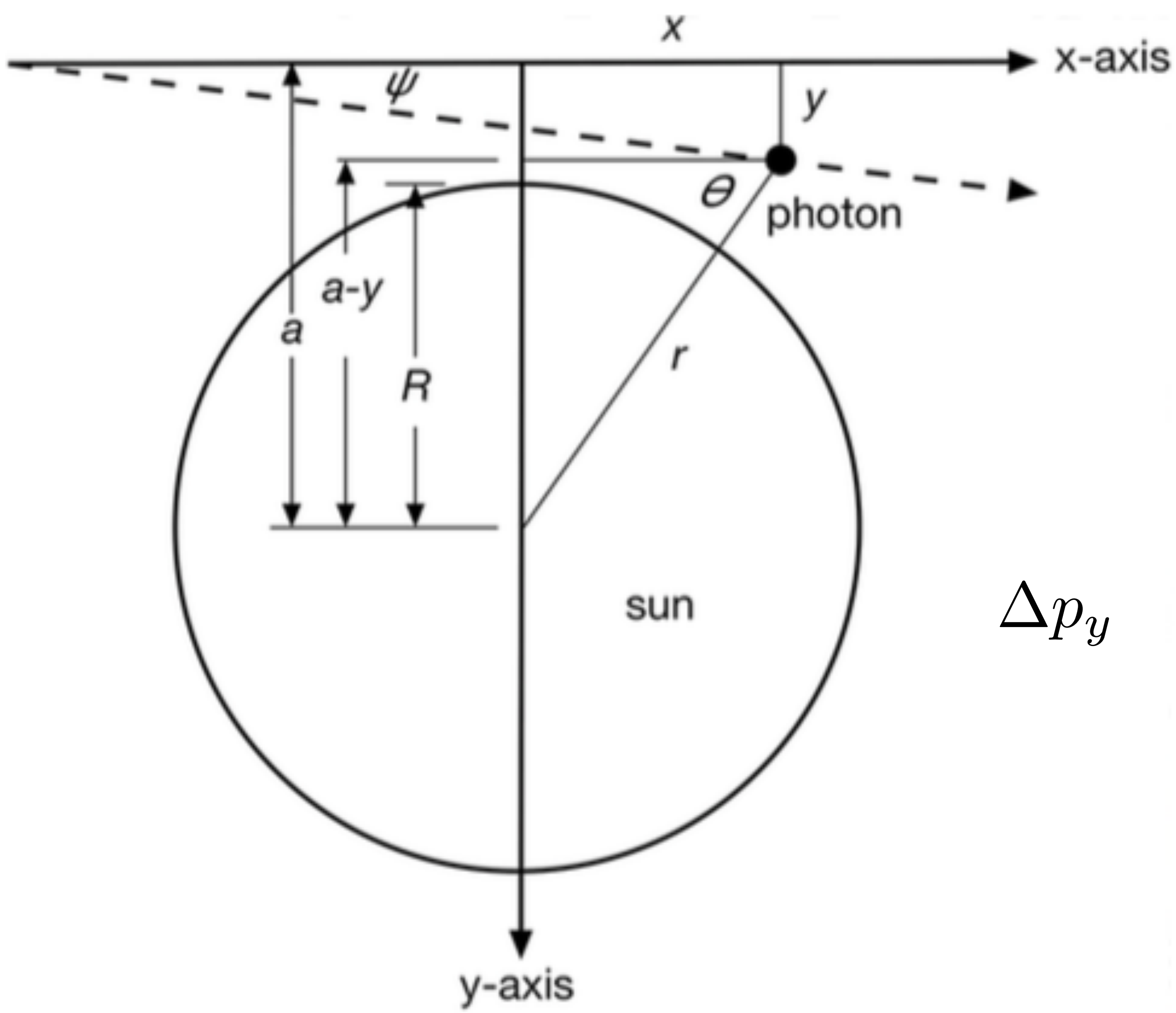

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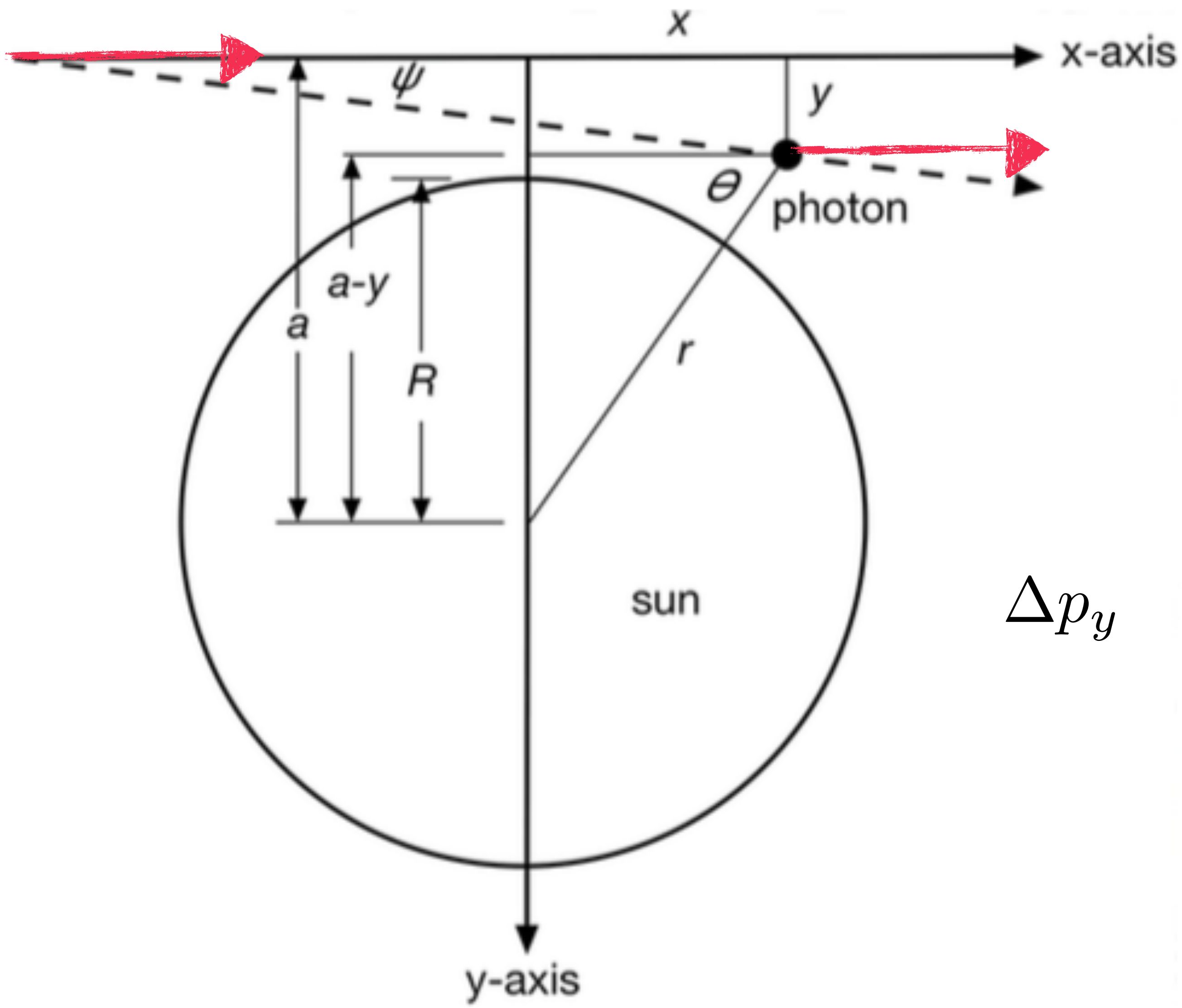

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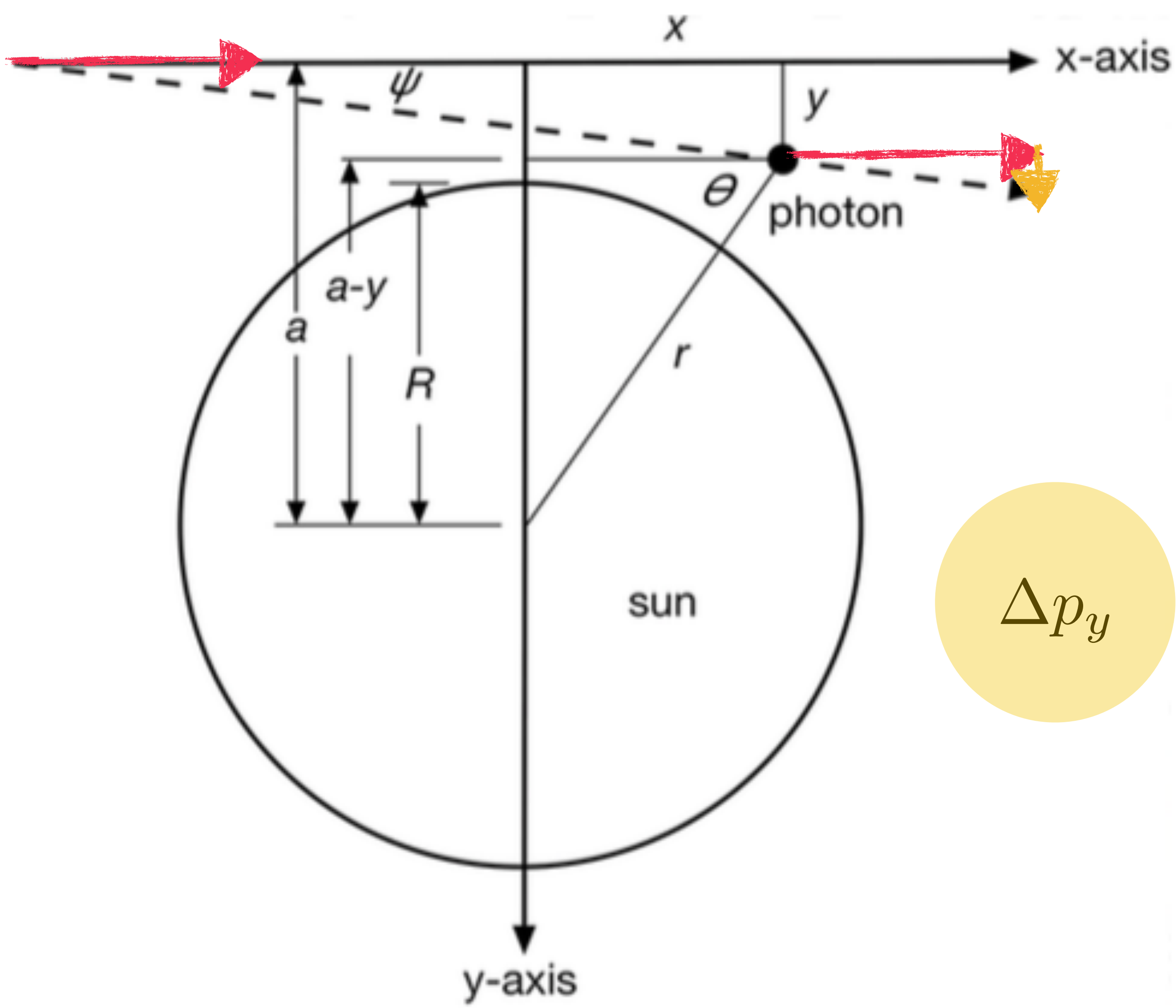

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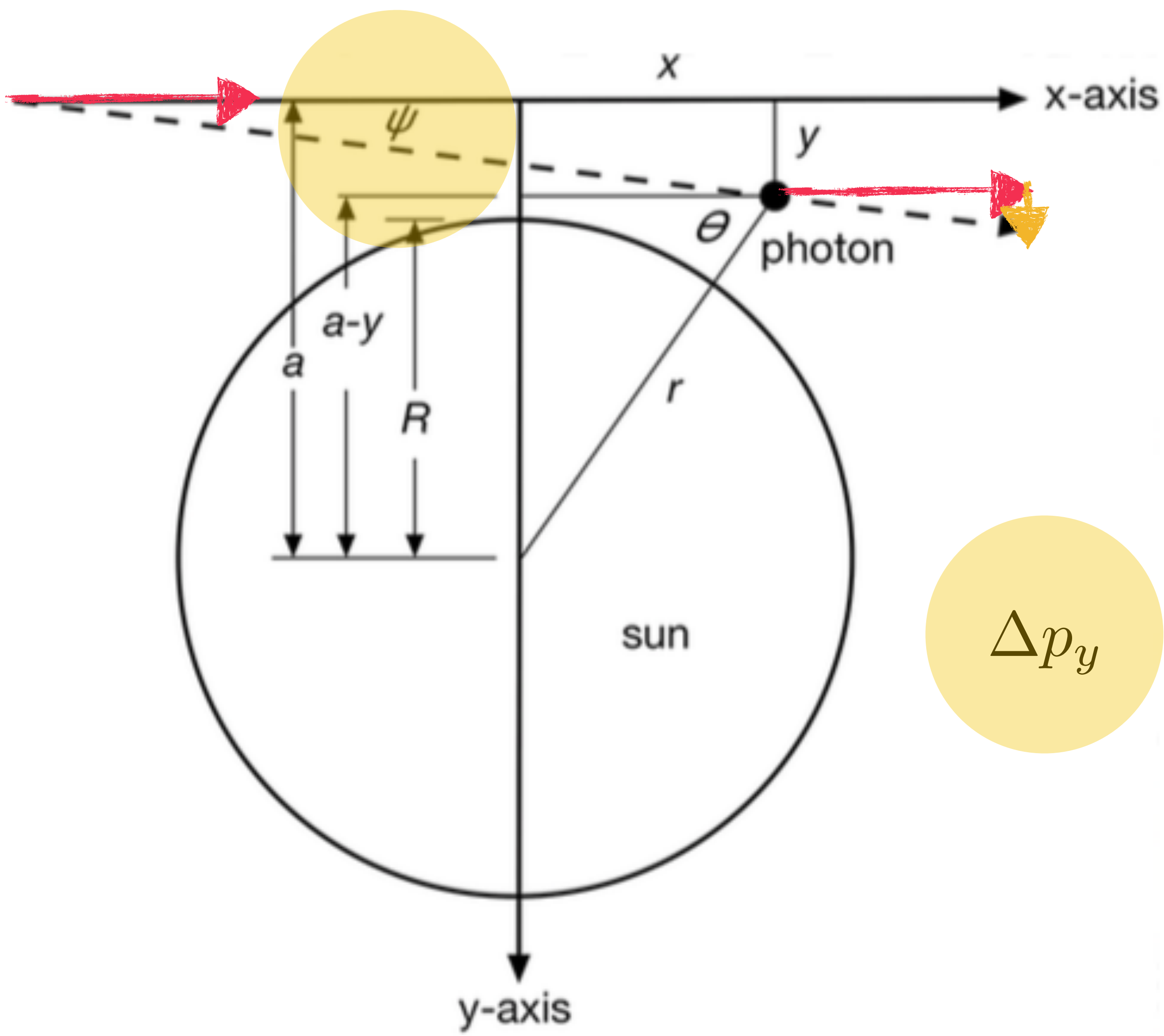

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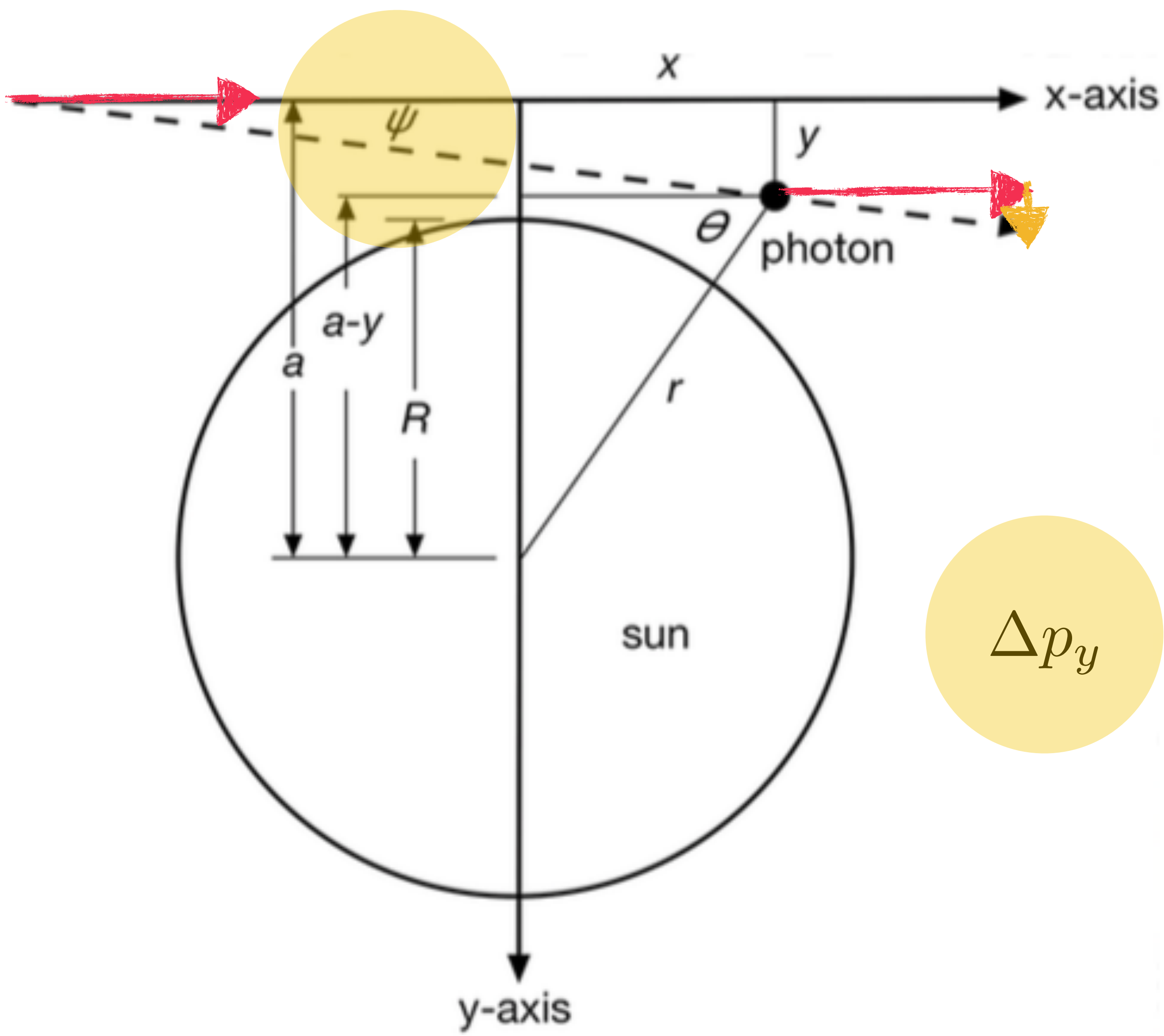

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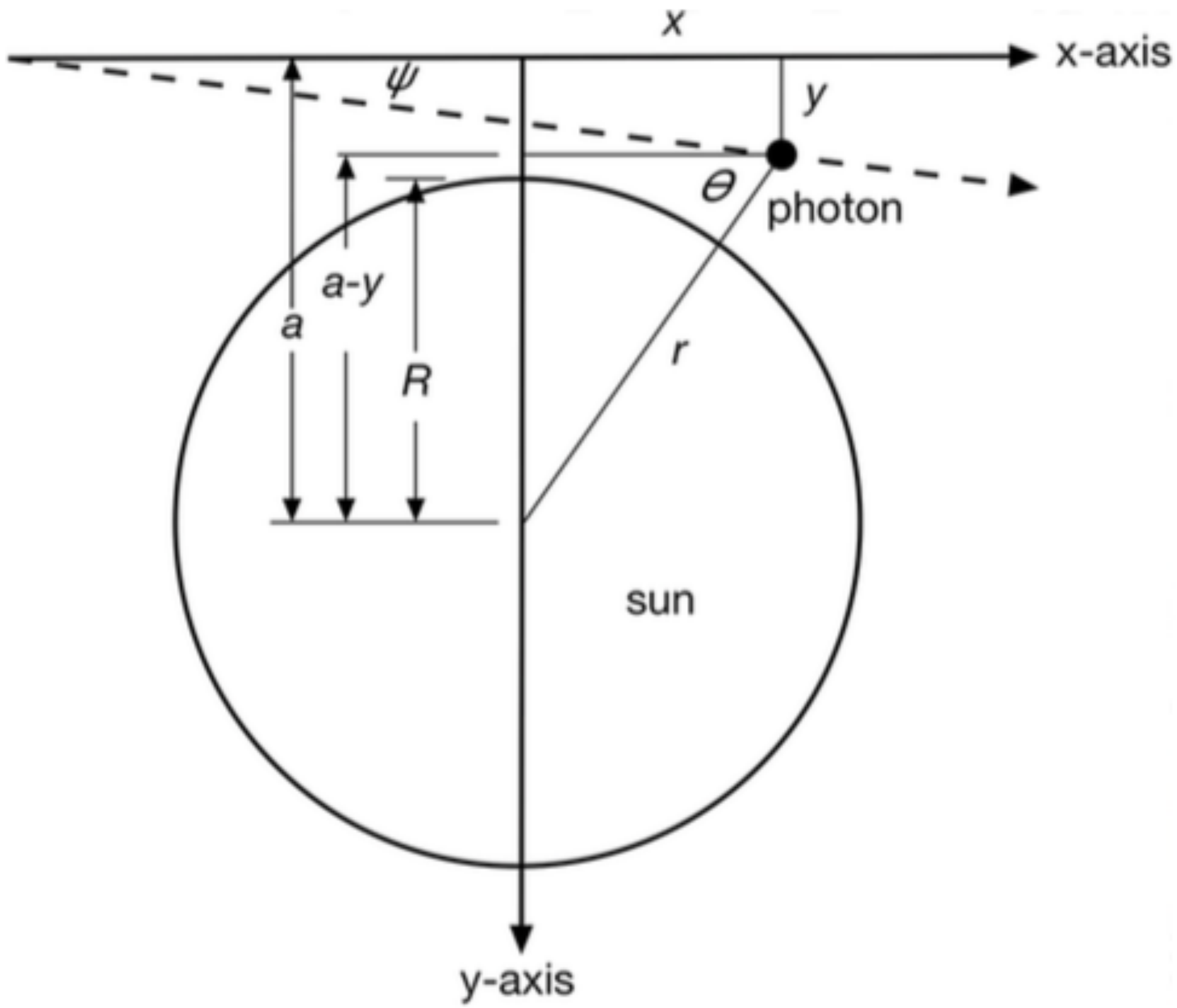
$$\psi = \frac{\Delta p_y}{p} = \frac{2GM}{c^2} \frac{1}{a-y}$$

DEFLECTION OF A LIGHT CORPUSCLE BY THE SUN



$$a - y = R_{\odot}$$

$$M = M_{\odot} = 1.989 \times 10^{30} kg$$

$$a - y = R_{\odot} = 6.96 \times 10^{8} m$$

$$\psi \approx 0.875''$$

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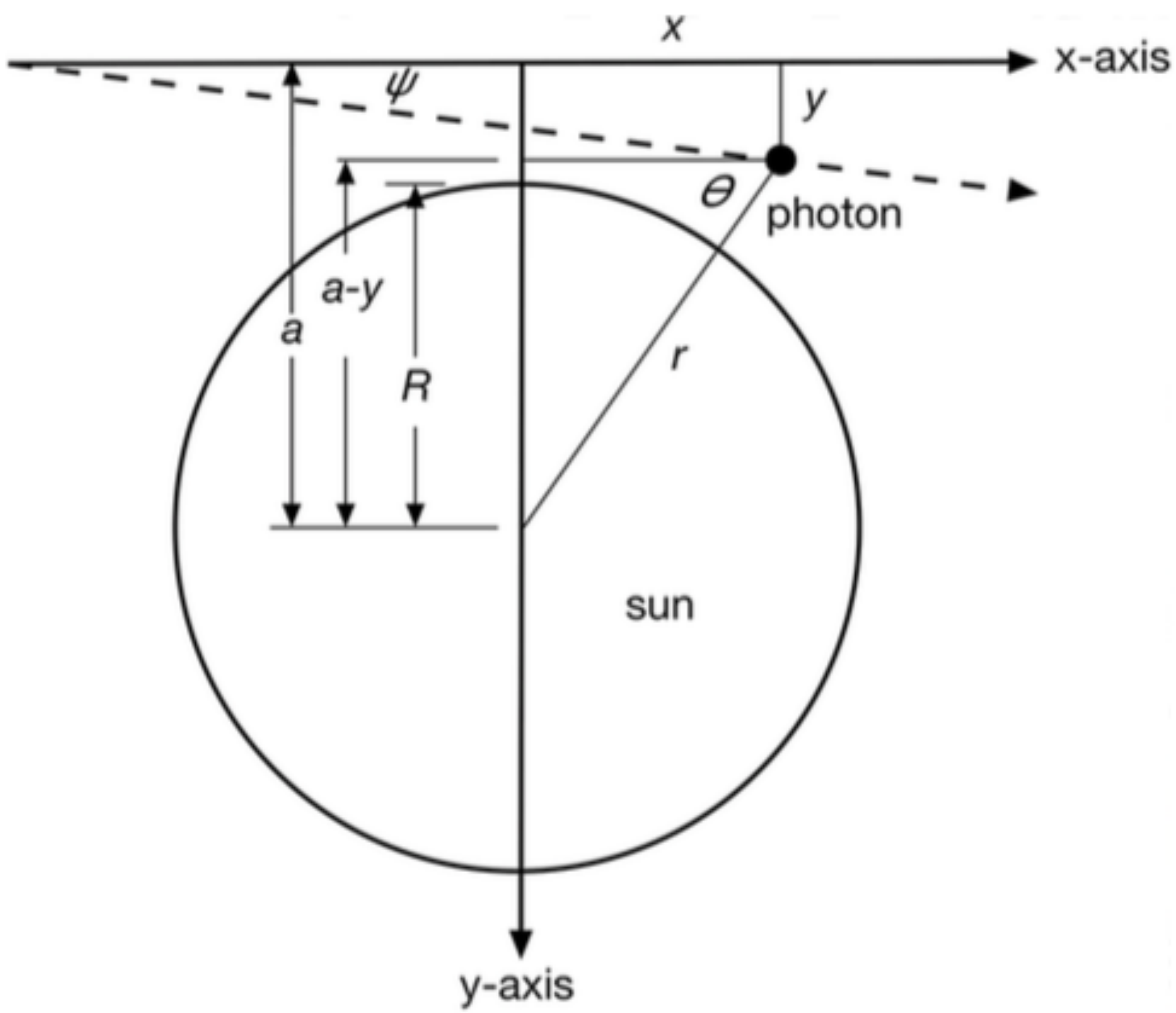

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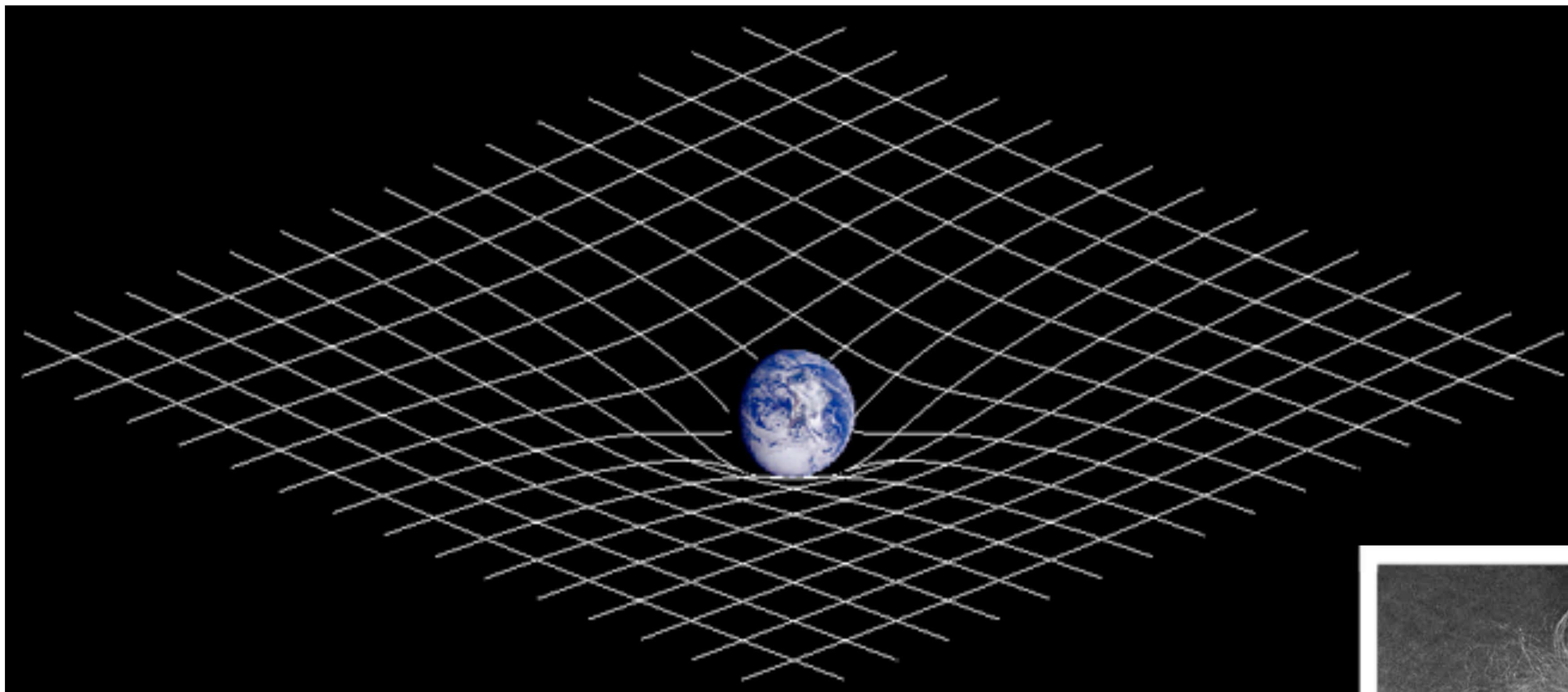
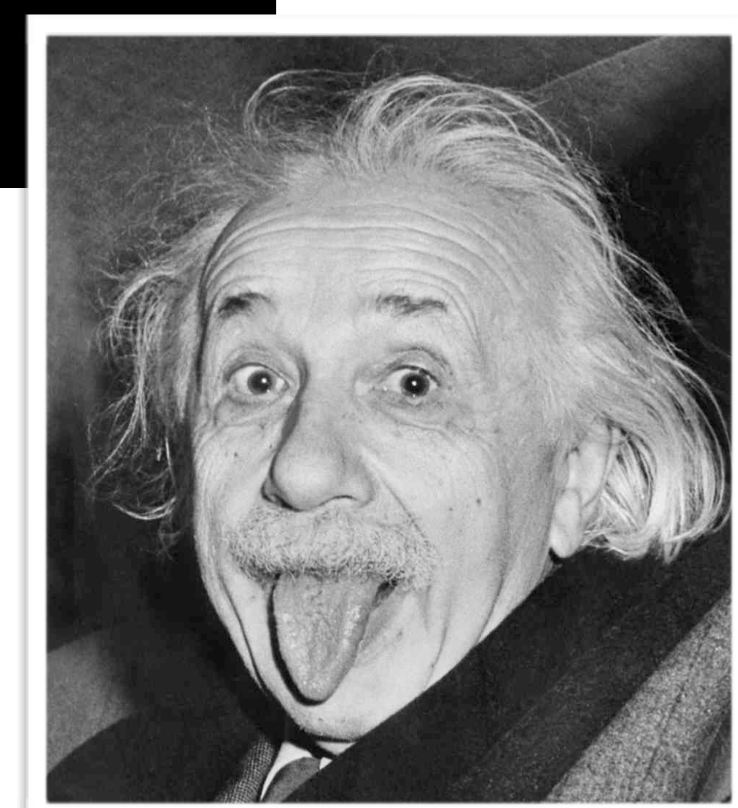
DEFLECTION OF LIGHT IN GENERAL RELATIVITY





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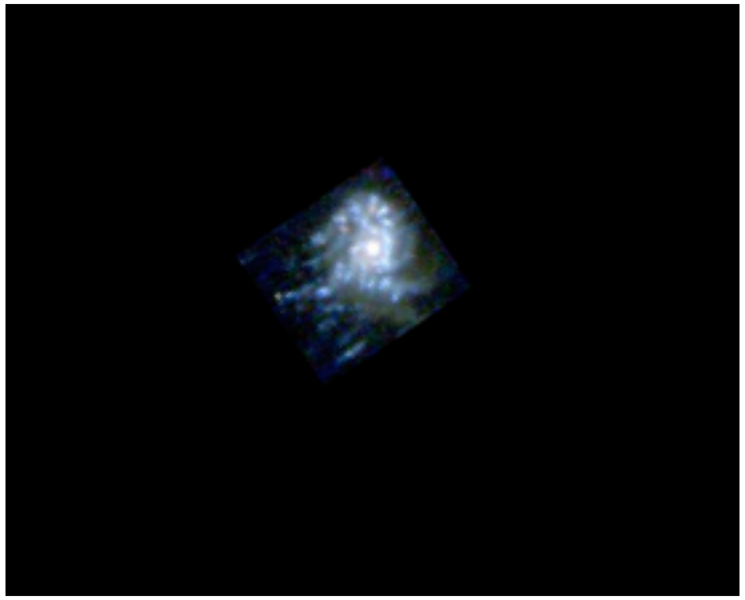
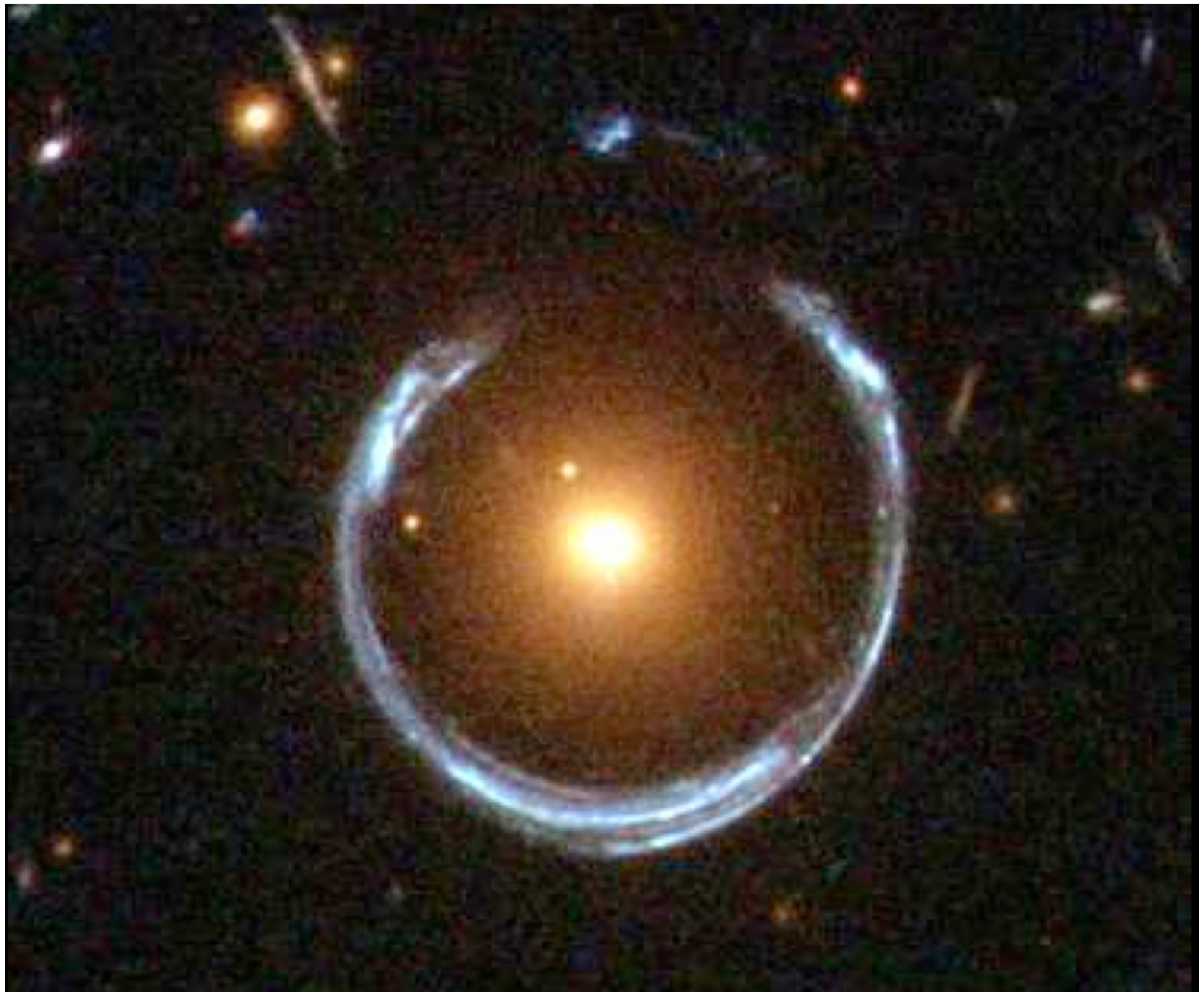
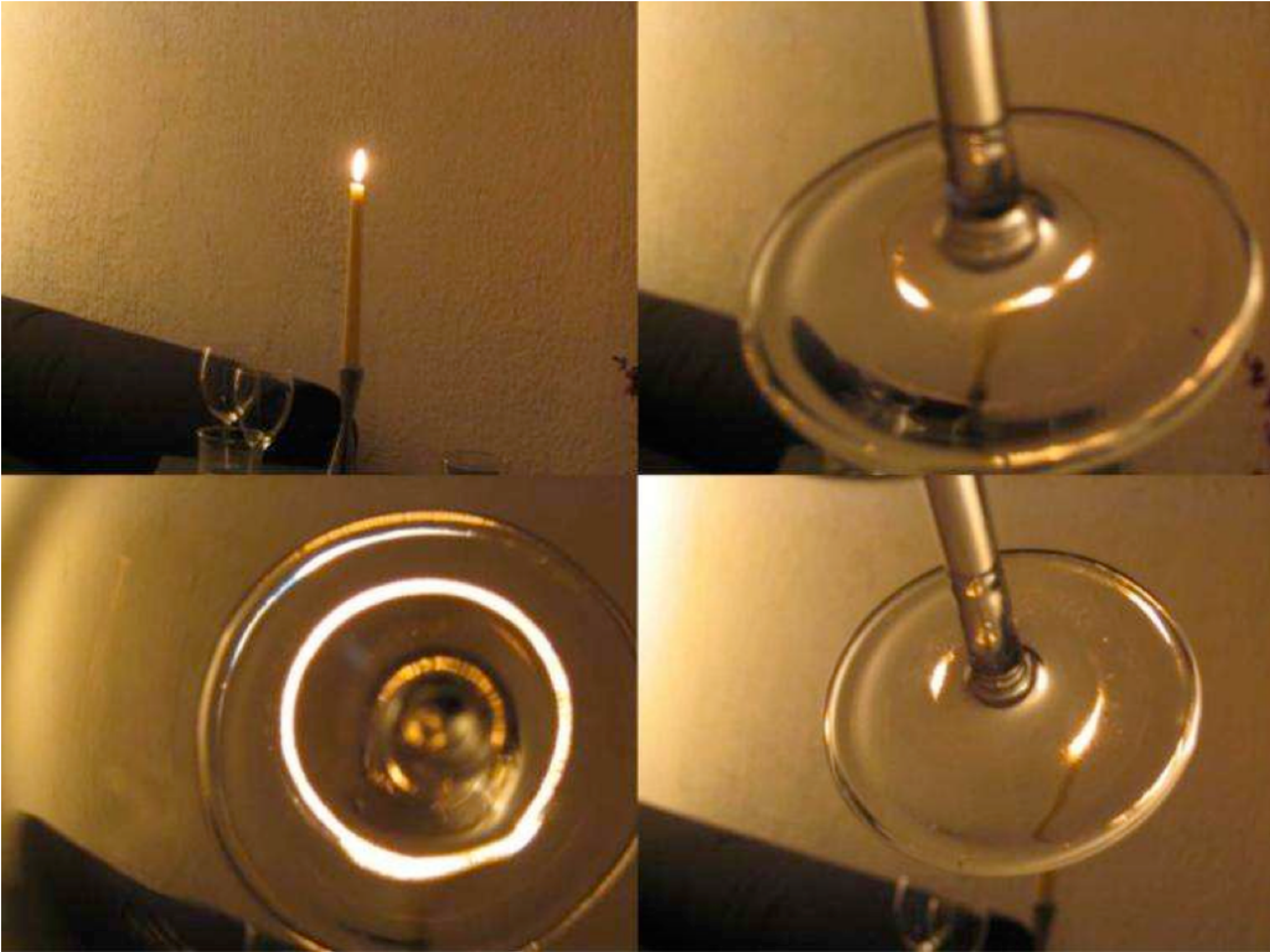


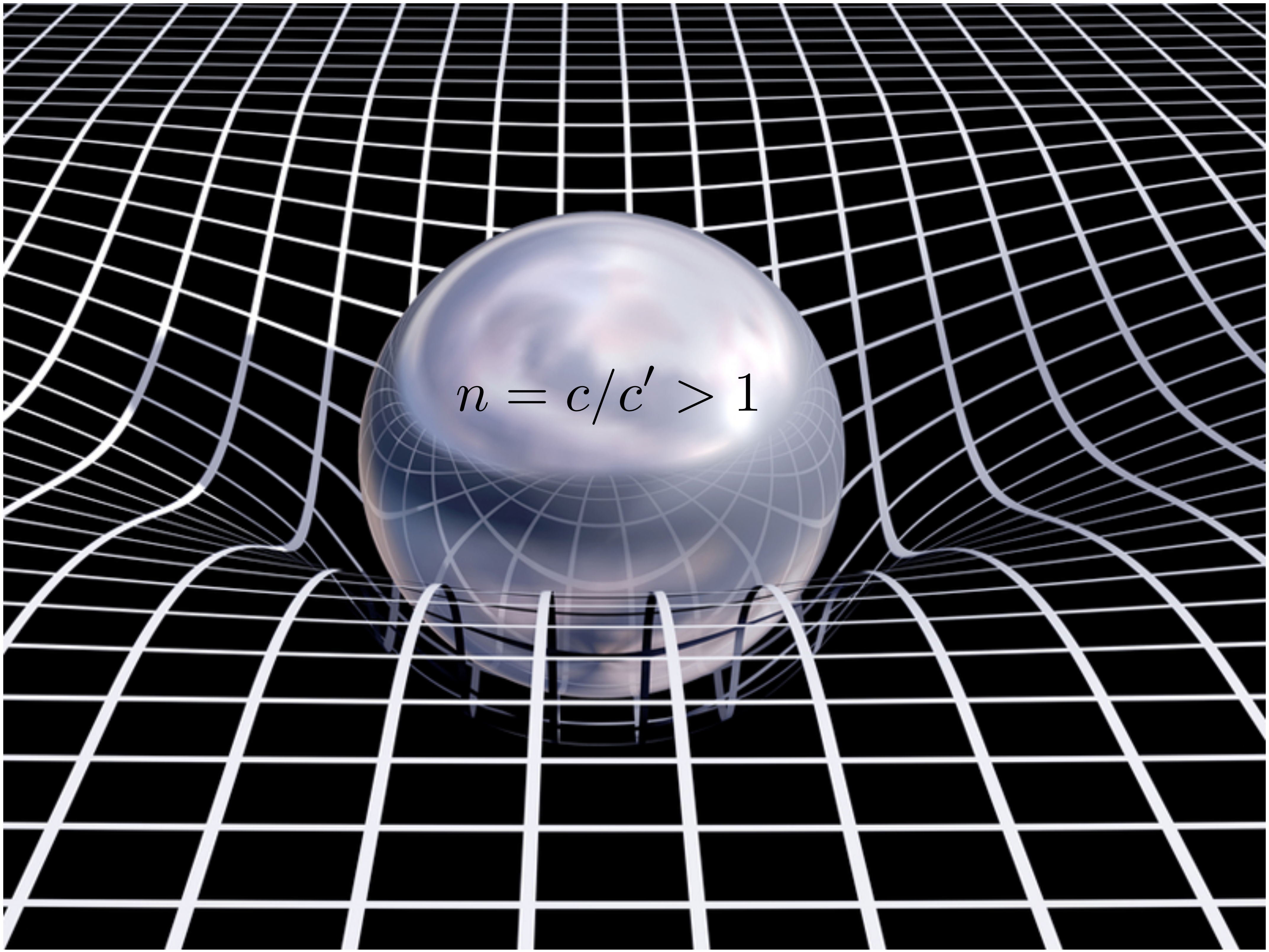


DEFLECTION OF LIGHT IN GENERAL RELATIVITY

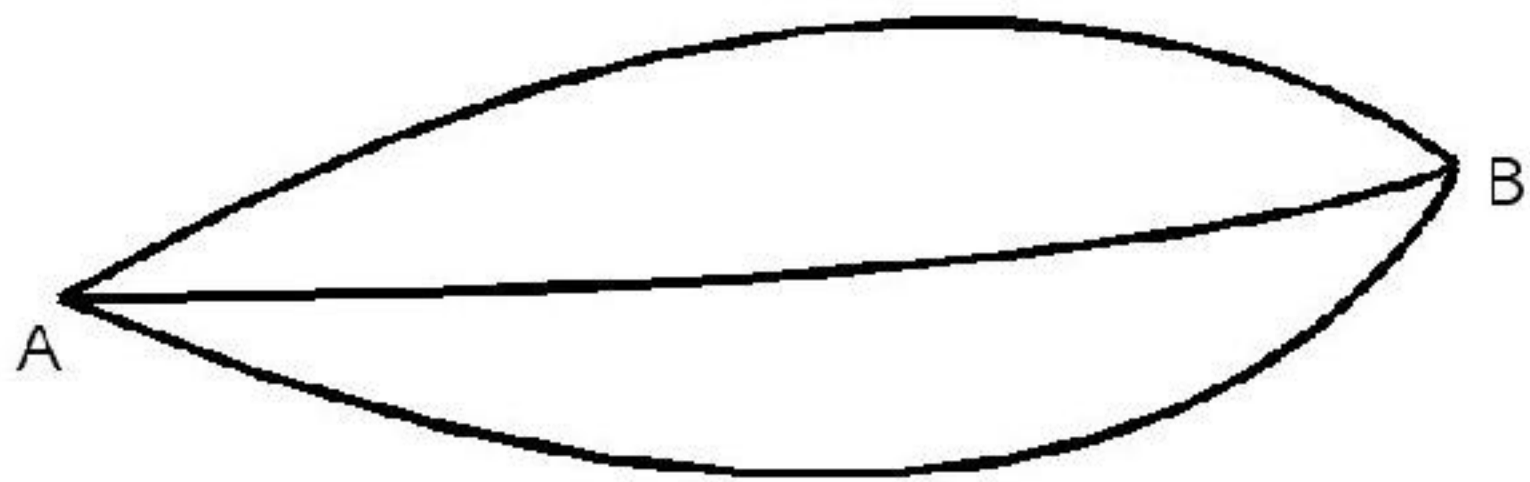
- We will now repeat the calculation of the deflection angle in the context of a locally curved space-time
- Assumptions:
  - the deflection occurs in small region of the universe and over time-scales where the expansion of the universe is not relevant
  - the weak-field limit can be safely applied: $|\Phi|/c^2 \ll 1$
  - perturbed region can be described in terms of an effective diffraction index
  - Fermat principle

DEFLECTION OF LIGHT IN GENERAL RELATIVITY



DEFLECTION OF LIGHT IN GENERAL RELATIVITY

Travel time $= \int \frac{n}{c} \mathrm{d}l$



Fermat principle: $\delta \int_A^B n dl = 0$

DEFLECTION OF LIGHT IN GENERAL RELATIVITY

How to define the effective diffraction index?

void = unperturbed space-time

described by the Minkowski metric

$$\eta_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\mathrm{d}s^2 = \eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = (\mathrm{d}x^0)^2 - (\mathrm{d}\vec{x})^2 = c^2\mathrm{d}t^2 - (\mathrm{d}\vec{x})^2$$

effective diffraction index >1 = perturbed space-time, described by the perturbed metric

$$\eta_{\mu\nu} \to g_{\mu\nu} = \begin{pmatrix} 1+\frac{2\Phi}{c^2} & 0 & 0 & 0 \\ 0 & -(1-\frac{2\Phi}{c^2}) & 0 & 0 \\ 0 & 0 & -(1-\frac{2\Phi}{c^2}) & \\ 0 & 0 & 0 & -(1-\frac{2\Phi}{c^2}) \end{pmatrix}$$

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = \left(1+\frac{2\Phi}{c^2}\right)c^2\mathrm{d}t^2 - \left(1-\frac{2\Phi}{c^2}\right)(\mathrm{d}\vec{x})^2$$

DEFLECTION OF LIGHT IN GENERAL RELATIVITY

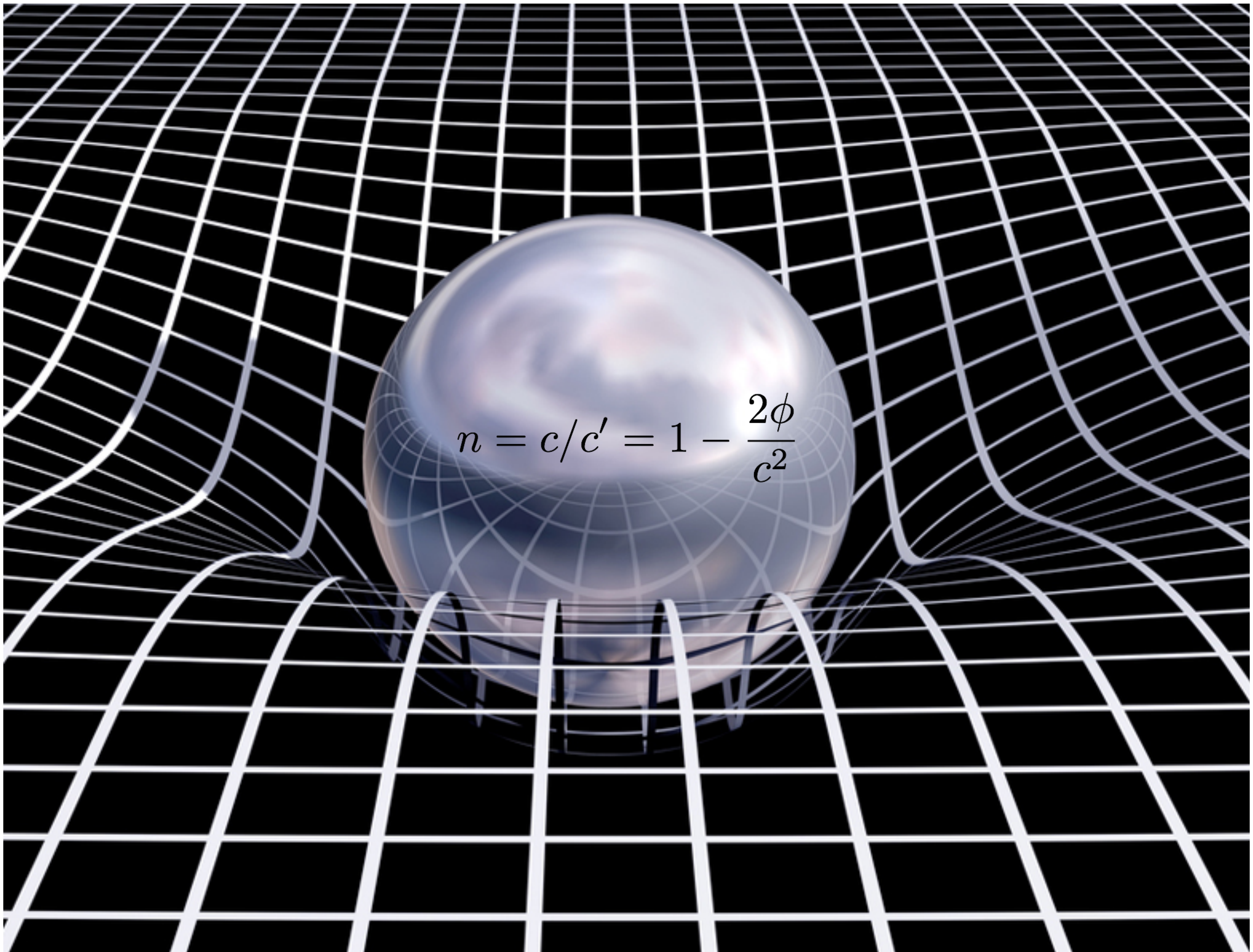
How to define the effective diffraction index?

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = \left(1 + \frac{2\Phi}{c^2}\right) c^2 \mathrm{d}t^2 - \left(1 - \frac{2\Phi}{c^2}\right) (\mathrm{d}\vec{x})^2$$

$$\left(1 + \frac{2\Phi}{c^2}\right) c^2 \mathrm{d}t^2 = \left(1 - \frac{2\Phi}{c^2}\right) (\mathrm{d}\vec{x})^2$$

$$c' = \frac{|\mathrm{d}\vec{x}|}{\mathrm{d}t} = c\sqrt{\frac{1 + \frac{2\Phi}{c^2}}{1 - \frac{2\Phi}{c^2}}} \approx c\left(1 + \frac{2\Phi}{c^2}\right)$$

DEFLECTION OF LIGHT IN GENERAL RELATIVITY



DEFLECTION OF LIGHT IN GENERAL RELATIVITY

Let's use the Fermat principle

$$\delta \int_A^B n[\vec{x}(l)]\mathrm{d}l = 0$$

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λ → *generalized coordinate*

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generalized velocity $\dot{\vec{x}} \equiv \frac{\mathrm{d}\vec{x}}{\mathrm{d}\lambda}$

generalized coordinate

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$$\delta \int_A^B n[\vec{x}(l)]\mathrm{d}l = 0$$

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$$\delta \int_{\lambda_A}^{\lambda_B} \mathrm{d}\lambda\, n[\vec{x}(\lambda)] \left|\frac{\mathrm{d}\vec{x}}{\mathrm{d}\lambda}\right| = 0$$

generalized velocity $\dot{\vec{x}} \equiv \frac{\mathrm{d}\vec{x}}{\mathrm{d}\lambda}$

generalized coordinate

$$\boxed{n[\vec{x}(\lambda)] \left|\frac{\mathrm{d}\vec{x}}{\mathrm{d}\lambda}\right| \equiv L(\dot{\vec{x}}, \vec{x}, \lambda)}$$

Langrangian!

DEFLECTION OF LIGHT IN GENERAL RELATIVITY

Let's use the Fermat principle

$$\delta \int_{\lambda_A}^{\lambda_B} \mathrm{d}\lambda \, n[\vec{x}(\lambda)] \left| \frac{\mathrm{d}\vec{x}}{\mathrm{d}\lambda} \right| = 0$$

Euler-Langrange equation: $$\frac{\mathrm{d}}{\mathrm{d}\lambda} \frac{\partial L}{\partial \dot{\vec{x}}} - \frac{\partial L}{\partial \vec{x}} = 0$$

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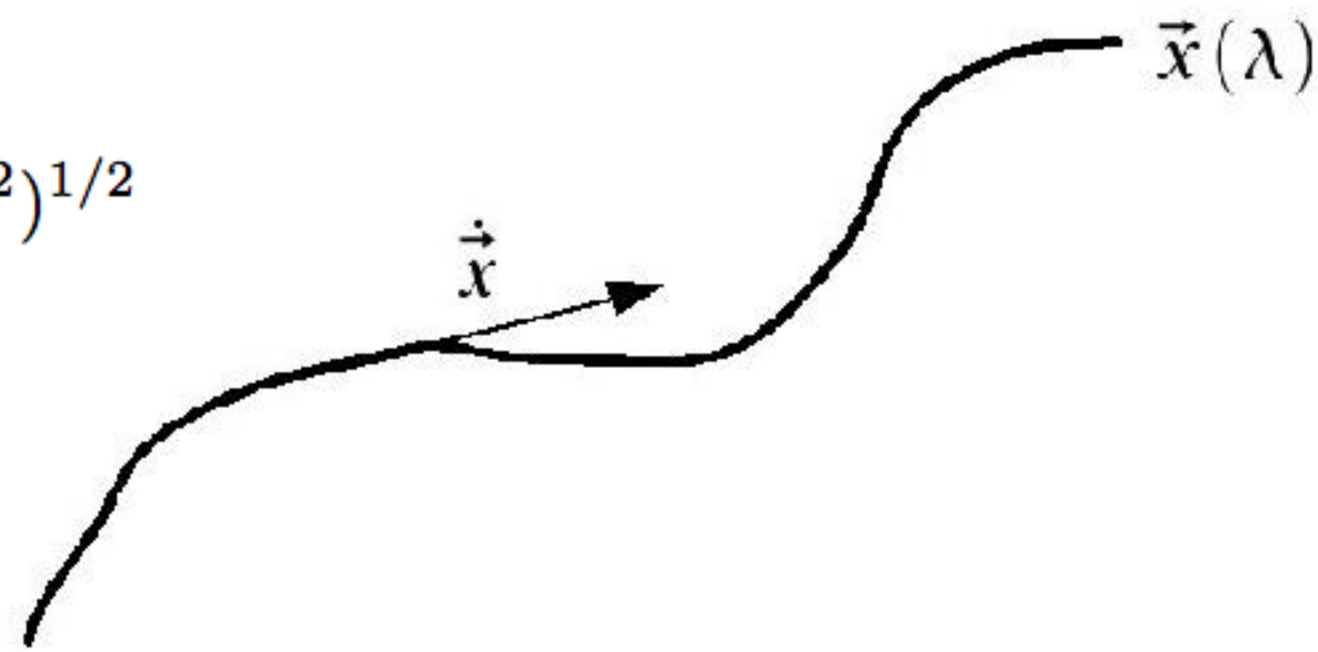
$$|\dot{\vec{x}}| = 1$$

$$\vec{e} \equiv \dot{\vec{x}}$$



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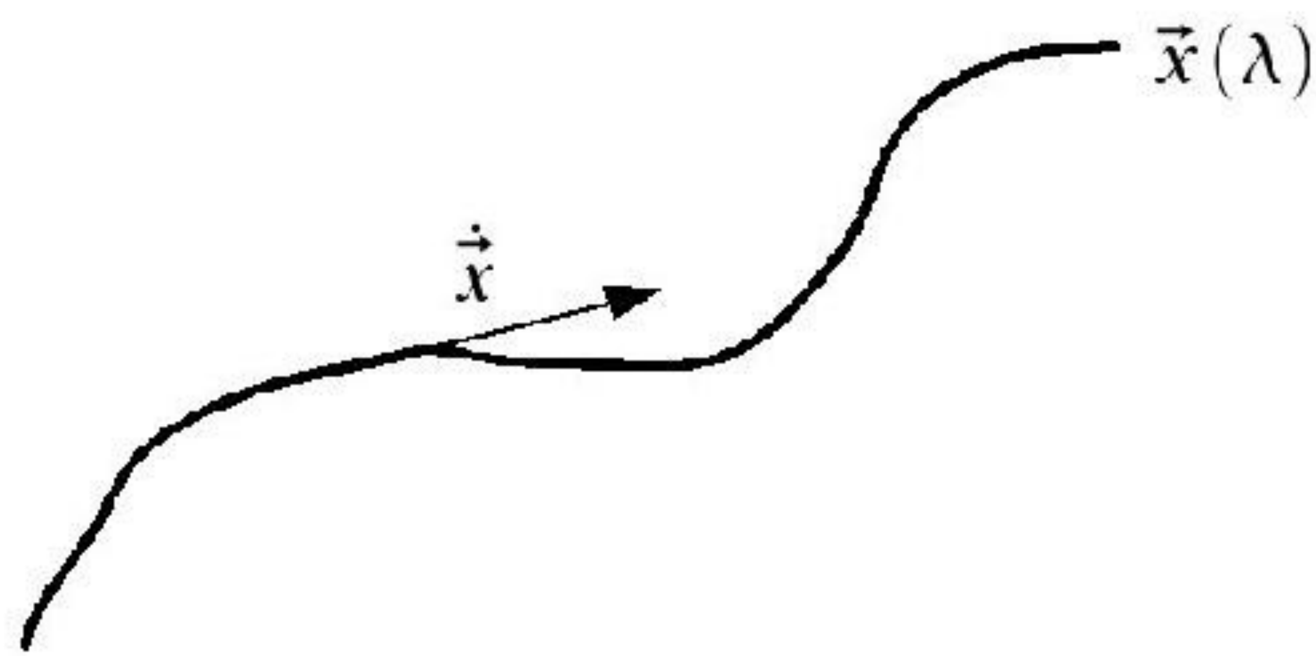

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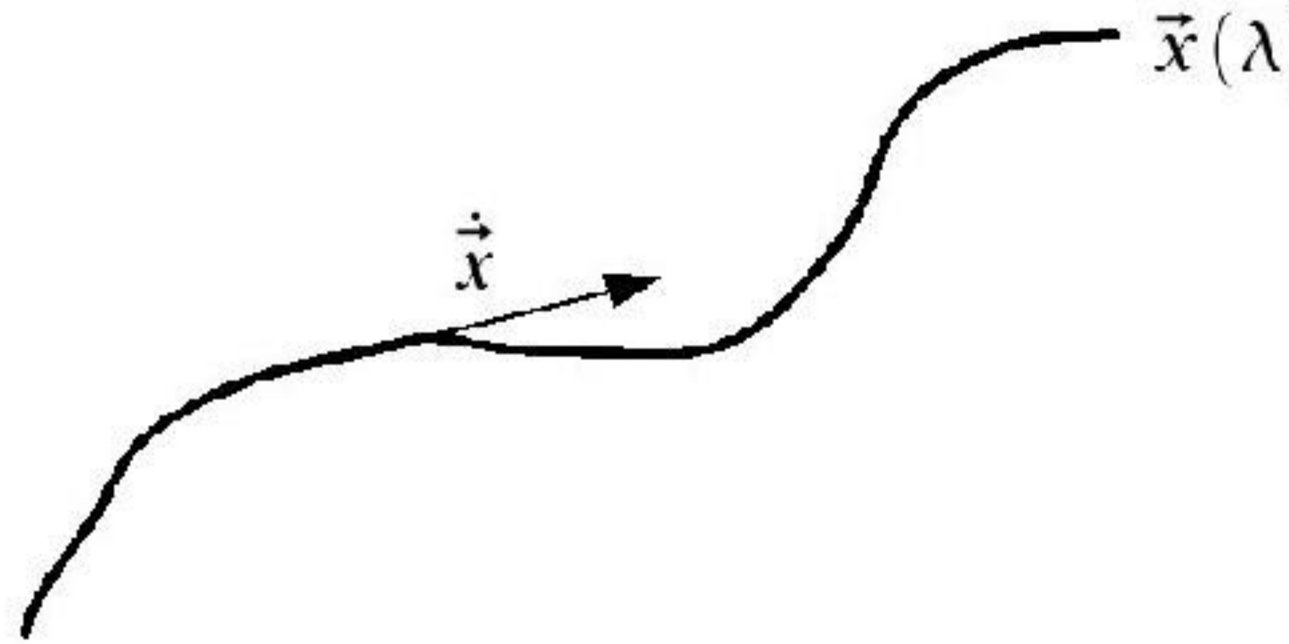

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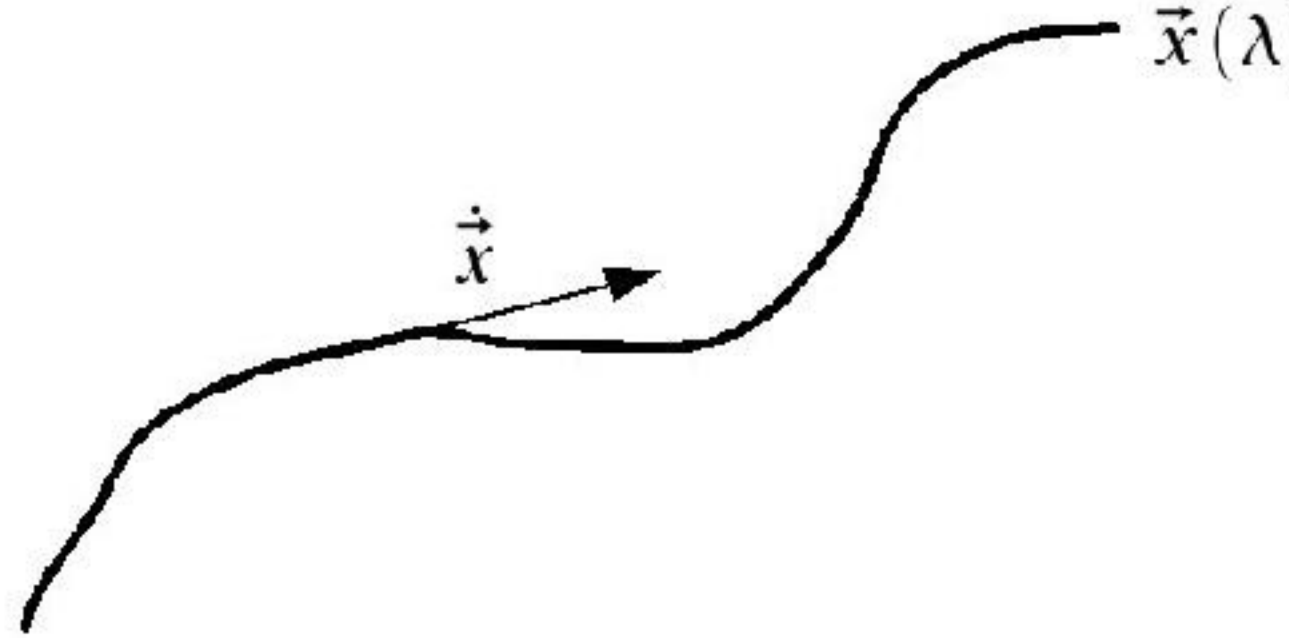
$$n\dot{\vec{e}} + \vec{e} \cdot [(\vec{\nabla} n)\dot{\vec{x}}] = \vec{\nabla} n \ ,$$

$$\Rightarrow n\dot{\vec{e}} = \vec{\nabla} n - \vec{e}(\vec{\nabla} n \cdot \vec{e})$$



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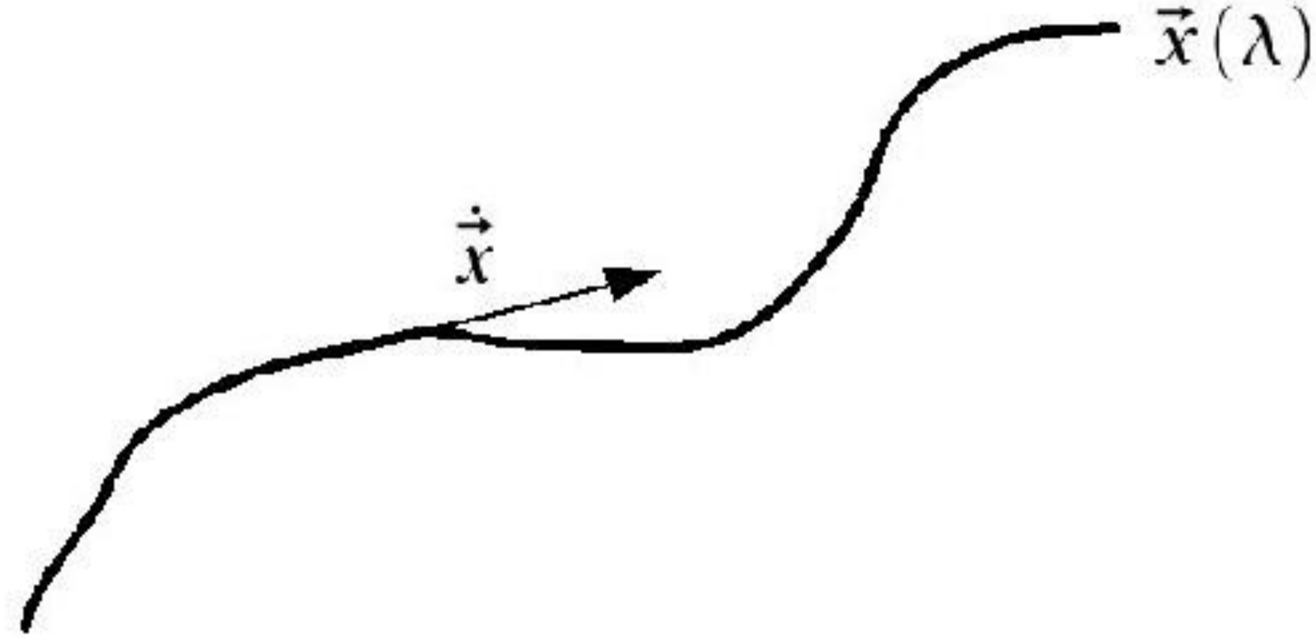

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$$\dot{\vec{e}} = \frac{1}{n} \vec{\nabla}_{\perp} n = \vec{\nabla}_{\perp} \ln n$$

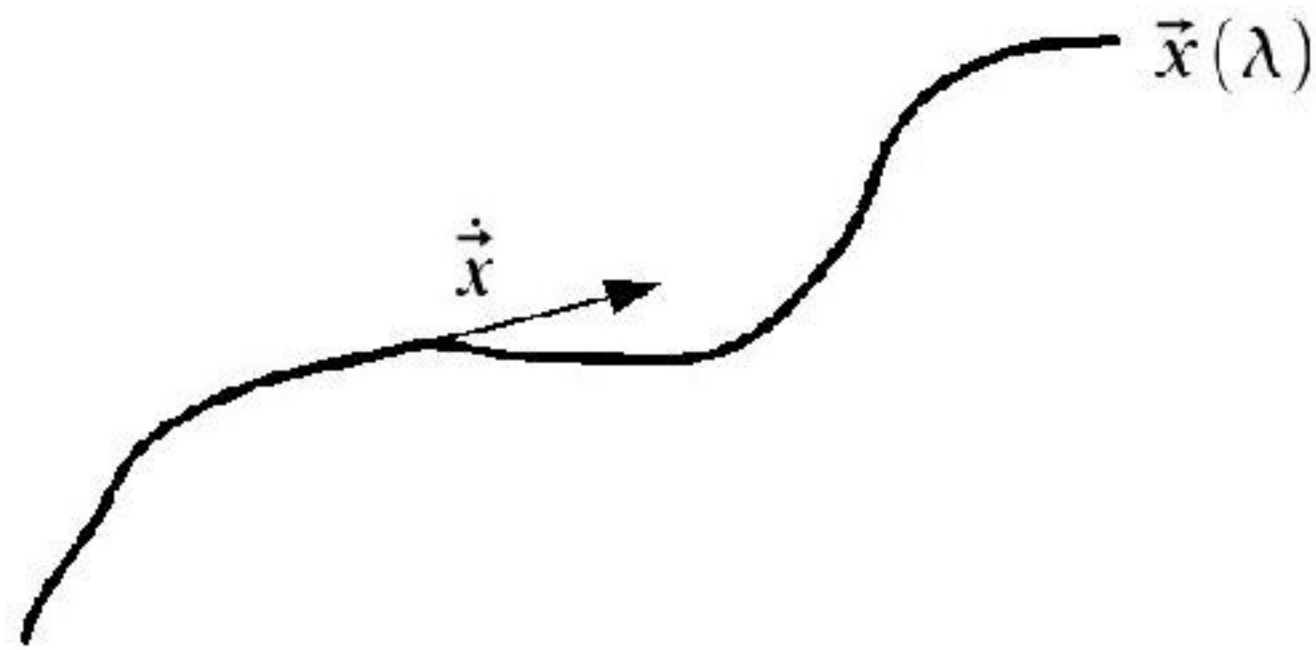
DEFLECTION OF LIGHT IN GENERAL RELATIVITY

$$\dot{\vec{e}} = \frac{1}{n}\vec{\nabla}_\perp n = \vec{\nabla}_\perp \ln n$$

$$n = 1 - \frac{\phi}{c^2} \qquad \frac{\phi}{c^2} \ll 1$$

$$\ln n \approx -2\frac{\phi}{c^2}$$

$$\dot{\vec{e}} \approx -\frac{2}{c^2}\vec{\nabla}_\perp \Phi$$



$$\hat{\vec{\alpha}} = \frac{2}{c^2}\int_{\lambda_A}^{\lambda_B} \vec{\nabla}_\perp \Phi \mathrm{d}\lambda$$

Deflection angle

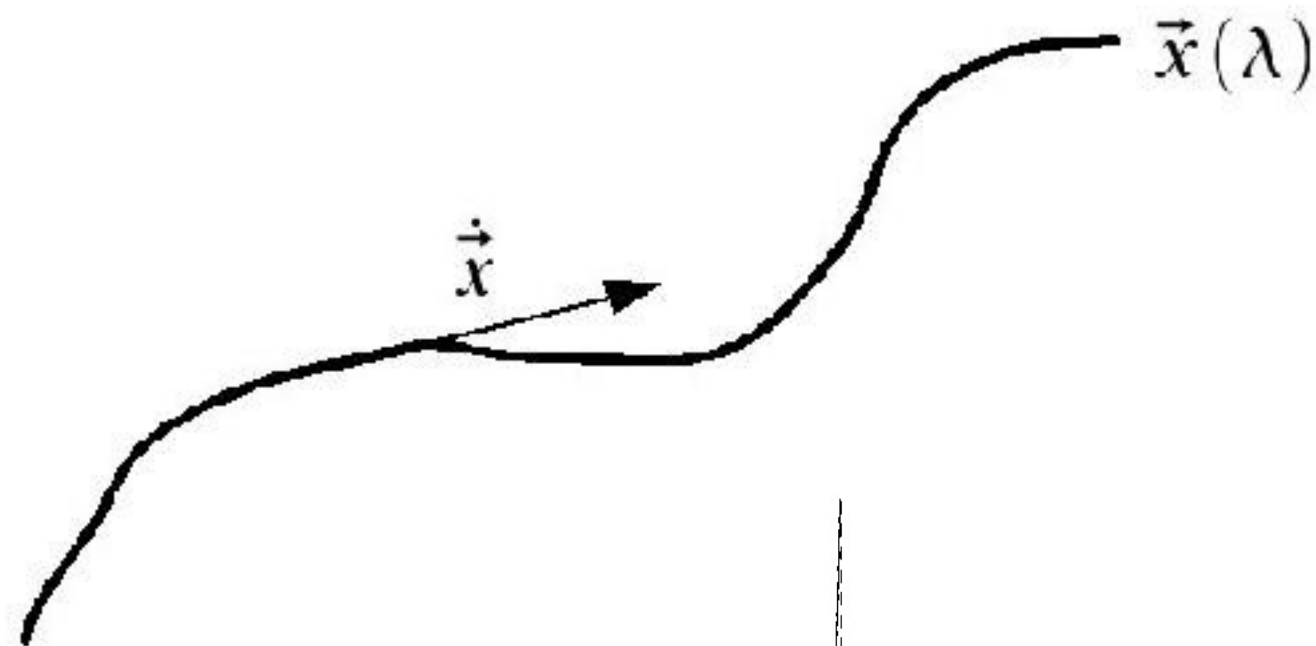
DEFLECTION OF LIGHT IN GENERAL RELATIVITY

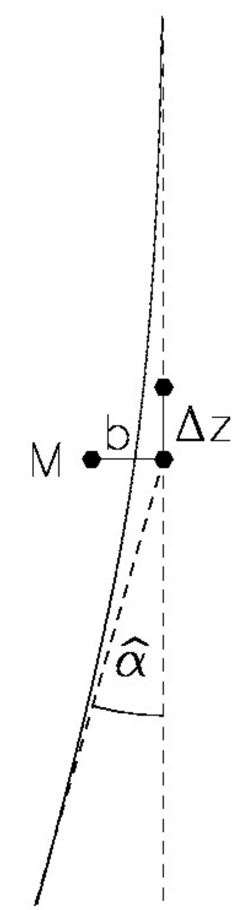
$$\hat{\vec{\alpha}} = \frac{2}{c^2} \int_{\lambda_A}^{\lambda_B} \vec{\nabla}_\perp \Phi \mathrm{d}\lambda$$

As it is written, this equation is not useful, as we would have to integrate over the actual light path.

Let's assume that the deflection is "istantaneous" (Born approximation):

$$\hat{\vec{\alpha}}(b) = \frac{2}{c^2} \int_{-\infty}^{+\infty} \vec{\nabla}_\perp \phi \mathrm{d}z$$





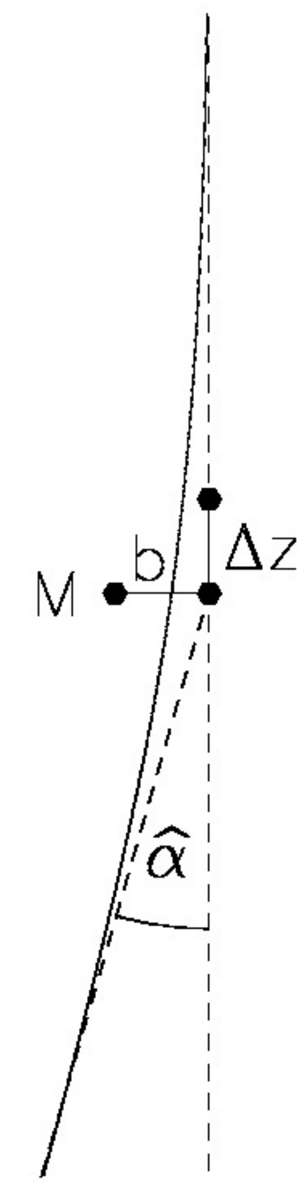
A PARTICULAR CASE: THE POINT MASS

$$\phi = -\frac{GM}{r}$$

$$r = \sqrt{x^2 + y^2 + z^2} = \sqrt{b^2 + z^2}$$

$$\vec{\nabla}_\perp \phi = \begin{pmatrix} \partial_x \phi \\ \partial_y \phi \end{pmatrix} = \frac{GM}{r^3} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{aligned} \hat{\vec{\alpha}}(b) &= \frac{2GM}{c^2} \begin{pmatrix} x \\ y \end{pmatrix} \int_{-\infty}^{+\infty} \frac{dz}{(b^2+z^2)^{3/2}} \\ &= \frac{4GM}{c^2} \begin{pmatrix} x \\ y \end{pmatrix} \left[\frac{z}{b^2(b^2+z^2)^{1/2}} \right]_0^\infty = \frac{4GM}{c^2 b} \begin{pmatrix} \cos\phi \\ \sin\phi \end{pmatrix} \end{aligned}$$



A LIGHT RAY GRAZING THE SURFACE OF THE SUN

General relativity:

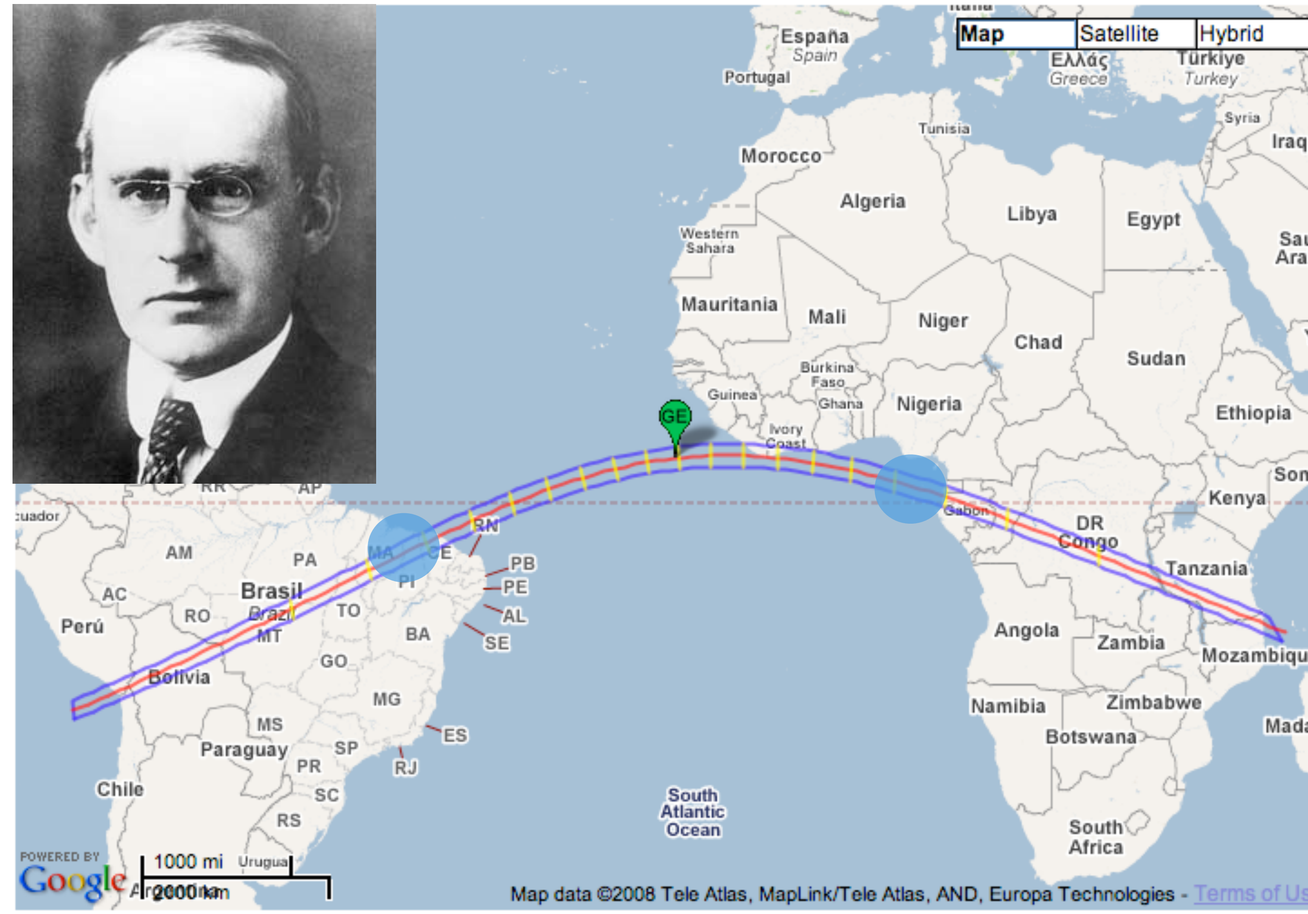
$$\hat{\alpha} = \frac{4GM_\odot}{c^2 R_\odot} = 1.75''$$

Newtonian gravity and corpuscolar light:

$$\hat{\alpha} = \frac{2GM_\odot}{c^2 R_\odot} = 0.875''$$

EDDINGTON EXPEDITIONS

- In 1919 Eddington organized due expeditions to observe a total solar eclipse (Principe Island and Sobral)
- The goal was to measure the lensing effect of the sun on background stars
- Very conveniently, the sun was well aligned with the Iades open cluster
- During the eclipse the expedition from Principe registered a shift in the apparent position of stars with respect to their night-time positions, which resulted to be consistent with the GR predictions
- The Sobral expedition measured a smaller deflection but this was interpreted as the result of a technical problem.



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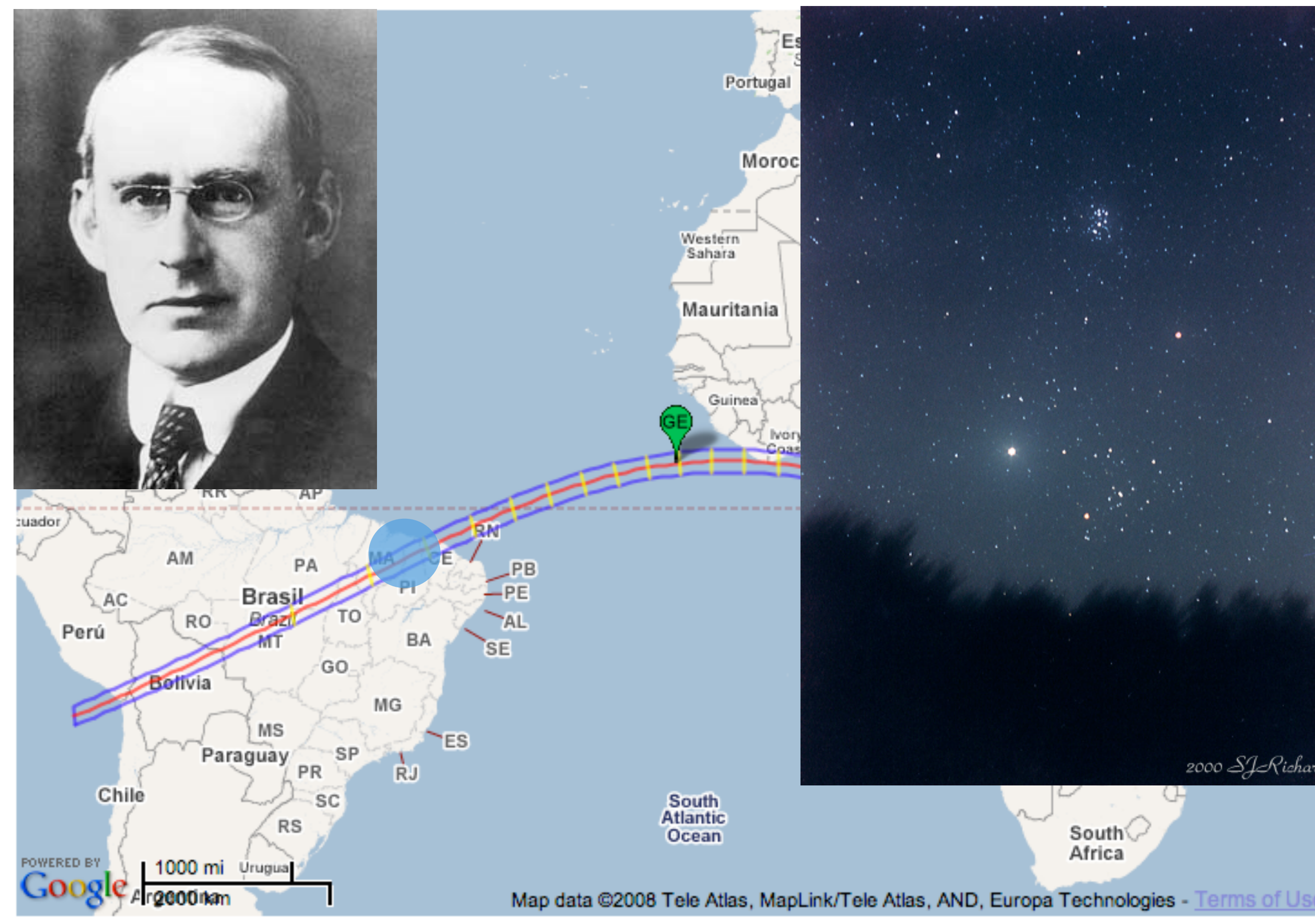

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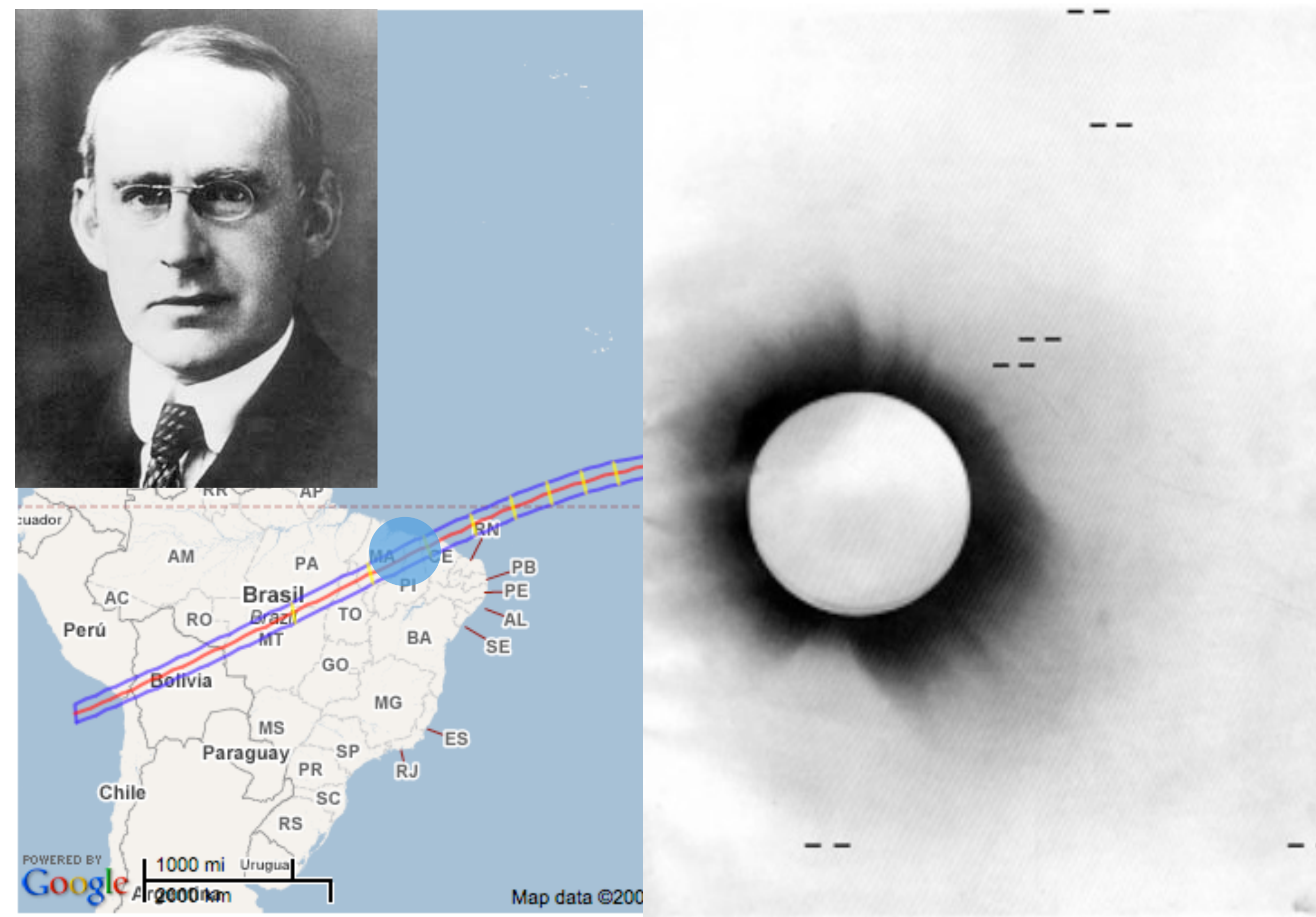

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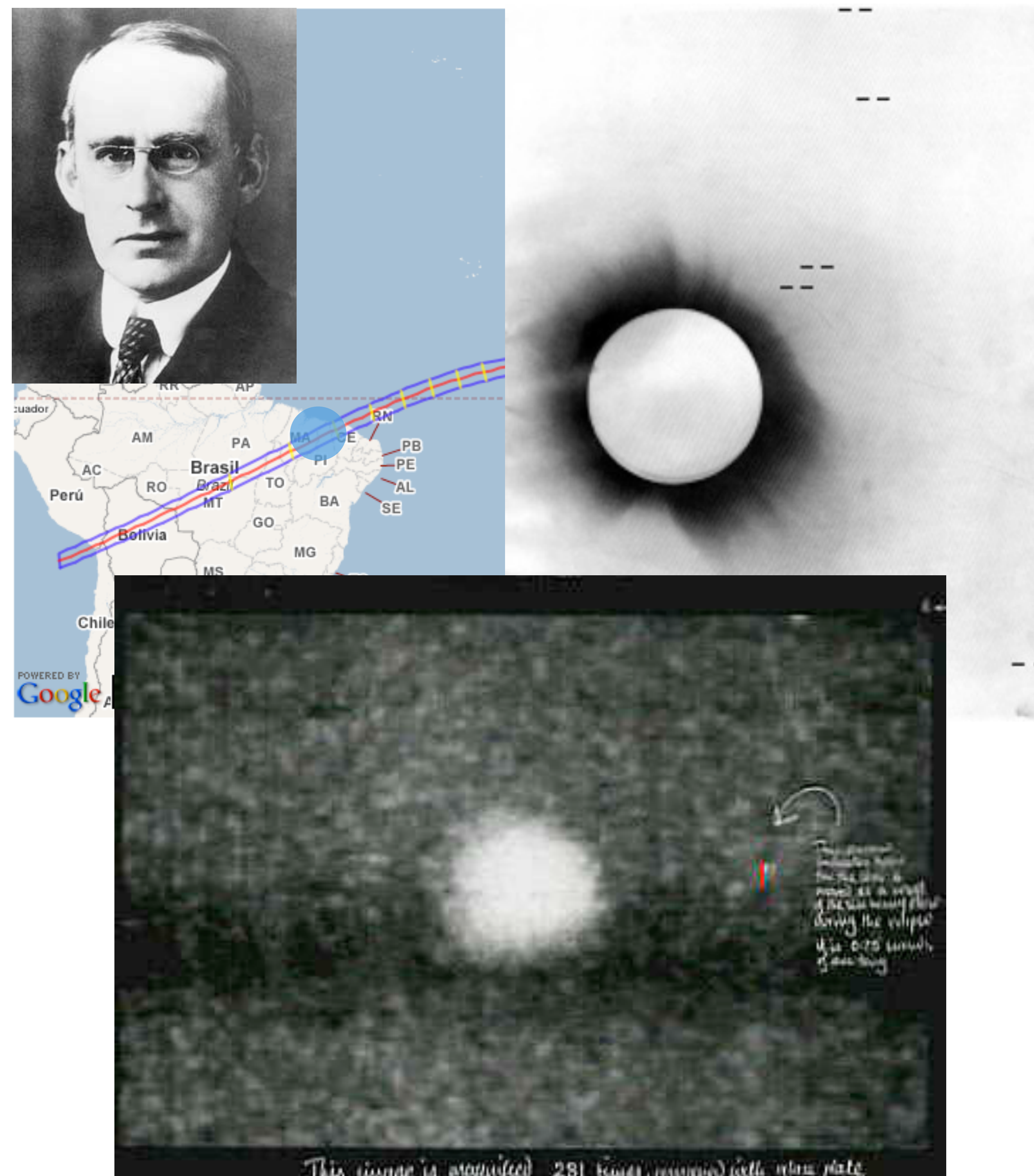

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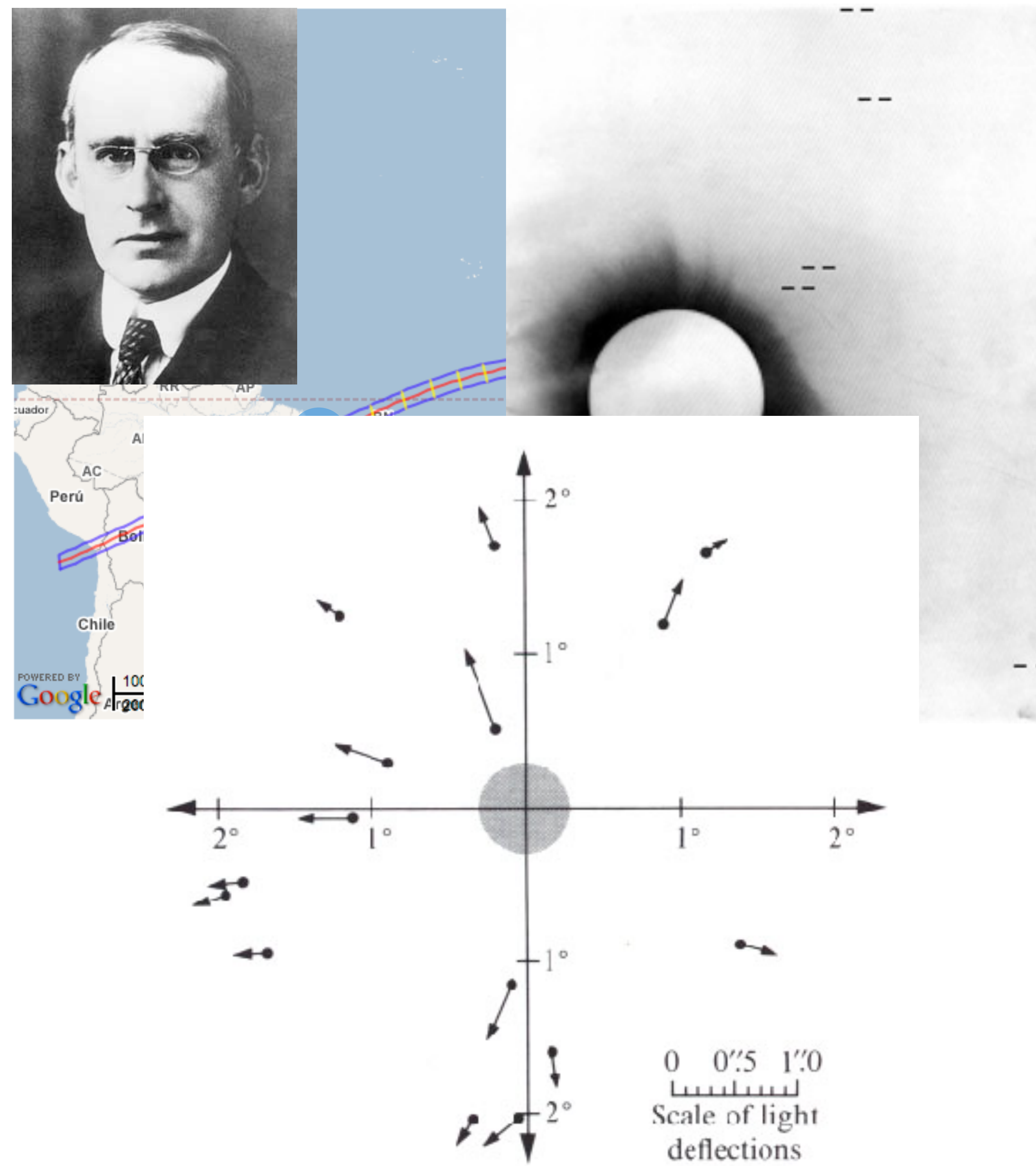

GRAVITATIONAL TIME DELAY

$$\Delta t = \int \frac{\mathrm{d}l}{c'} - \int \frac{\mathrm{d}l}{c} = \frac{1}{c}\int (n-1)\mathrm{d}l = -\frac{2}{c^3}\int \Phi \mathrm{d}l$$

Travel time in un-perturbed space time

Travel time in perturbed space time

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