GRAVITATIONAL LENSING

LECTURE 11

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AA 2015-2016

TODAY'S LECTURE

- Lensing by multiple point masses
  - Binary lenses

COMPLEX LENS EQUATION

- For a system of N-lenses we obtained:

$$z_s = z - \sum_{i=1}^{N} \frac{m_i}{z^* - z_i^*}$$

- Taking the conjugate:

$$z_s^* = z^* - \sum_{i=1}^{N} \frac{m_i}{z - z_i}$$

- We obtain z* and substitute it back into the original equation, which results in a (N^2+1)th order complex polynomial equation

$$p(z) = \sum_{i=0}^{N} c_i z^i$$

- This equation can be solved only numerically, even in the case of a binary lens

COMPLEX LENS EQUATION

- Note that the solutions are not necessarily solutions of the lens equations (spurious solutions)
- One has to check if the solutions are solutions of the lens equation
- Rhie 2001,2003: maximum number of images is 5(N-1) for N>2

MAGNIFICATION

- In the complex form, the magnification can still be derived from the lensing Jacobian:

$$\det A = \left(\frac{\partial z_s}{\partial z}\right)^2 - \frac{\partial z_s}{\partial z^*}\left(\frac{\partial z_s}{\partial z^*}\right)^* = 1 - \frac{\partial z_s}{\partial z^*}\left(\frac{\partial z_s}{\partial z^*}\right)^*$$

$$\frac{\partial z_s}{\partial z^*} = \sum_{i=1}^{N} \frac{m_i}{(z^* - z_i^*)^2}$$

$$\det A = 1 - \left|\sum_{i=1}^{N} \frac{m_i}{(z^* - z_i^*)^2}\right|^2$$

CRITICAL LINES AND CAUSTICS

- Therefore the critical lines form where

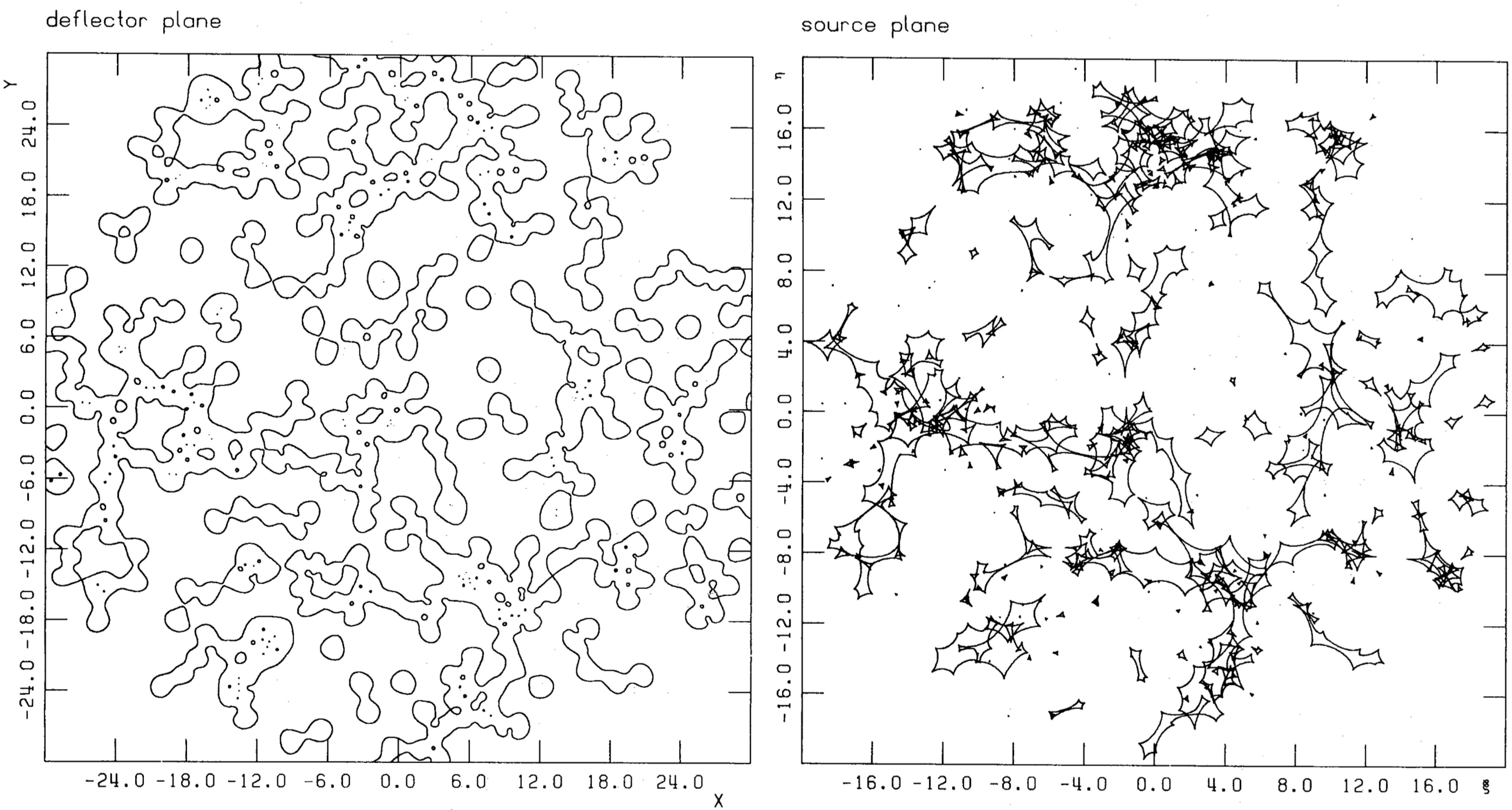
$$\left|\sum_{i=1}^{N} \frac{m_i}{(z^* - z_i^*)^2}\right|^2 = 1$$

- Thus, to find the critical points we solve

$$\sum_{i=1}^{N} \frac{m_i}{(z^* - z_i^*)^2} = e^{i\phi} \qquad\qquad \phi \in [0, 2\pi]$$

- Again, this can be turned into a complex polynomial of order 2N: for N lenses, there are 2N critical lines and caustics. The solutions can be found numerically.

CRITICAL LINES AND CAUSTICS



critical lines and caustics originated by 400 stars

Witt, 1990, A&A, 236, 311

BINARY LENSES

- Lens equation: $$z_s = z - \frac{m_1}{z^* - z_1^*} - \frac{m_2}{z^* - z_2^*}$$

- determinant of the Jacobian: $$\det A = 1 - \left|\frac{\partial z_s}{\partial z^*}\right|^2$$

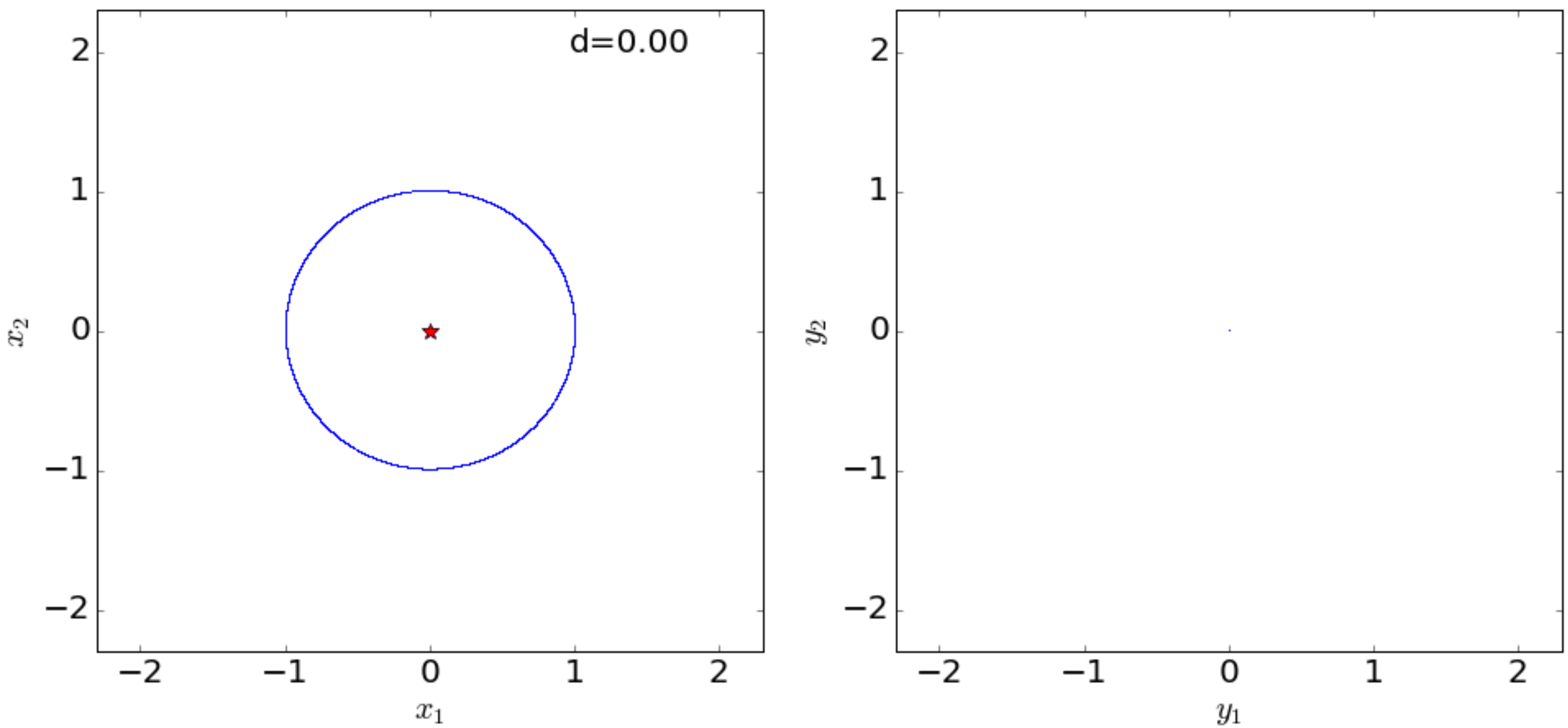
$$\frac{\partial z_s}{\partial z^*} = \frac{m_1}{(z^* - z_1^*)^2} + \frac{m_2}{(z^* - z_2^*)^2}$$

- condition for critical points: $$\frac{\partial z_s}{\partial z^*} = e^{i\phi}$$

- resulting fourth grade polynomial:

$$z^4 - z^2(2z_1^{*2} + e^{i\phi}) - zz_1^* 2(m_1 - m_2)e^{i\phi} + z_1^{*2}(z_1^{*2} - e^{i\phi}) = 0$$

BINARY LENSES:
TWO LENSES WITH THE SAME MASS (Q=1) AND VARYING DISTANCE



critical lines

caustics

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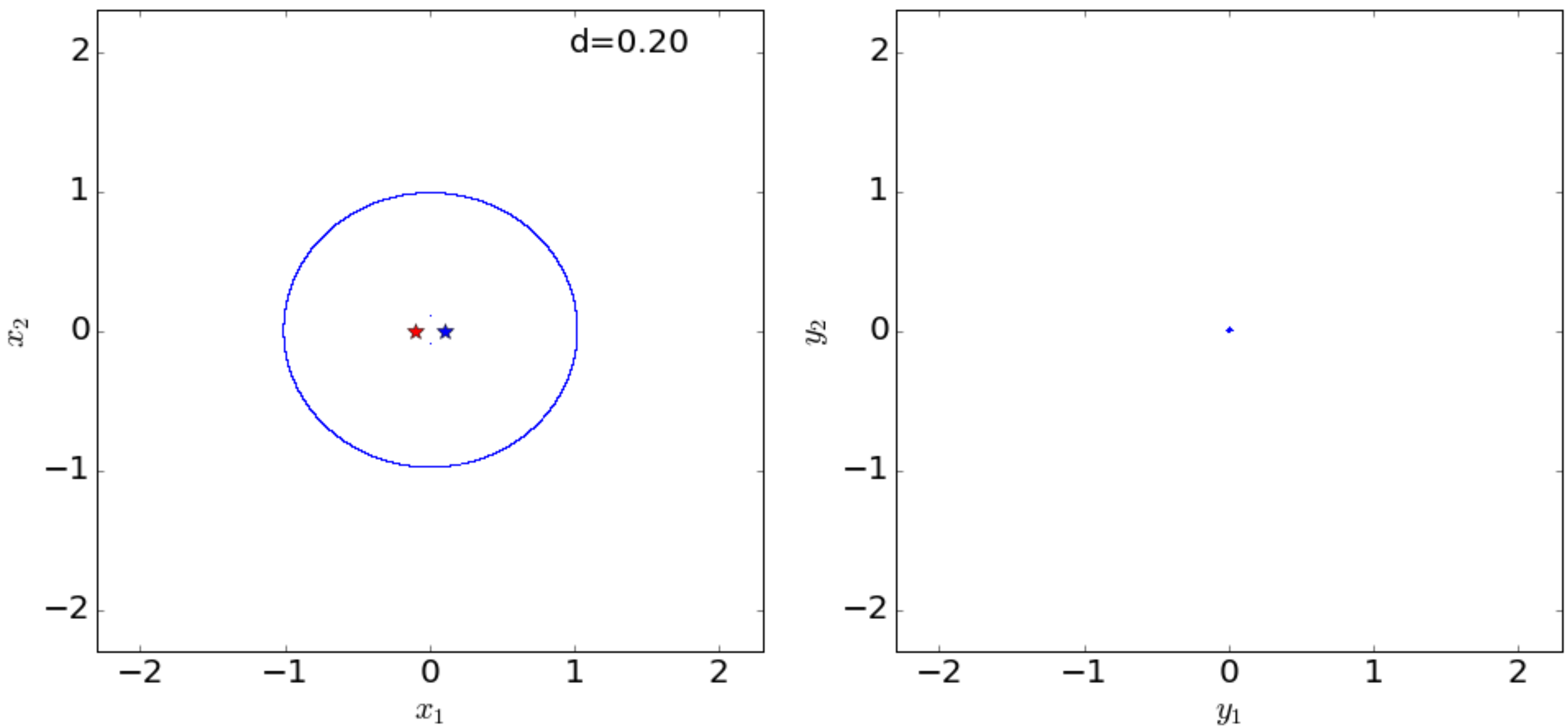

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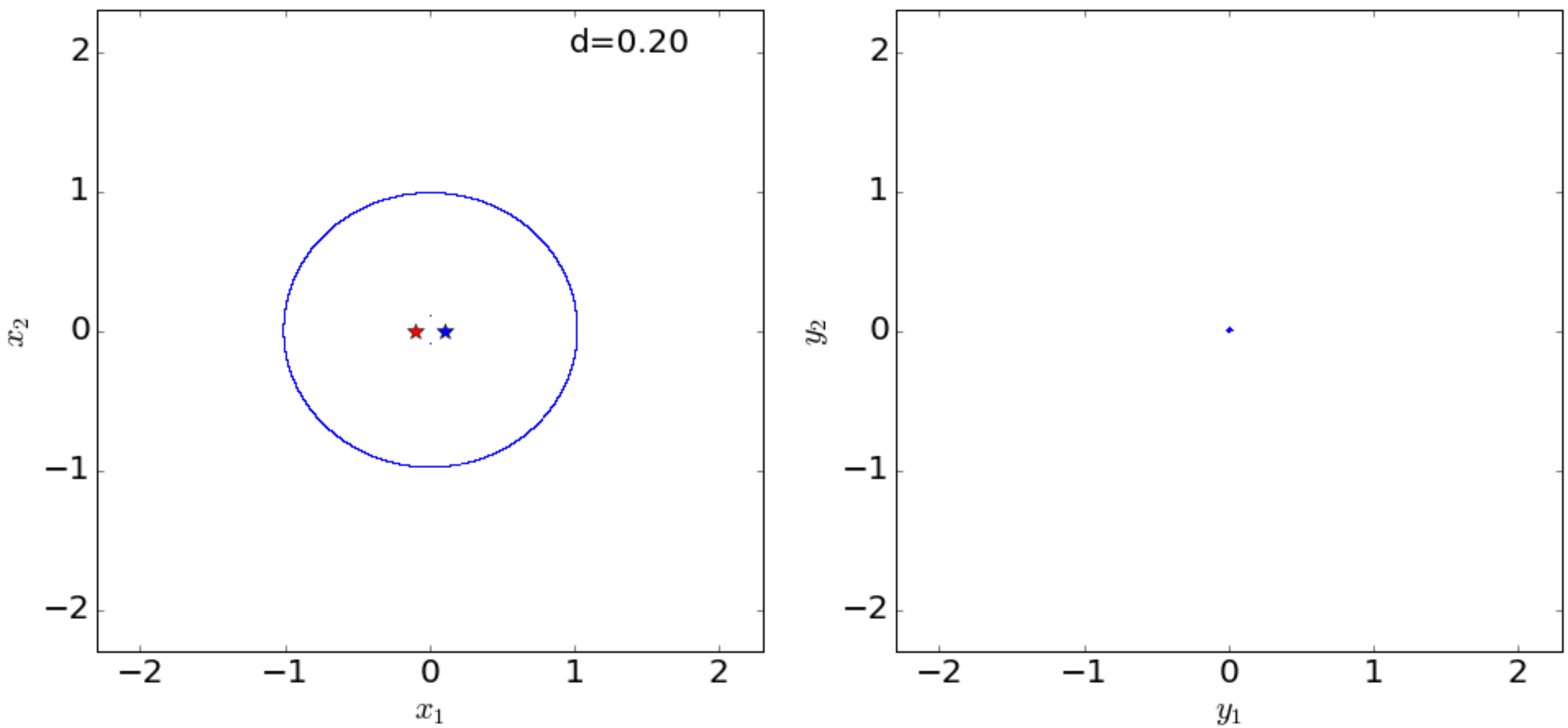

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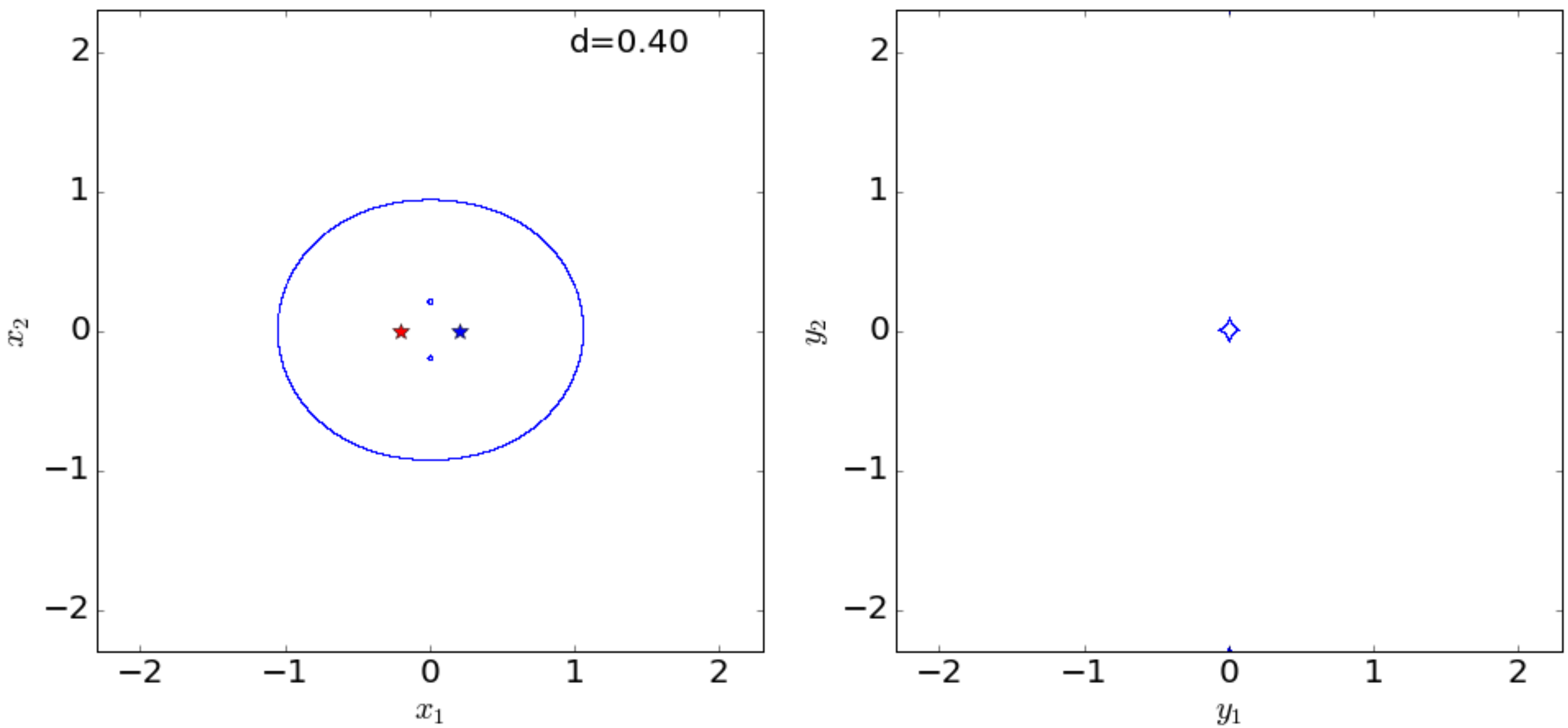

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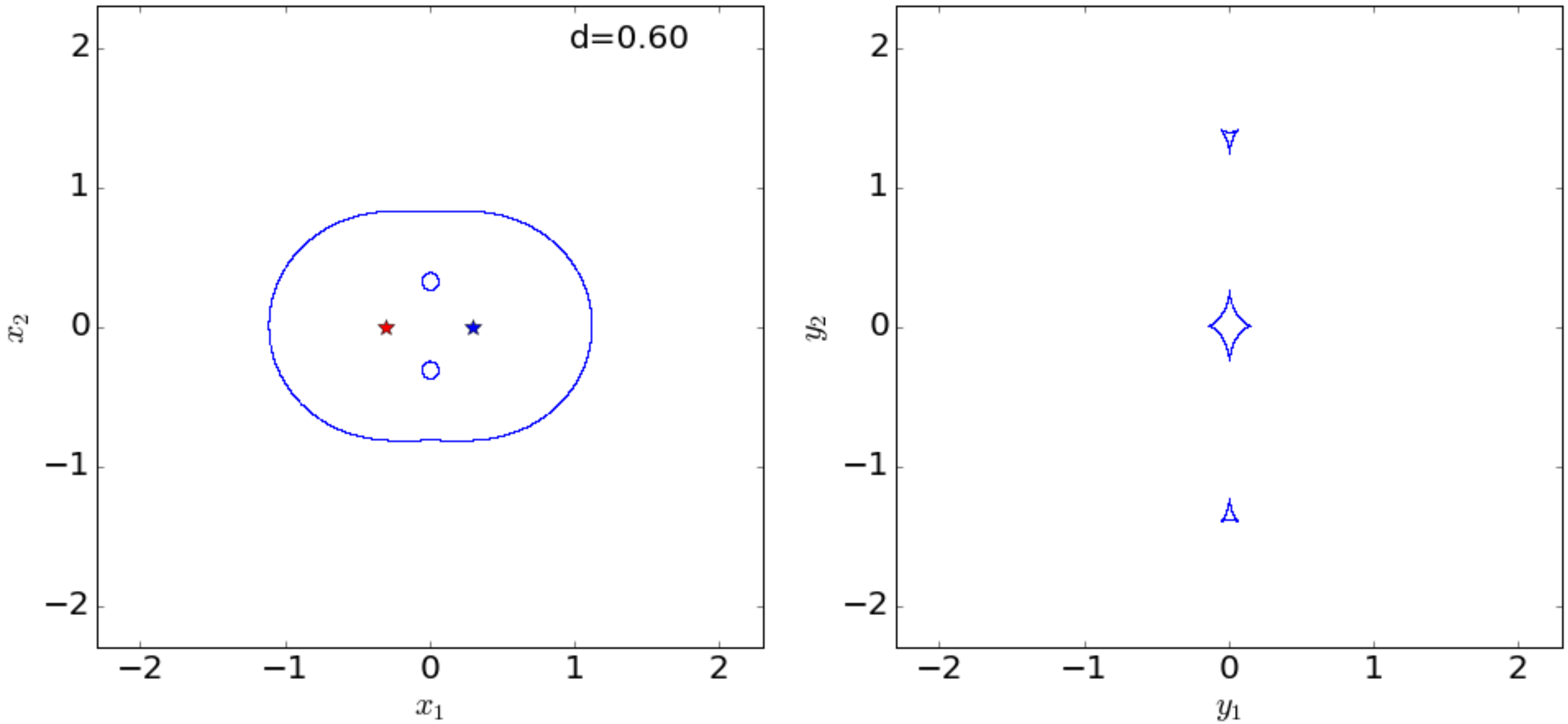

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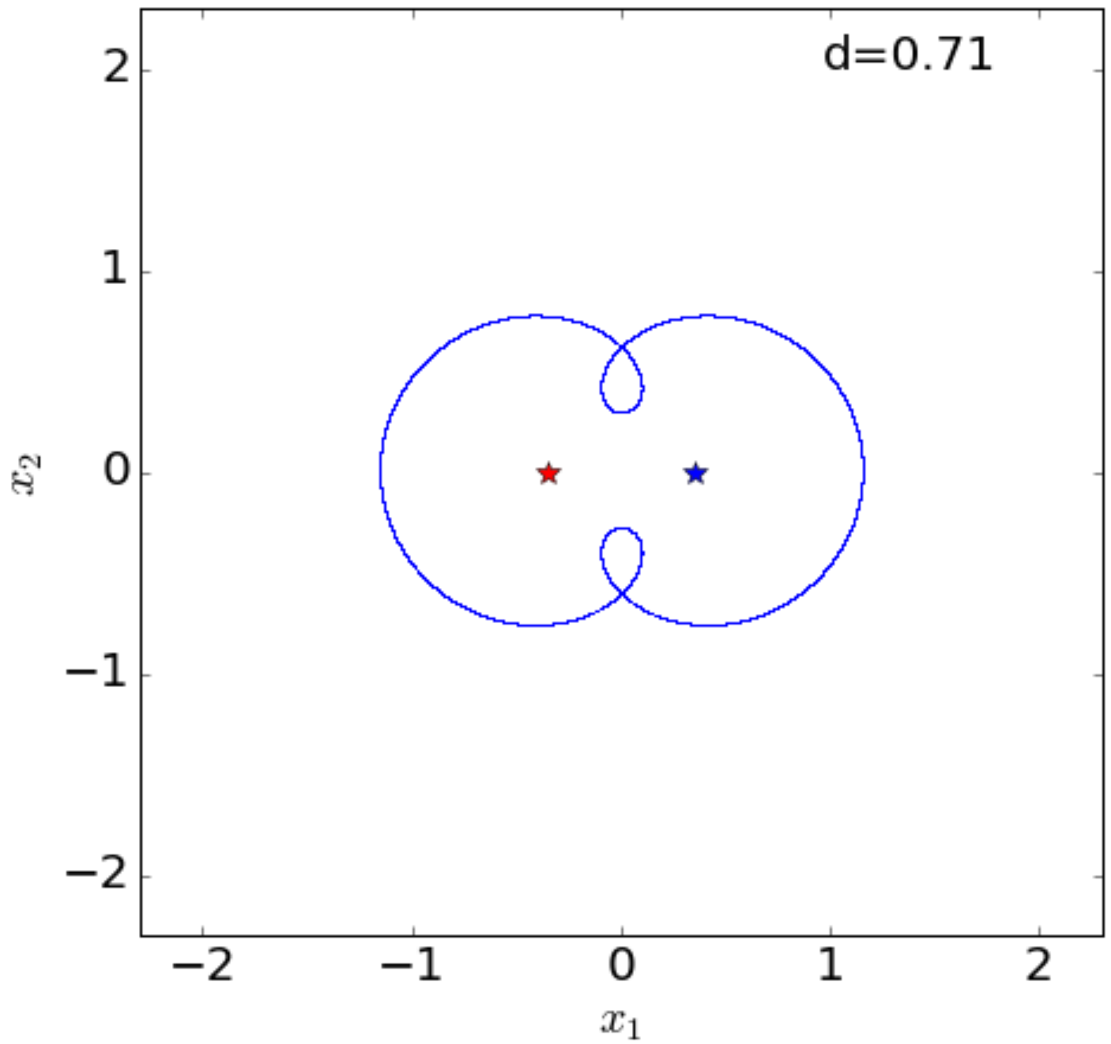
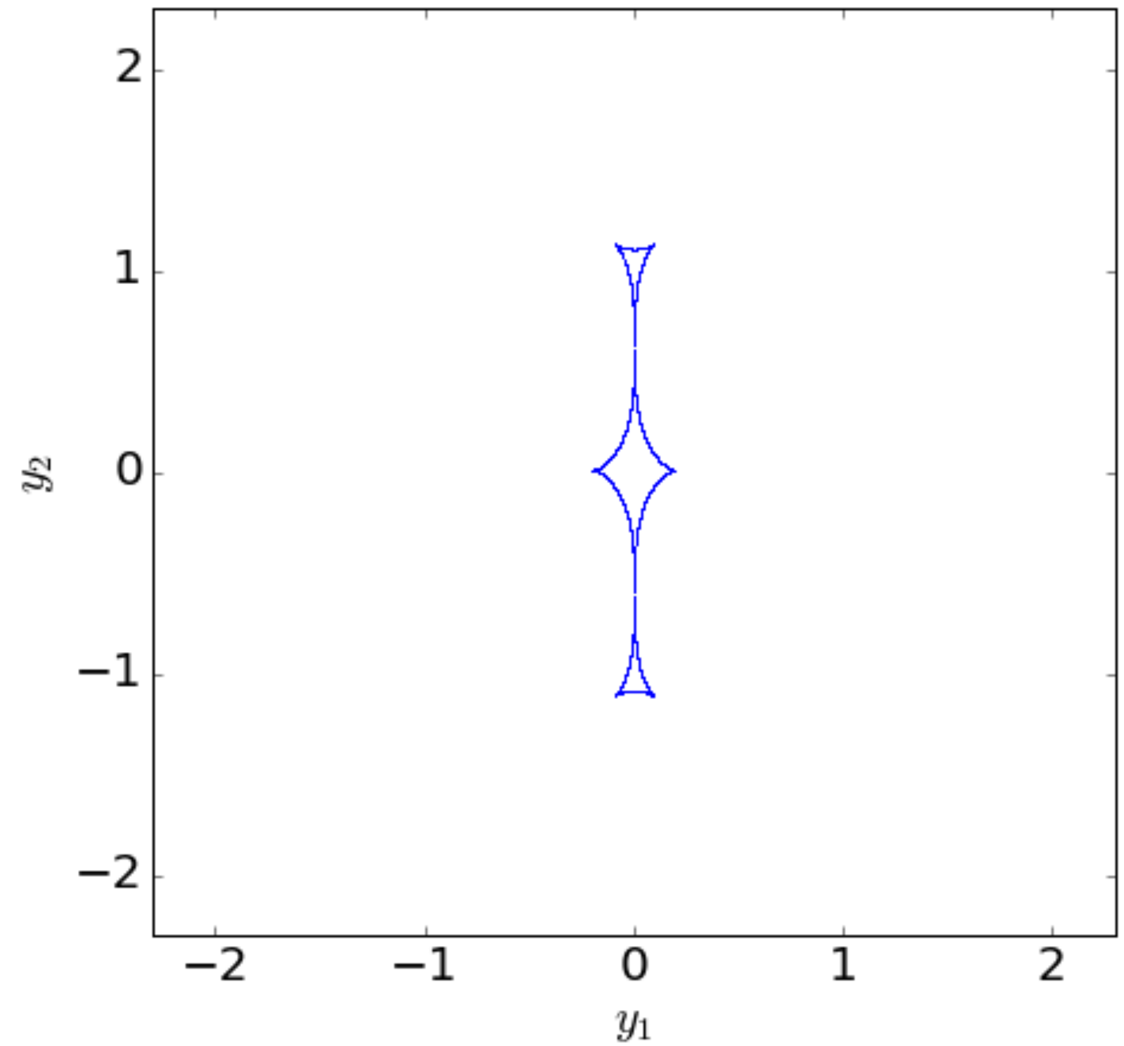


critical lines



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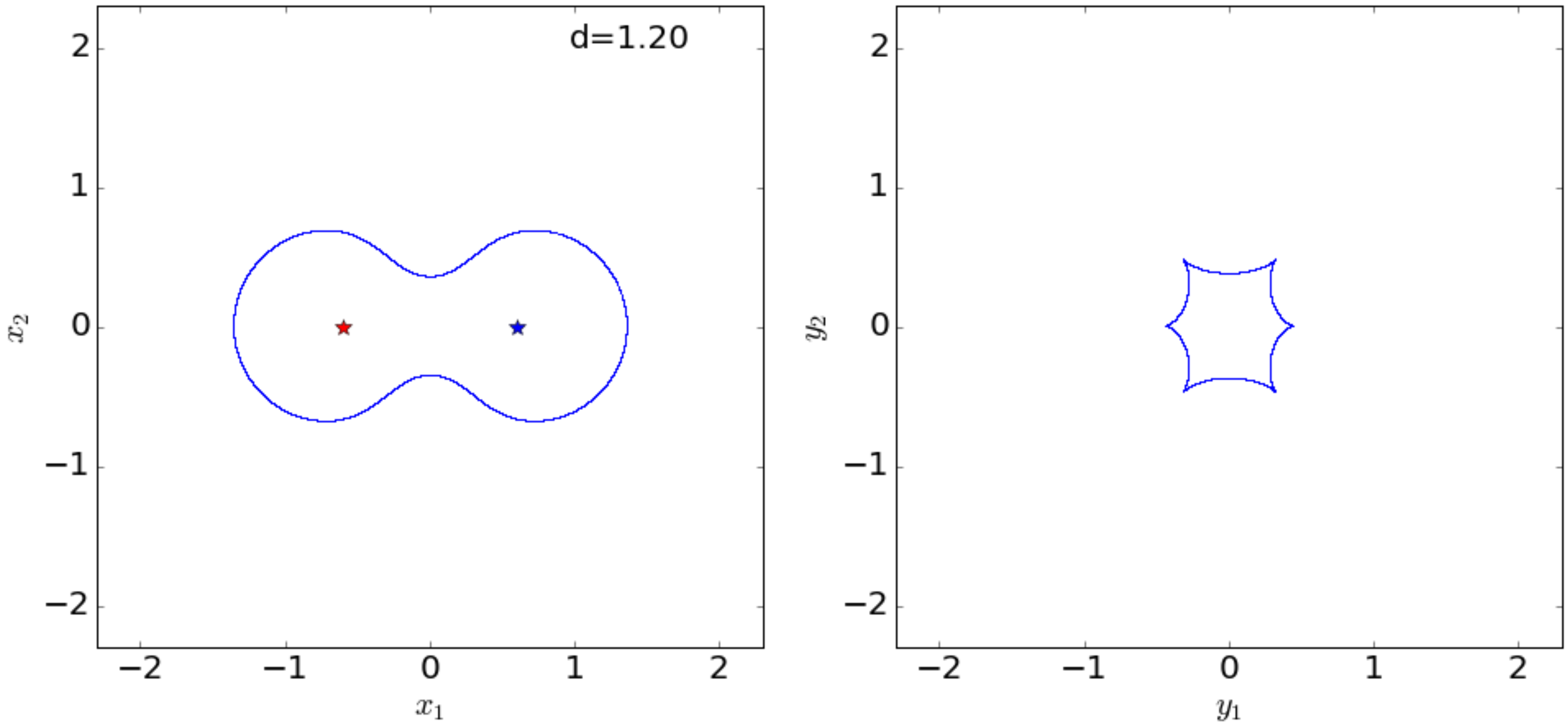

critical lines *caustics*

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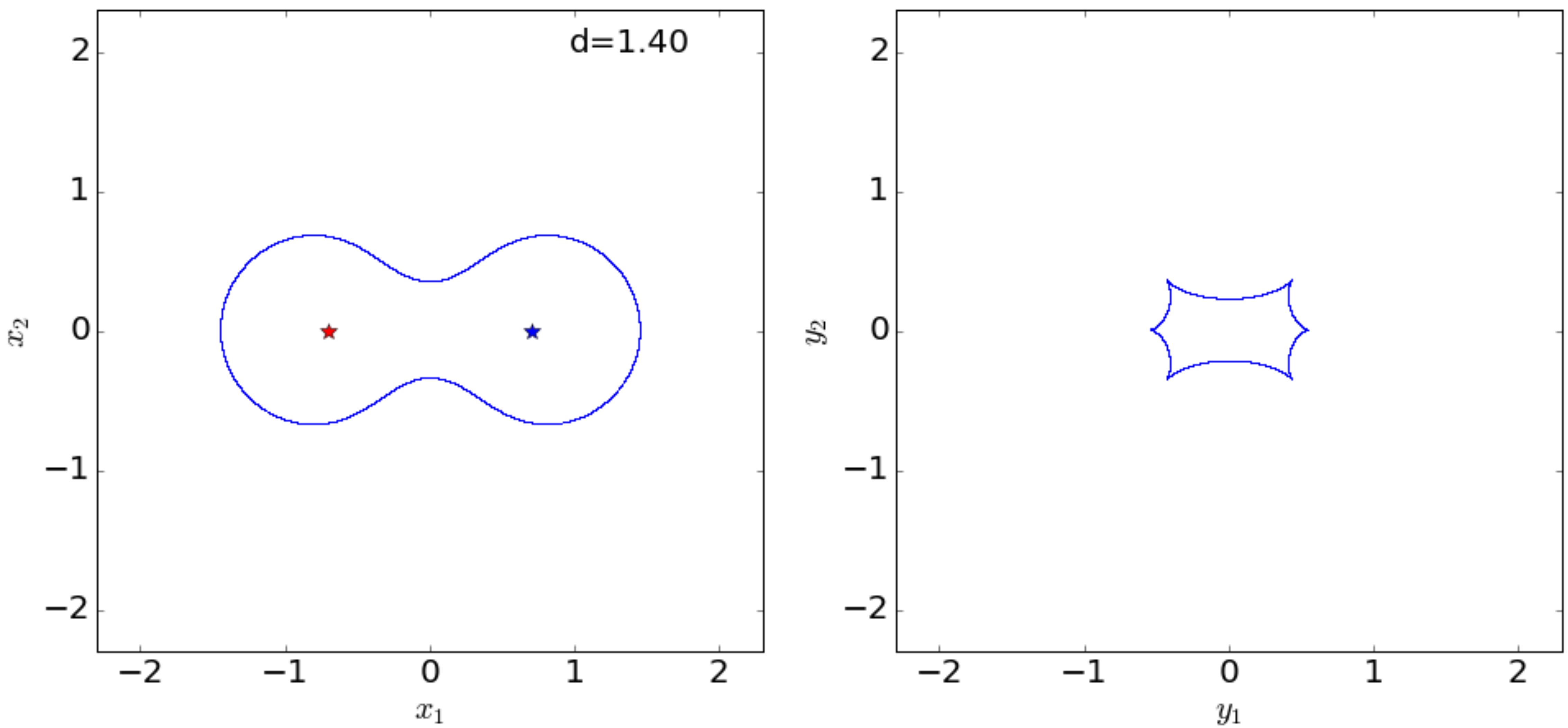

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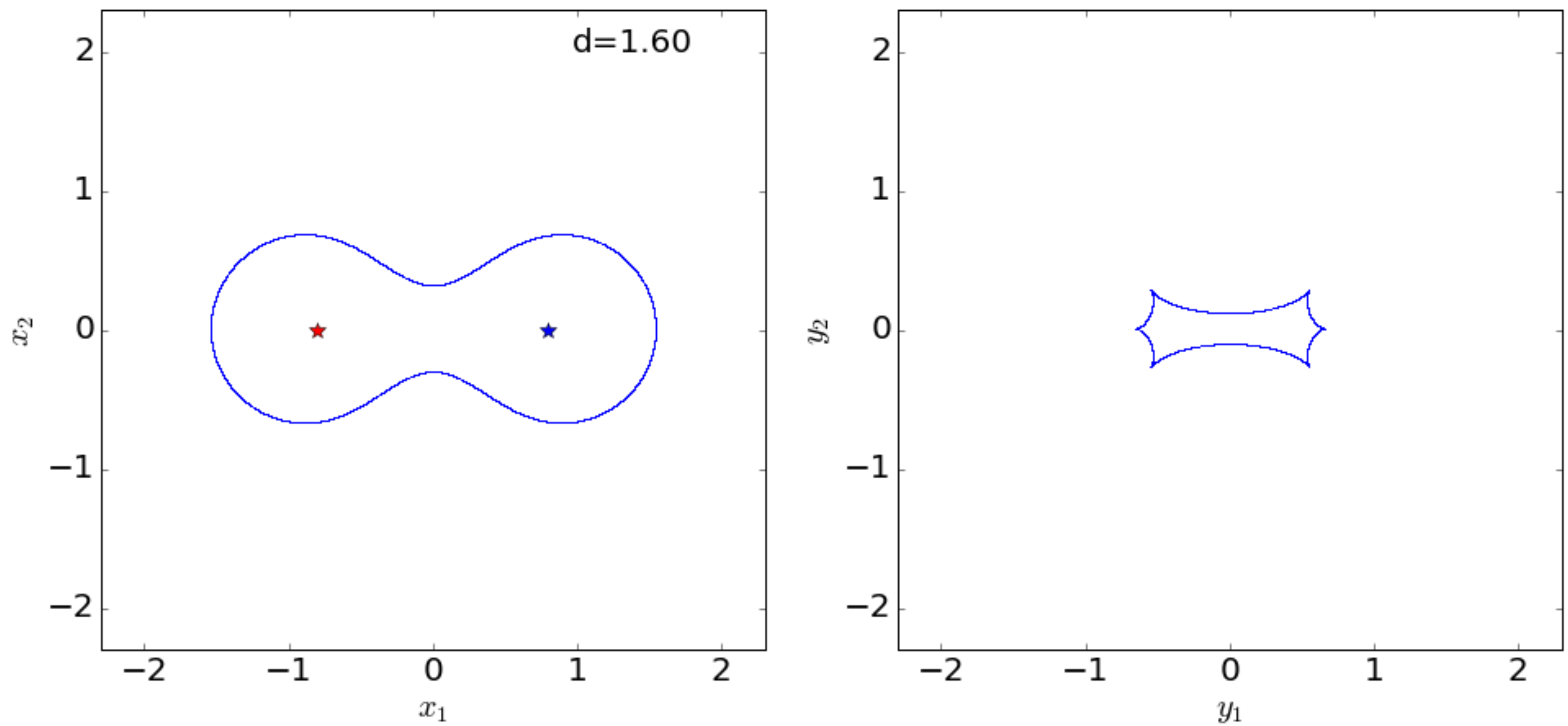

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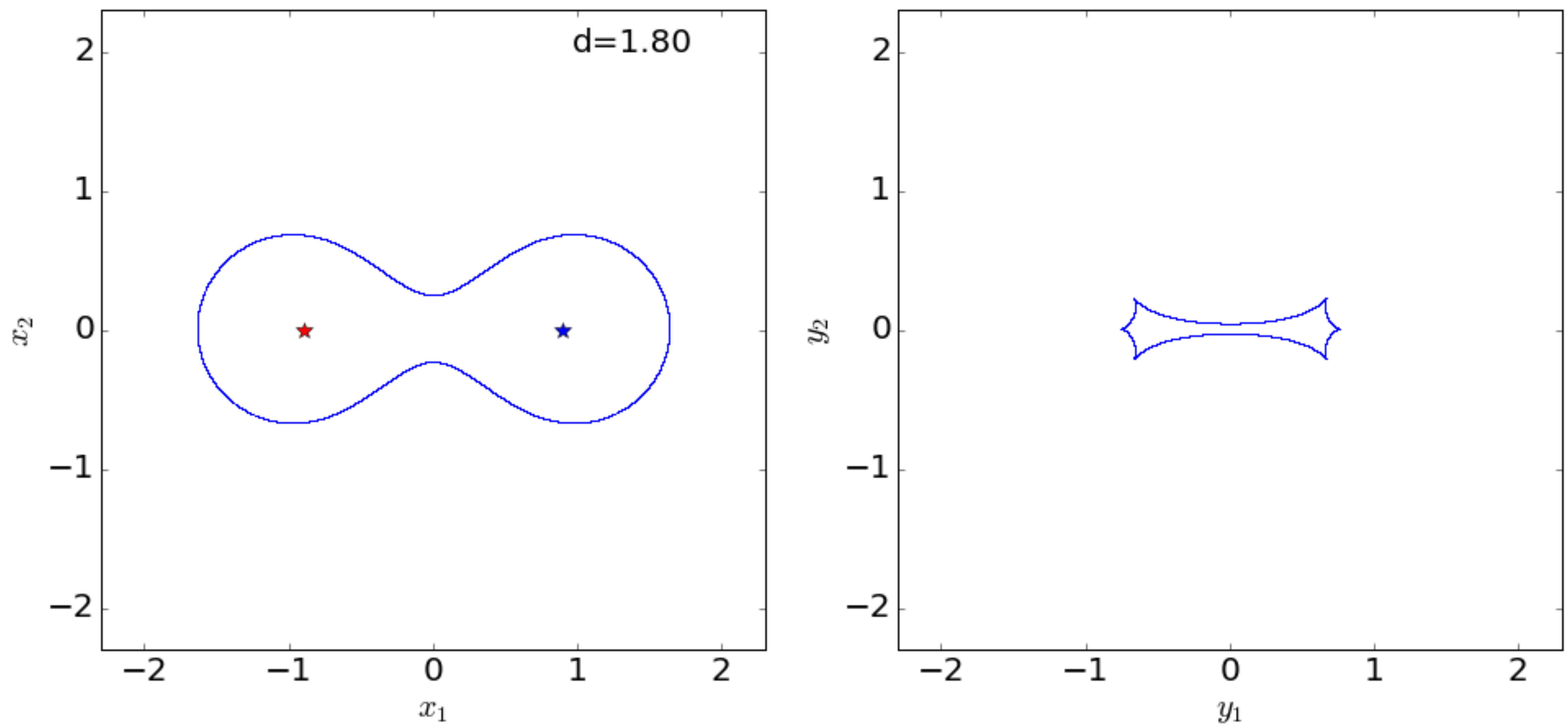

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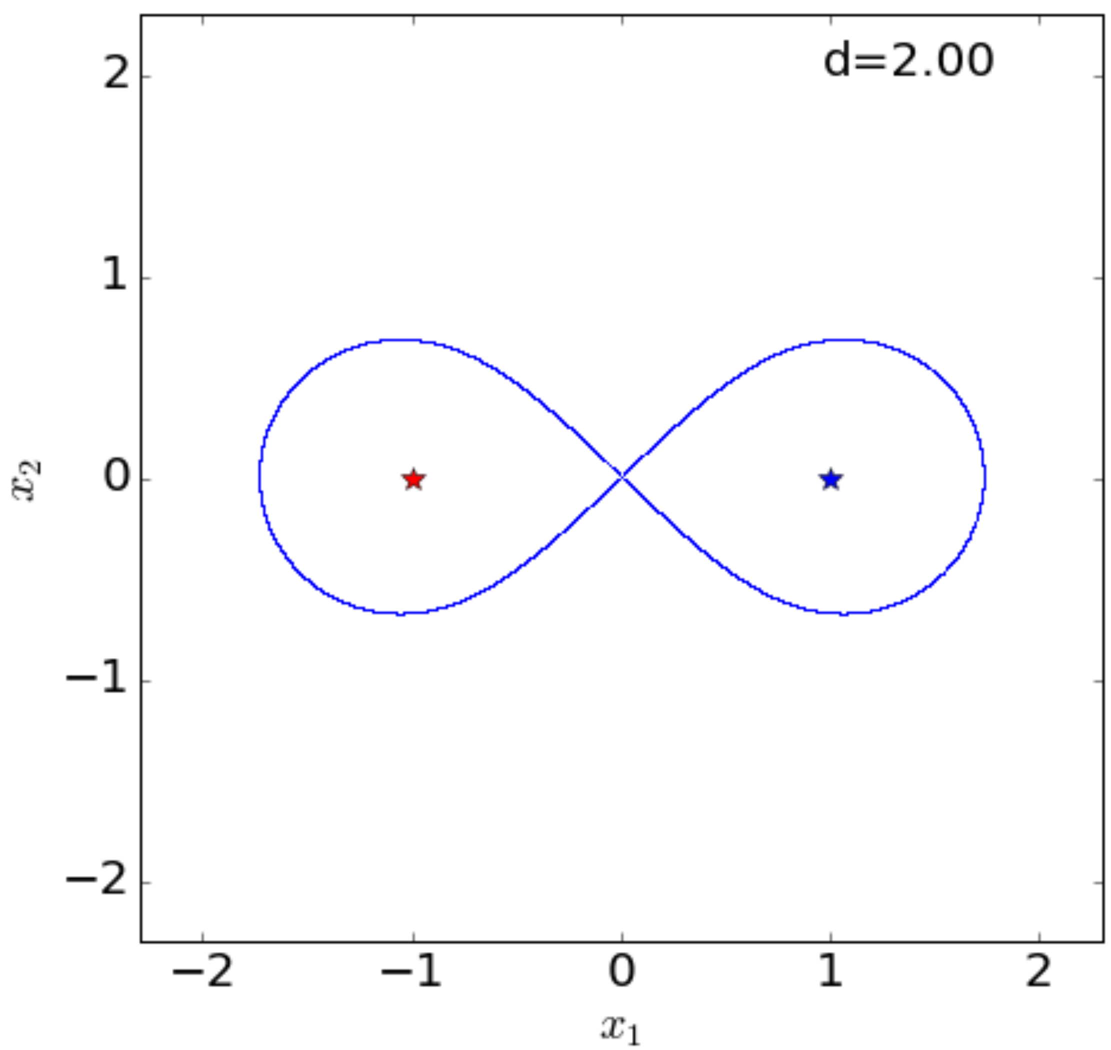
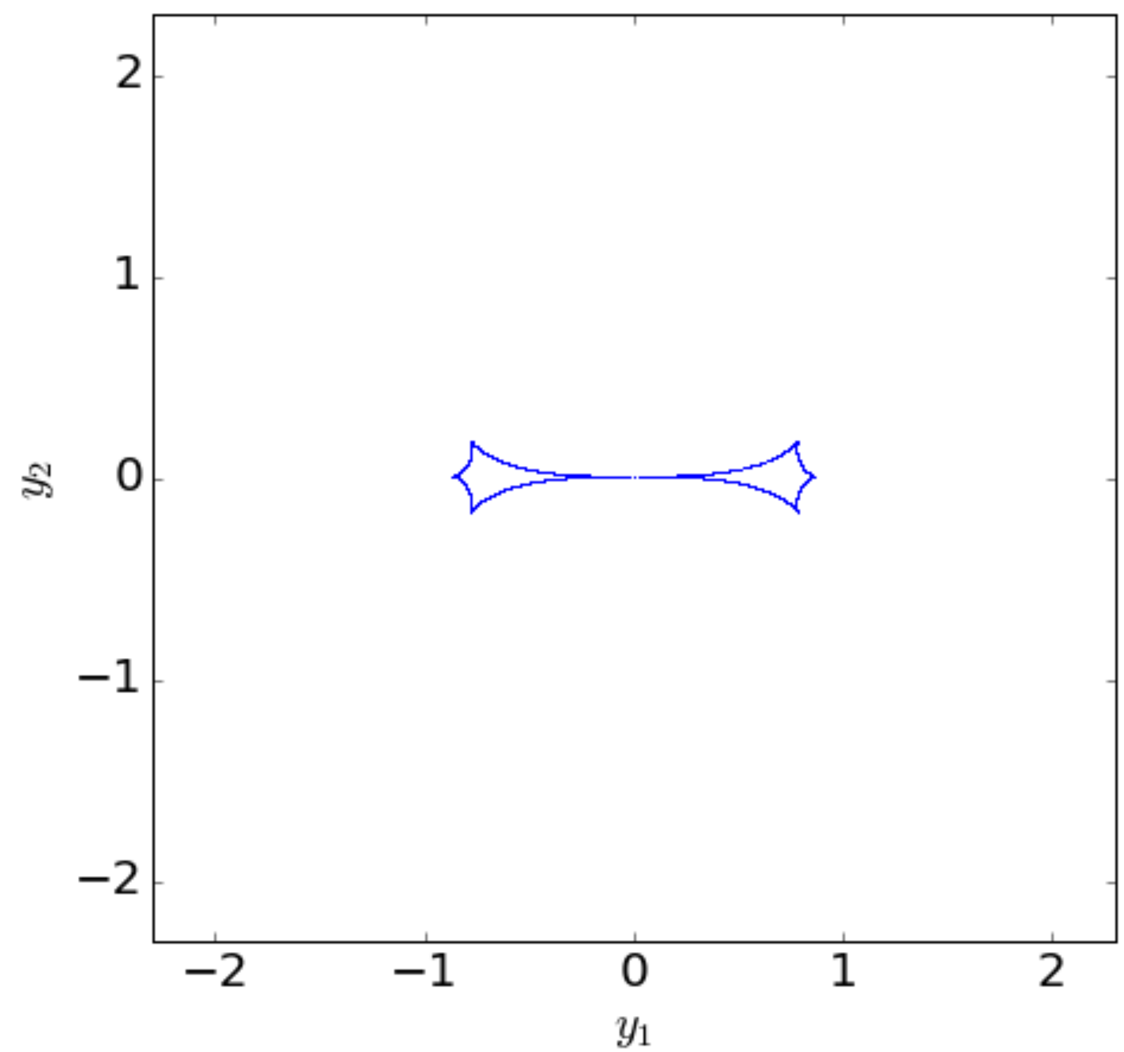


critical lines



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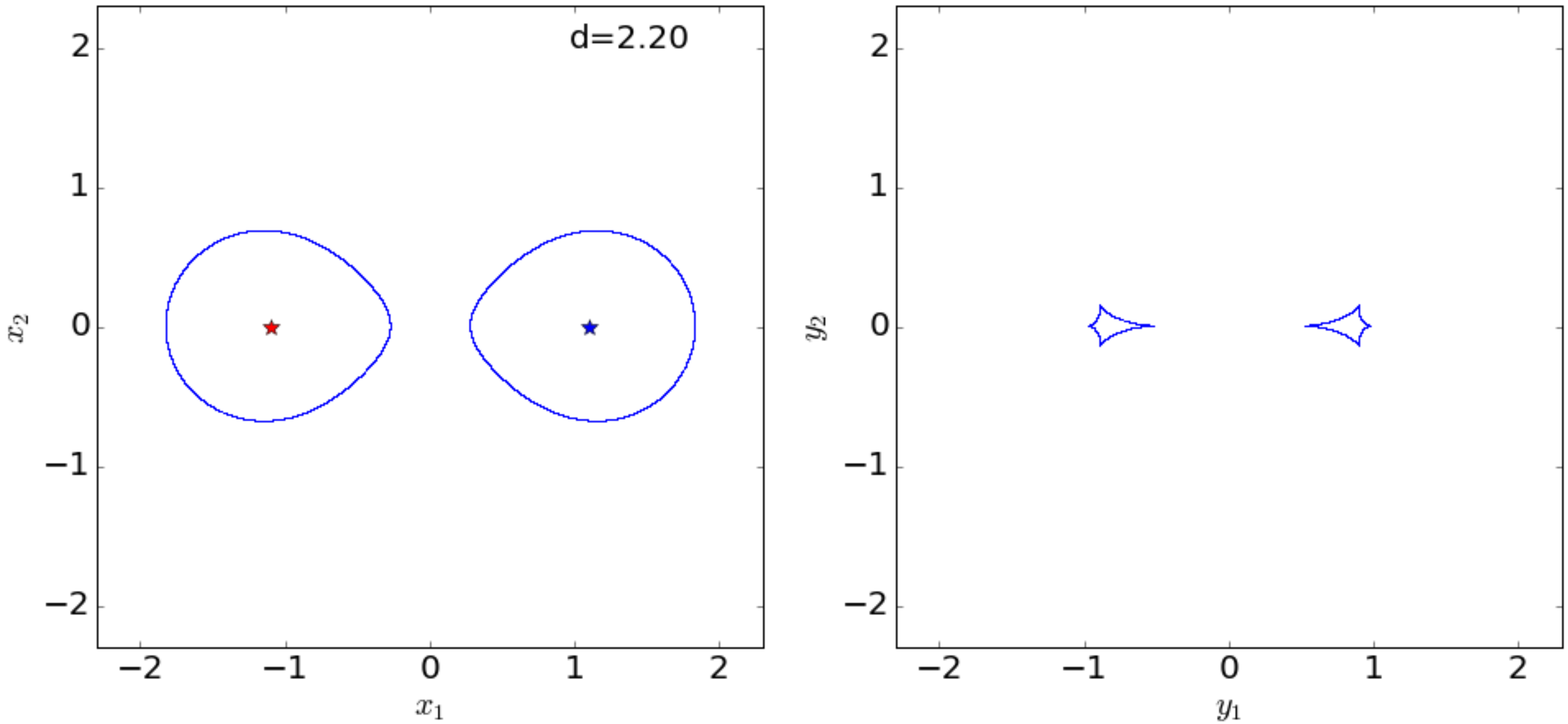

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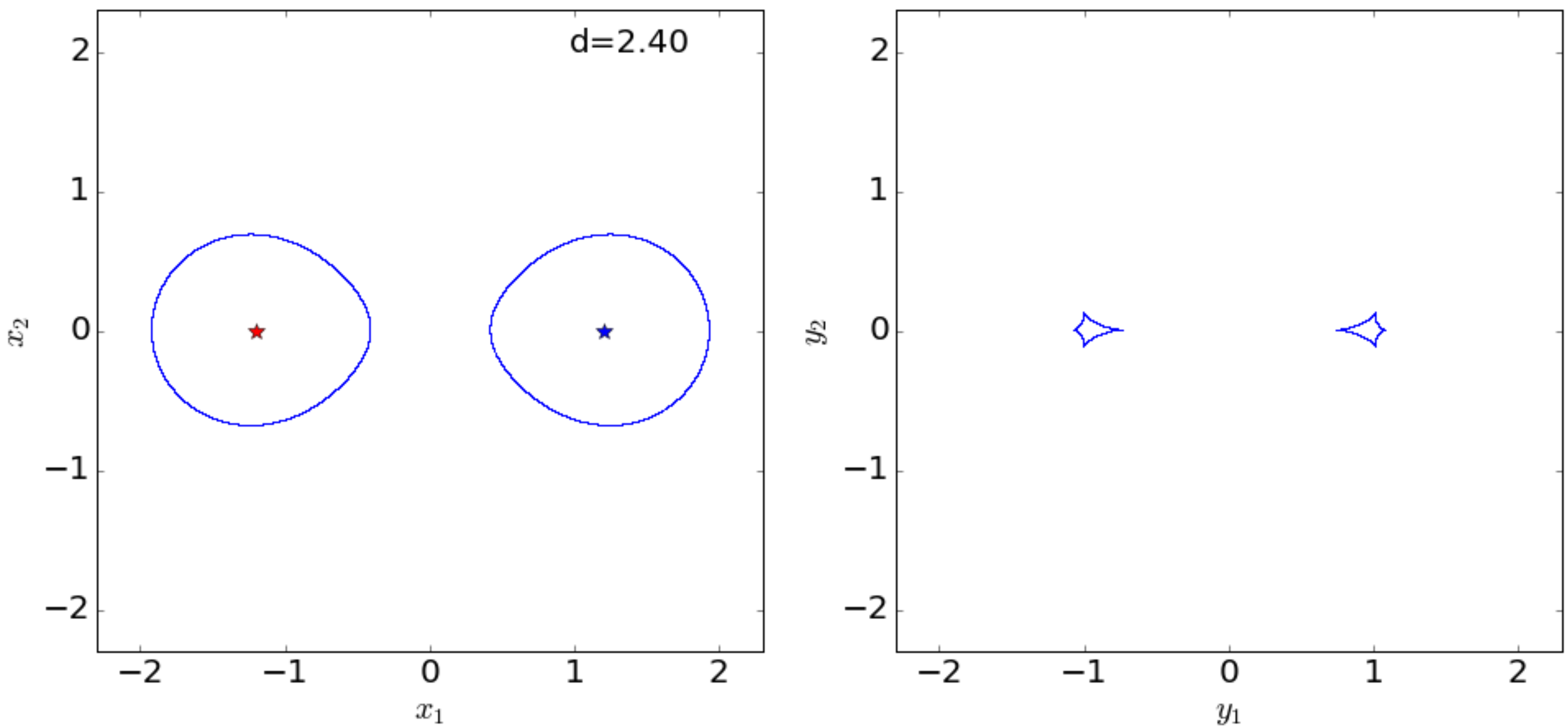

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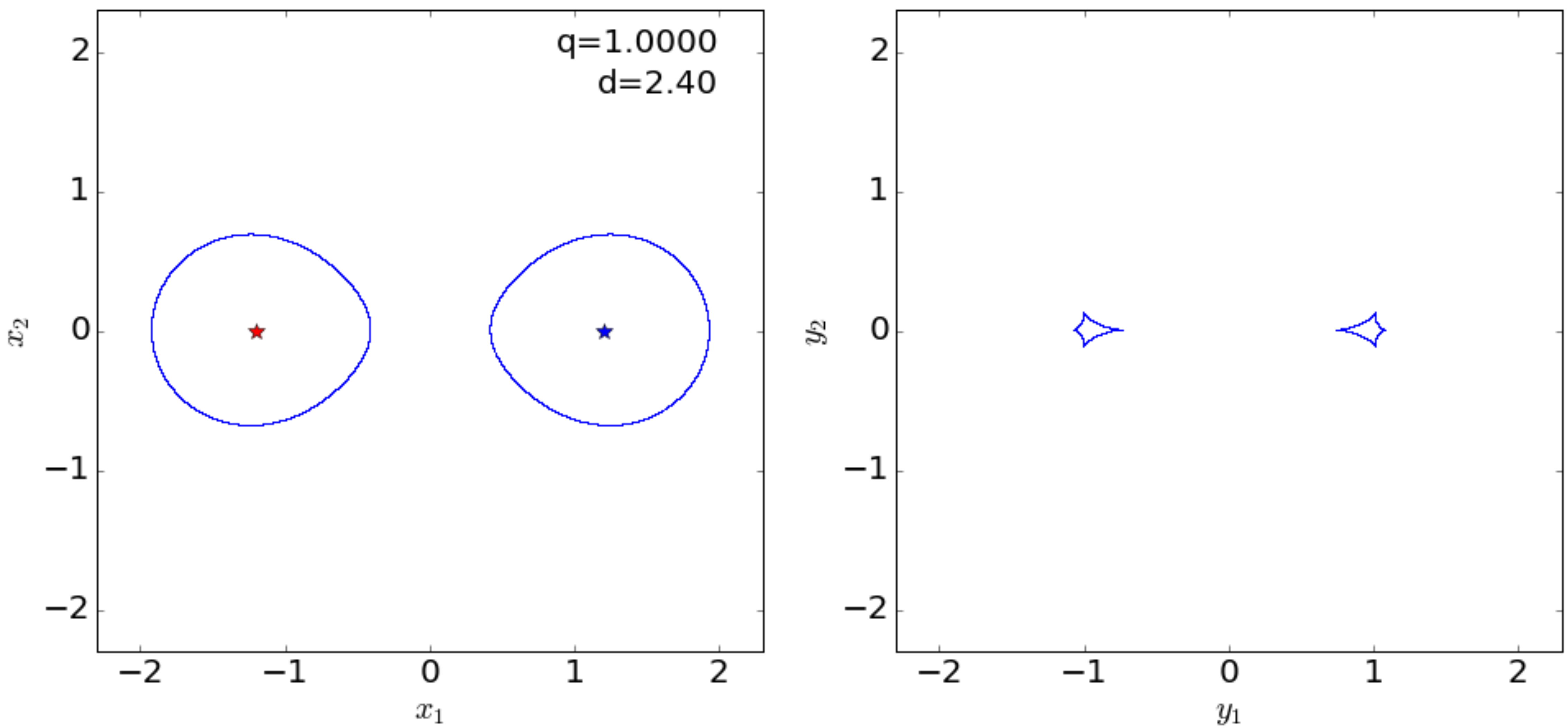

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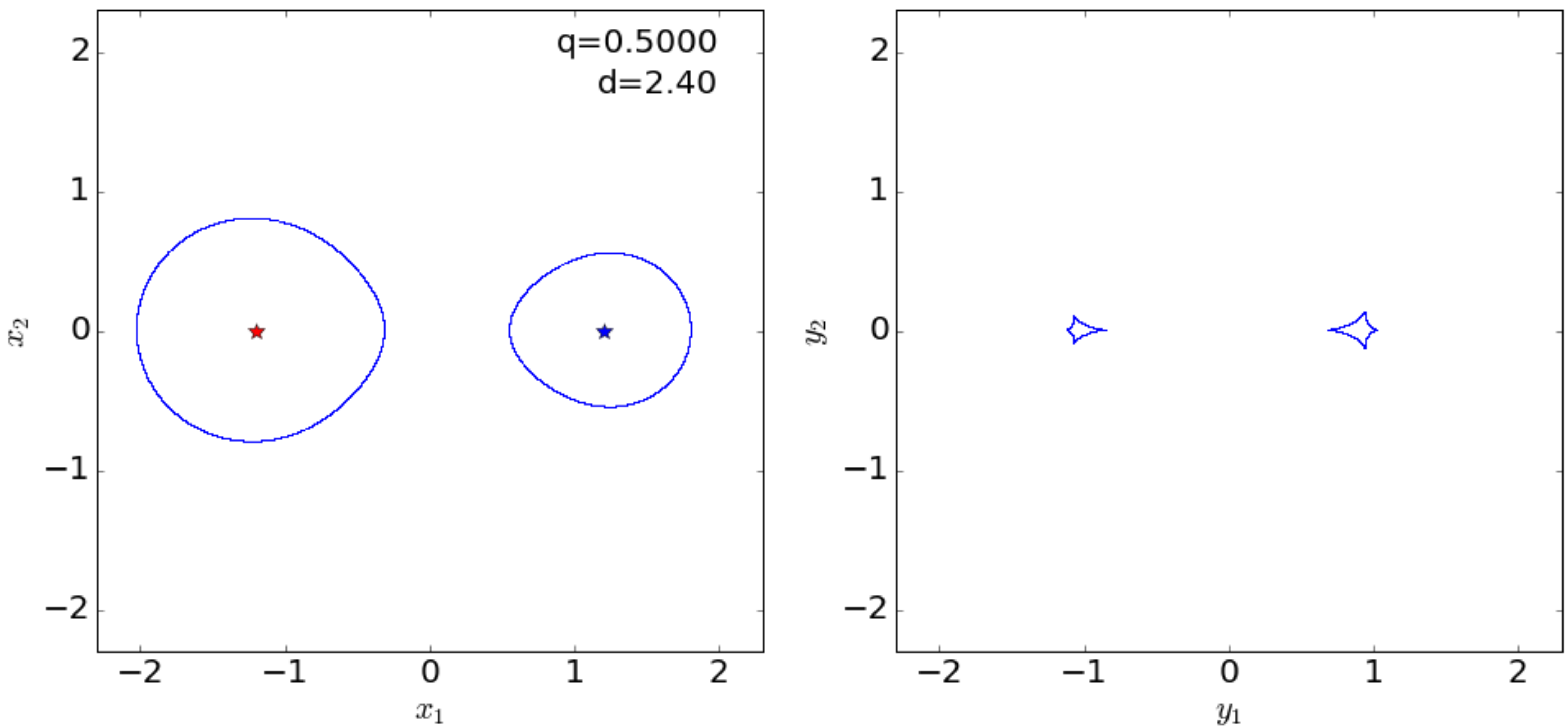

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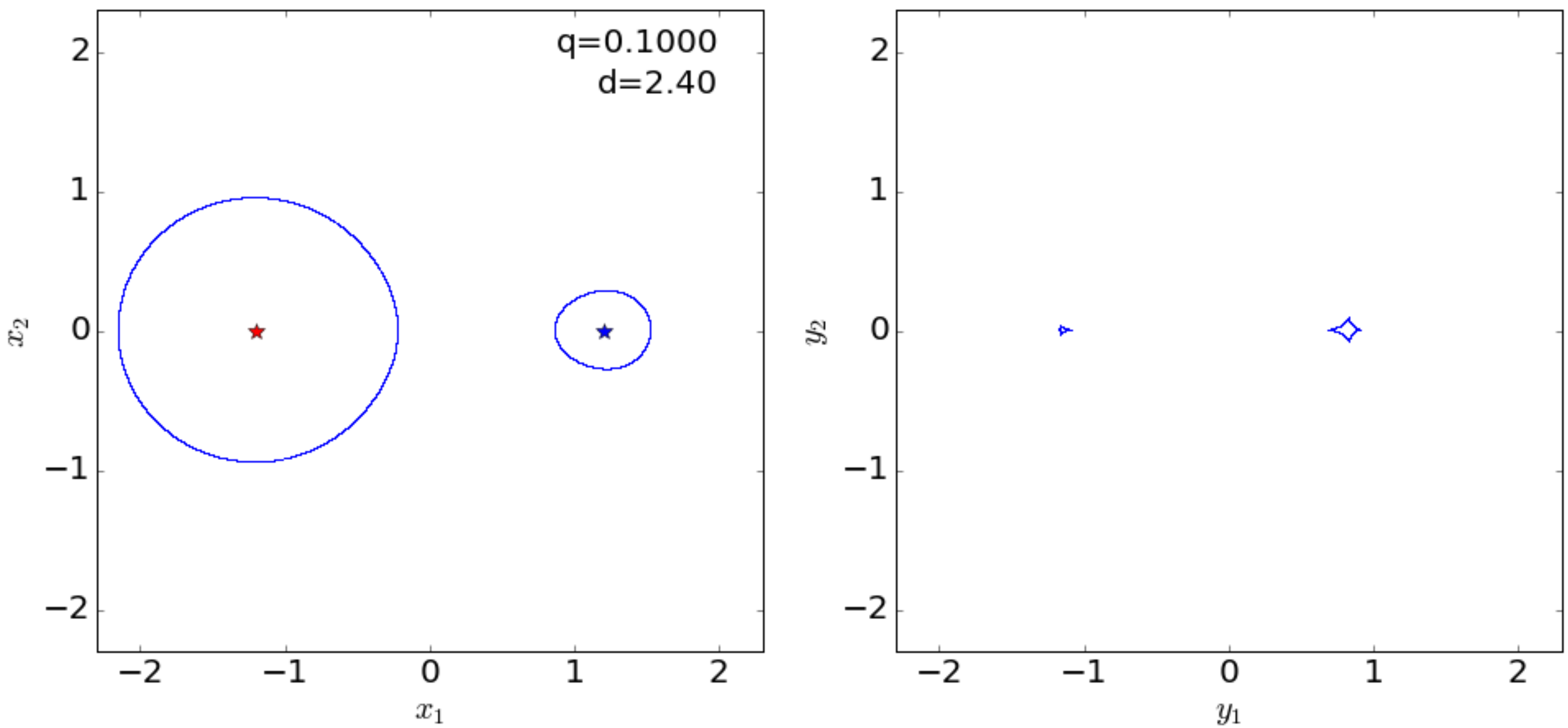

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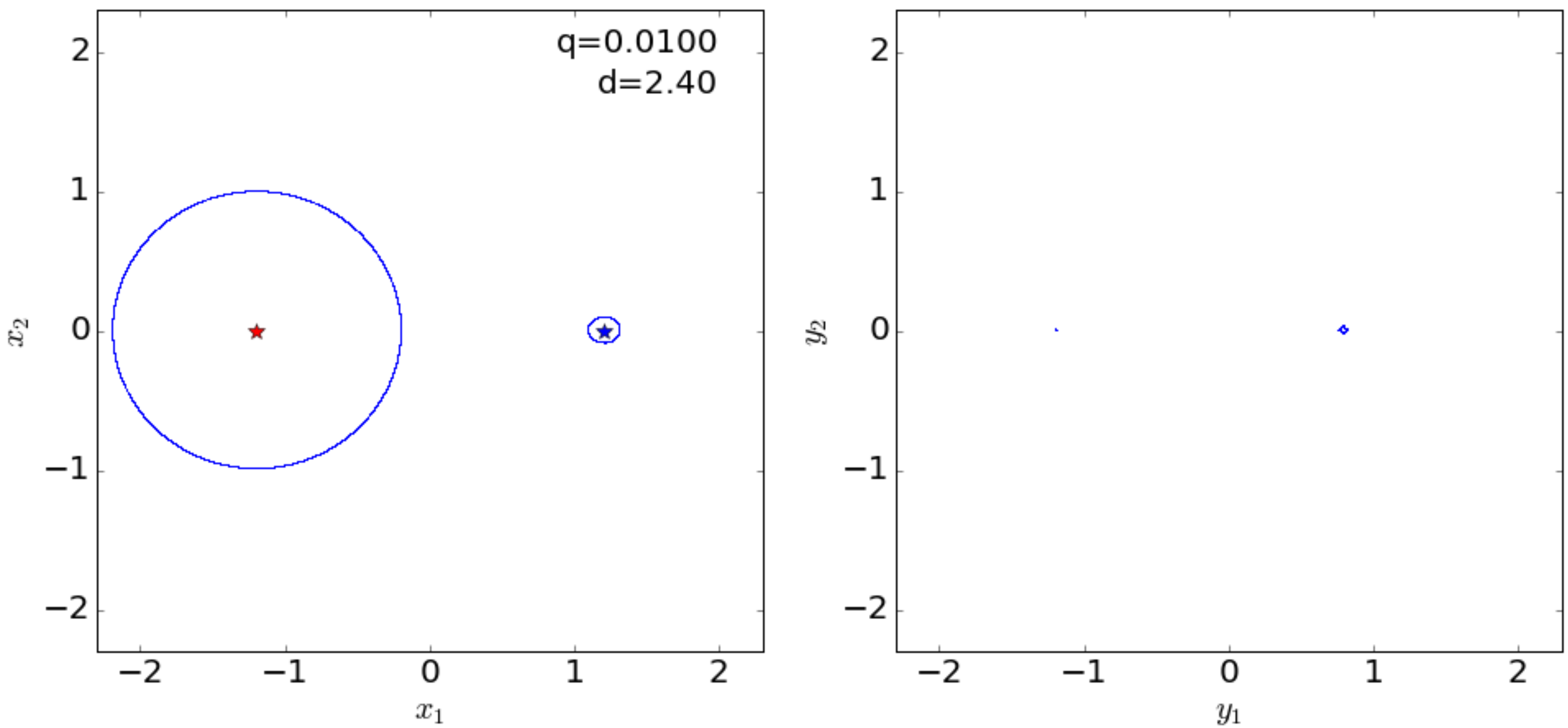

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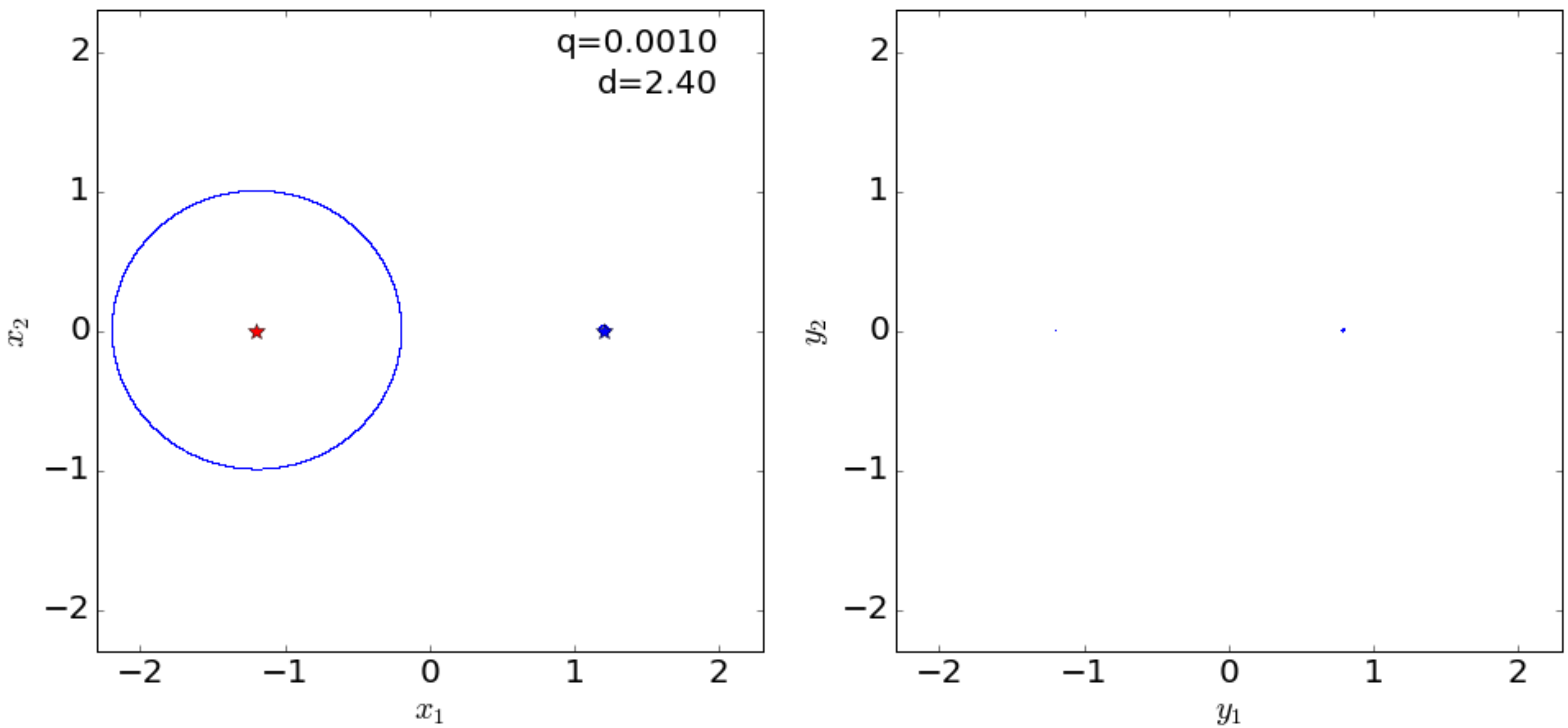

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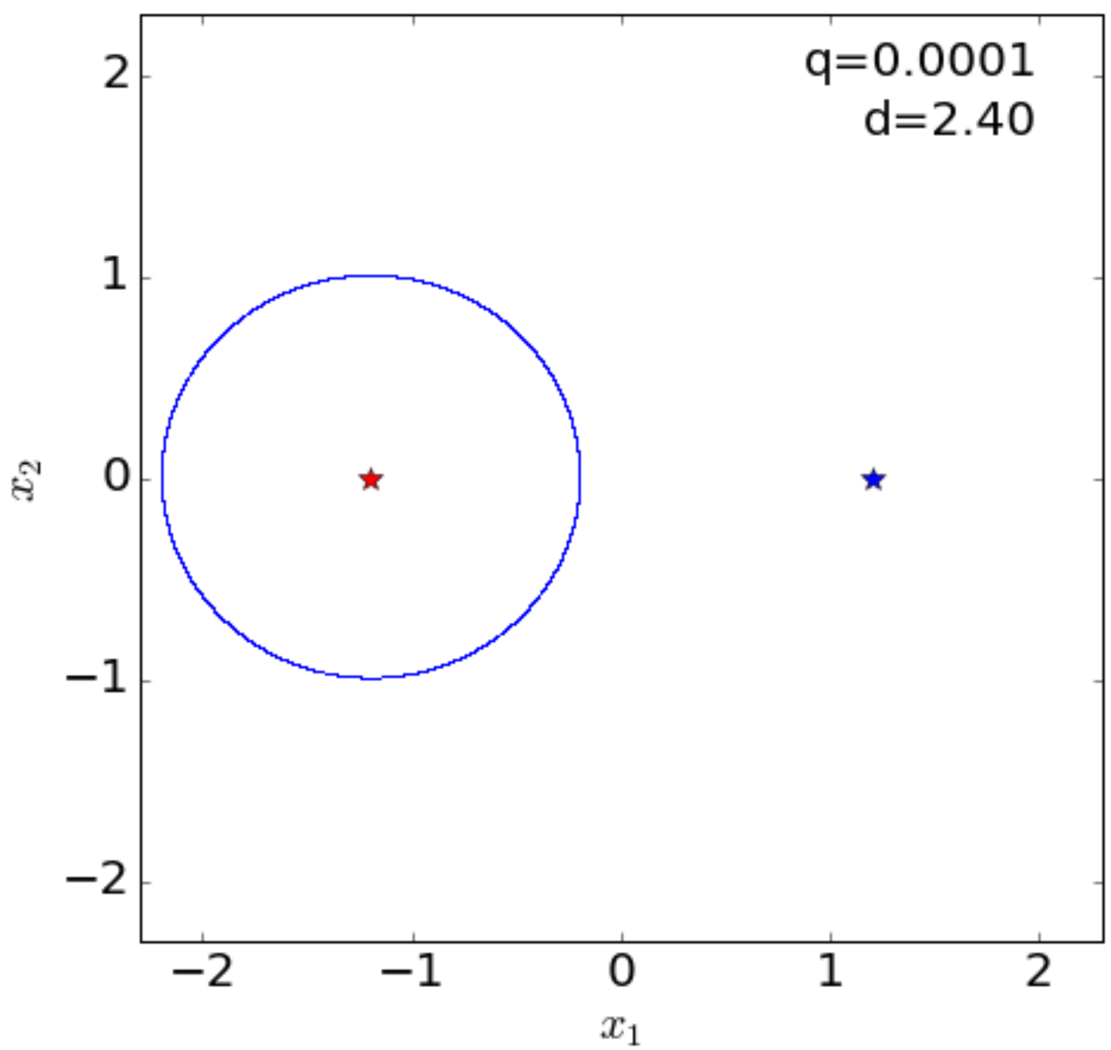

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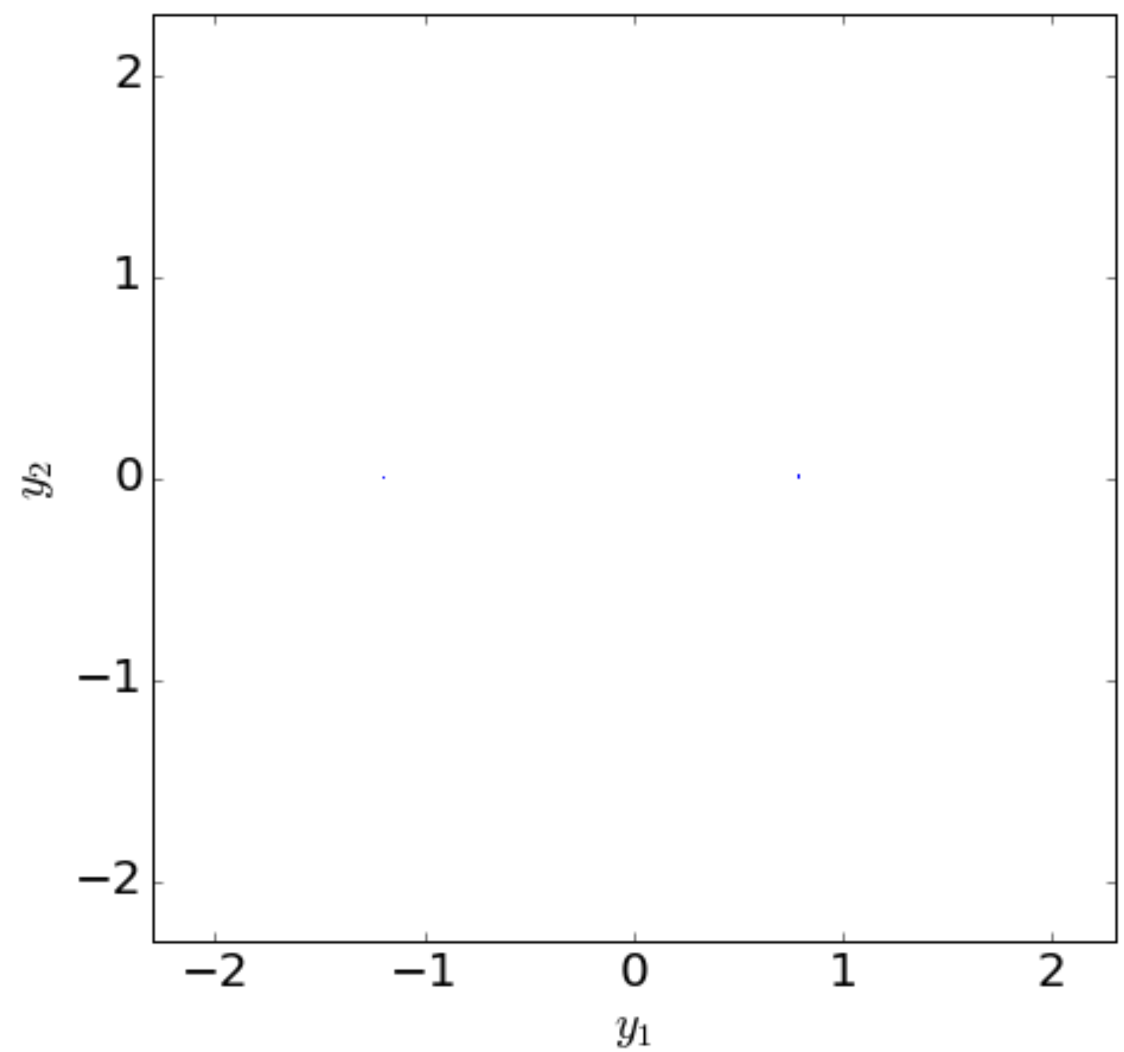

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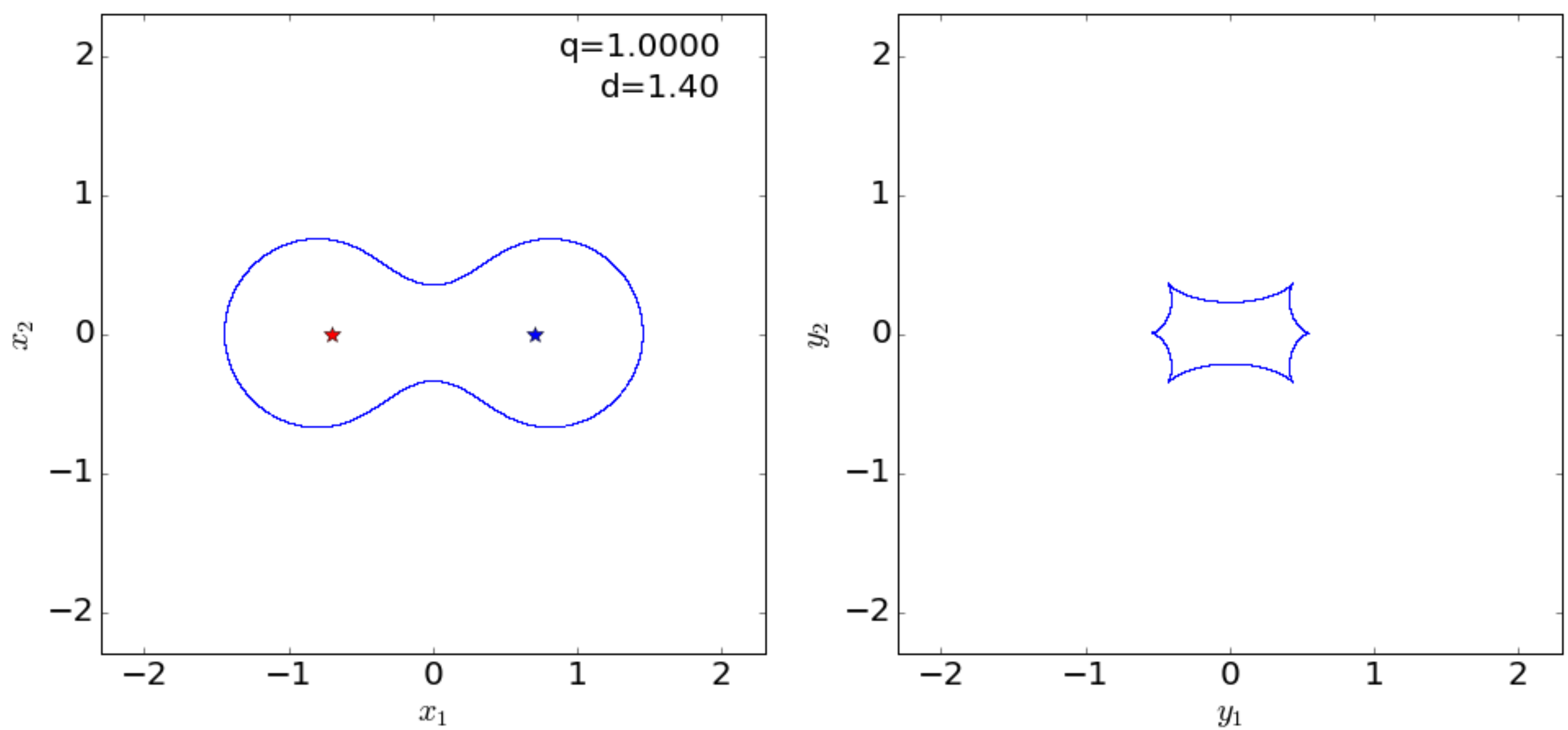

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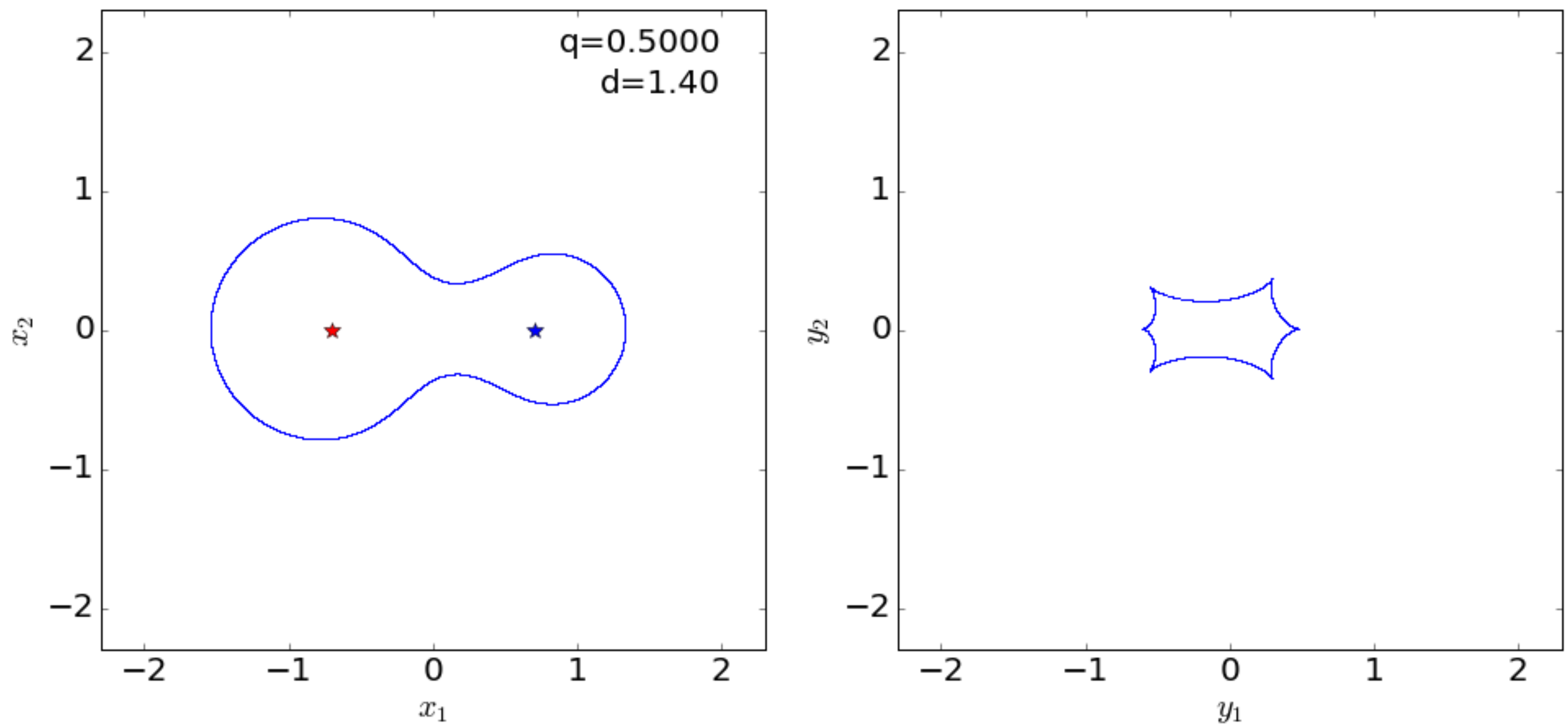

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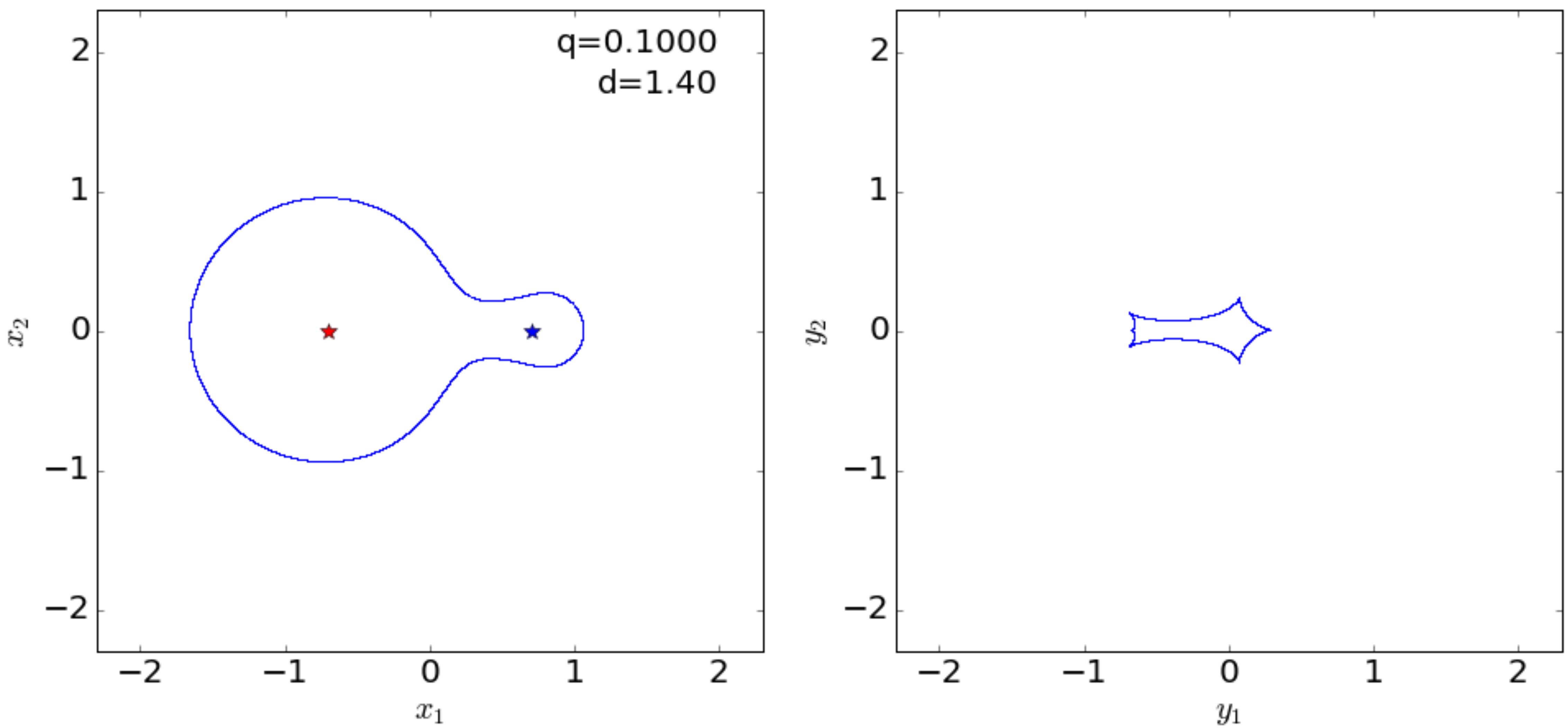

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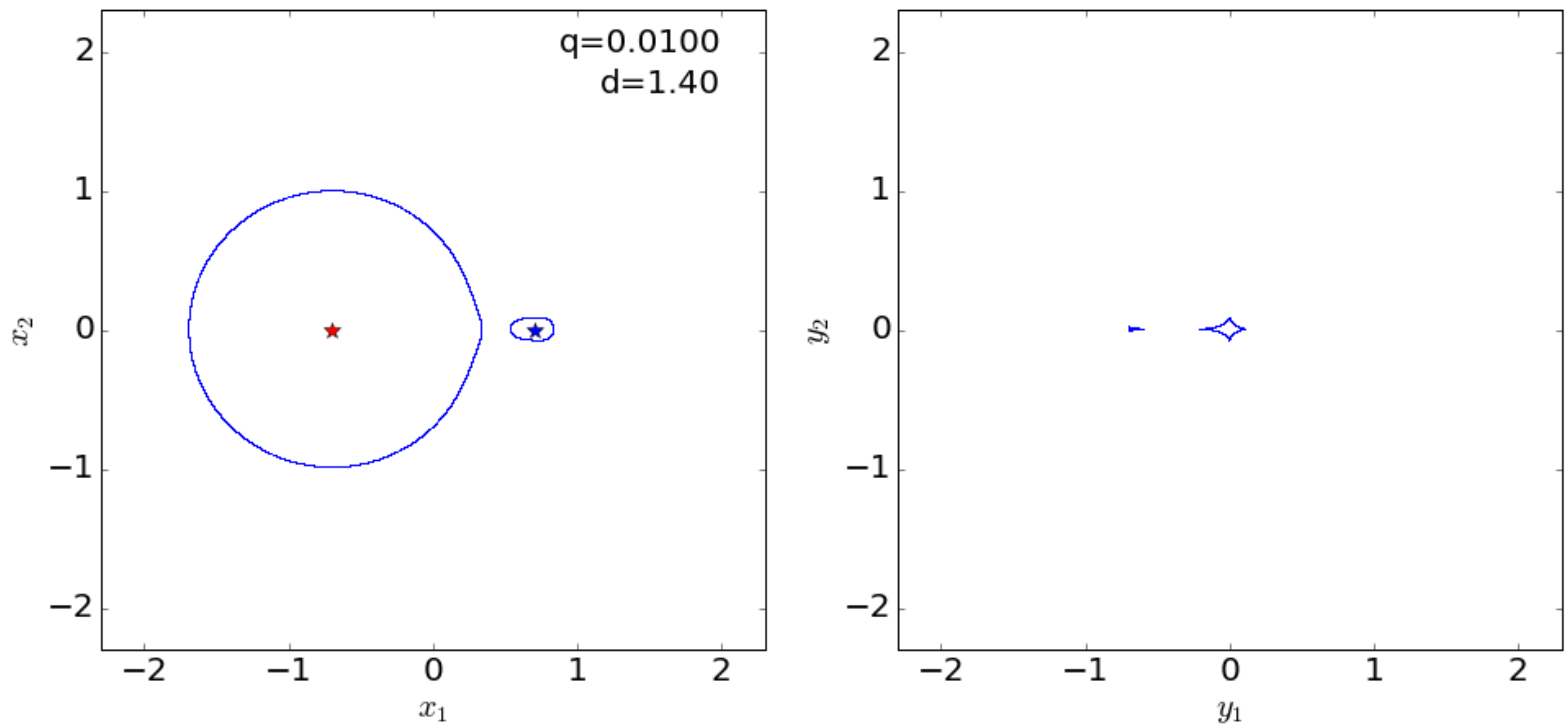

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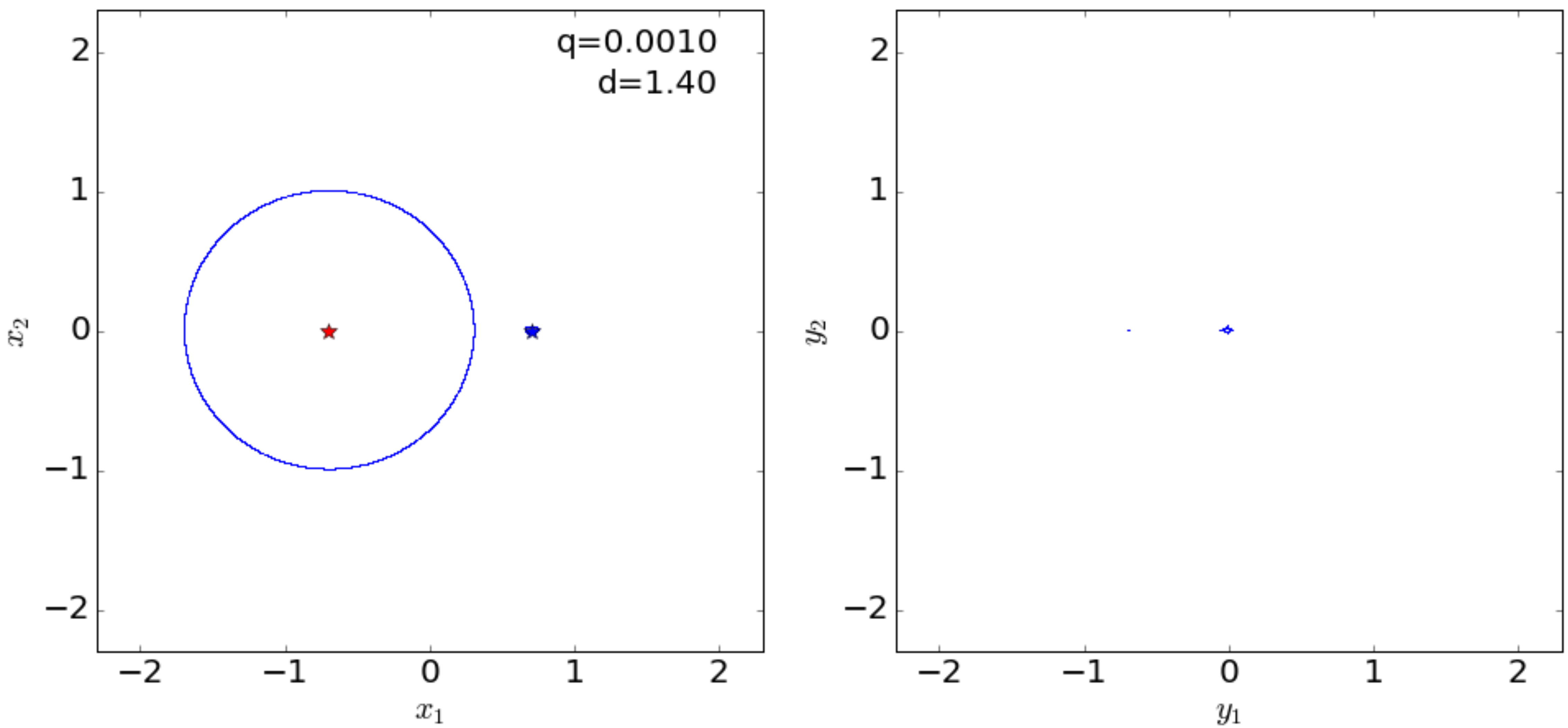

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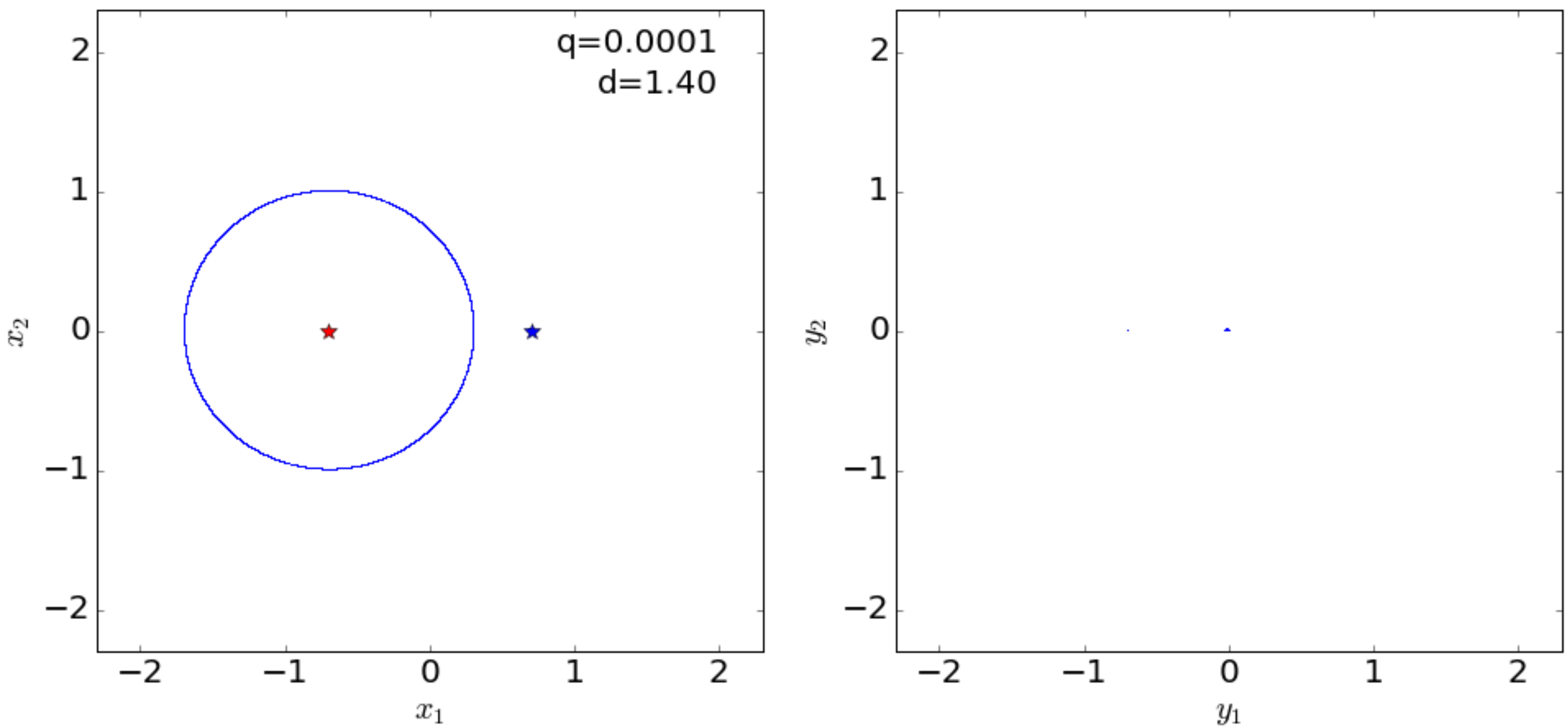

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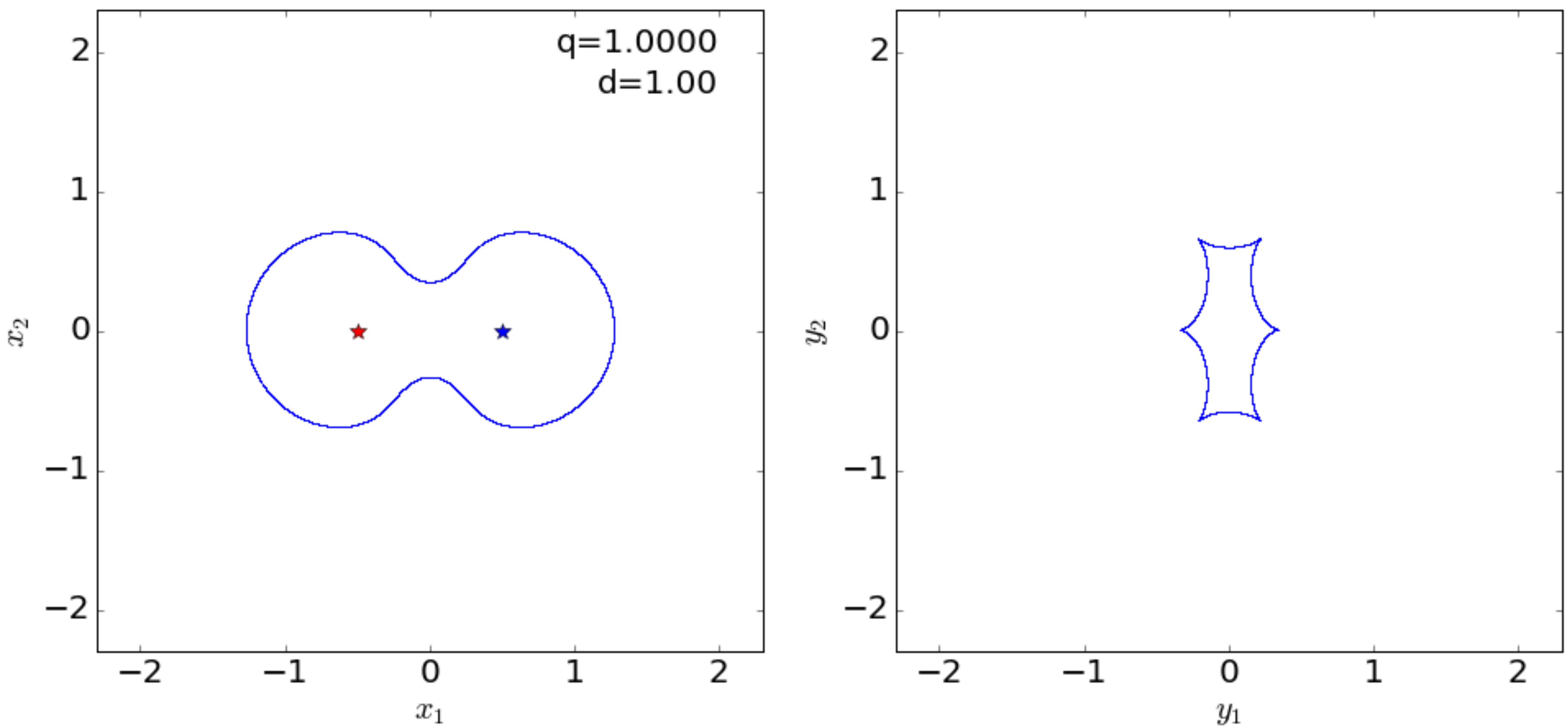

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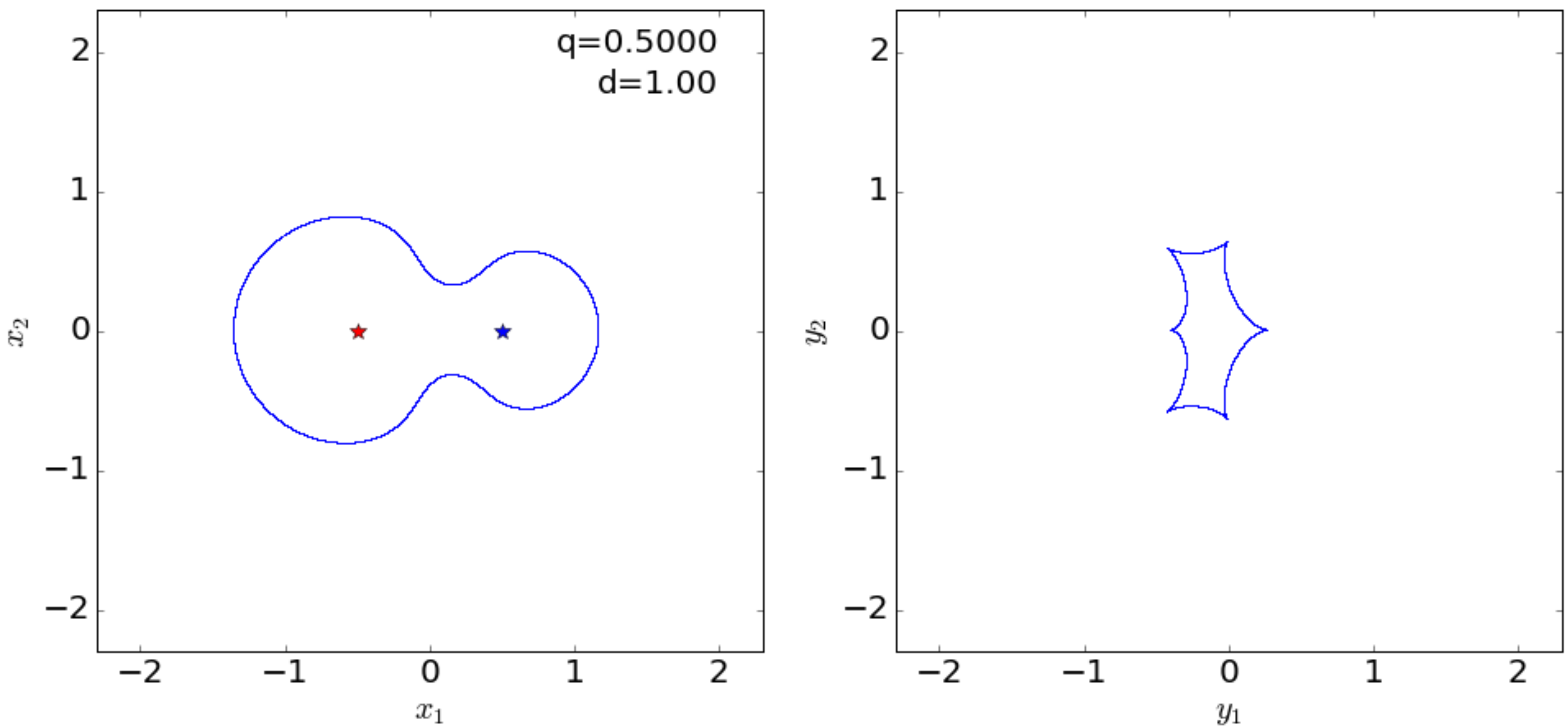

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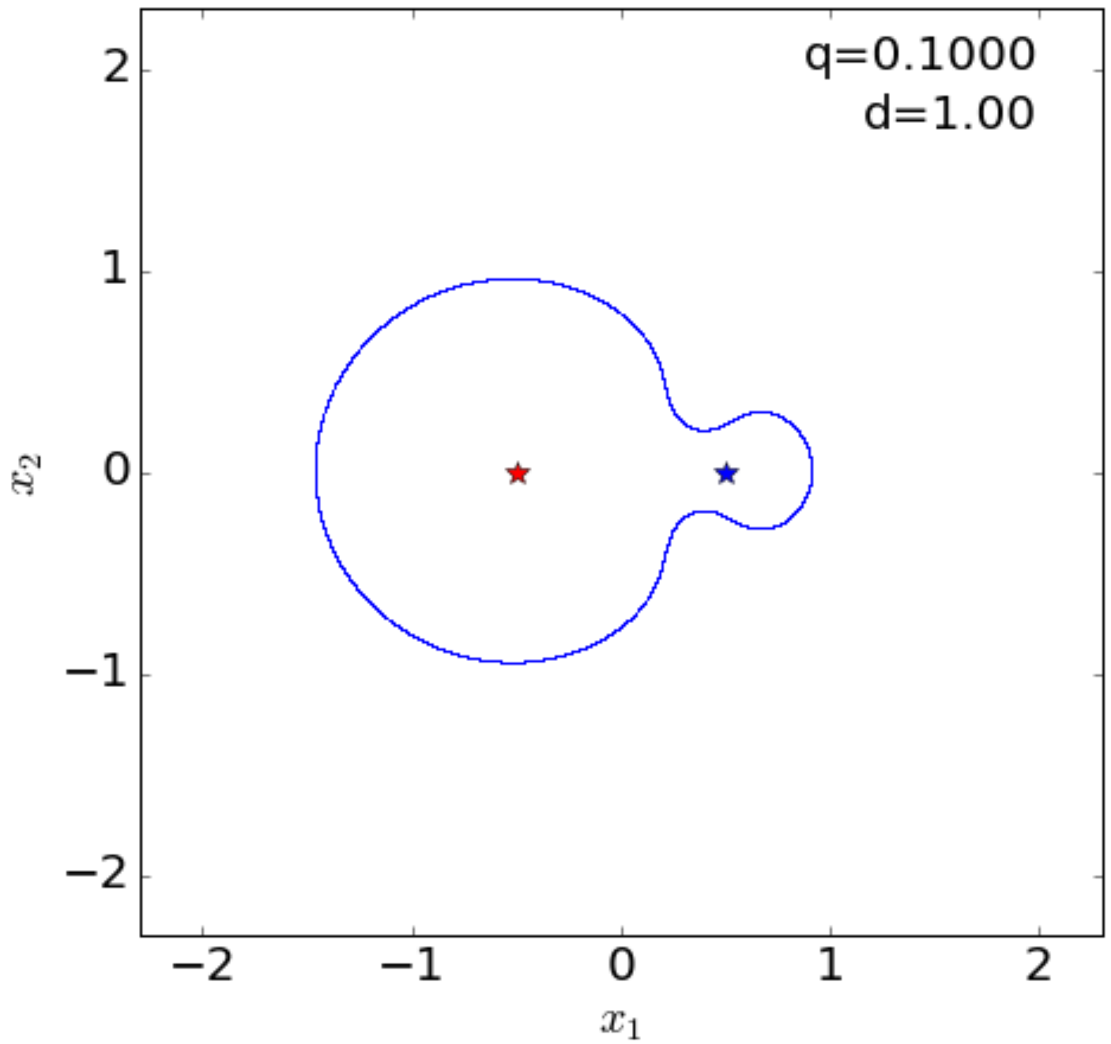

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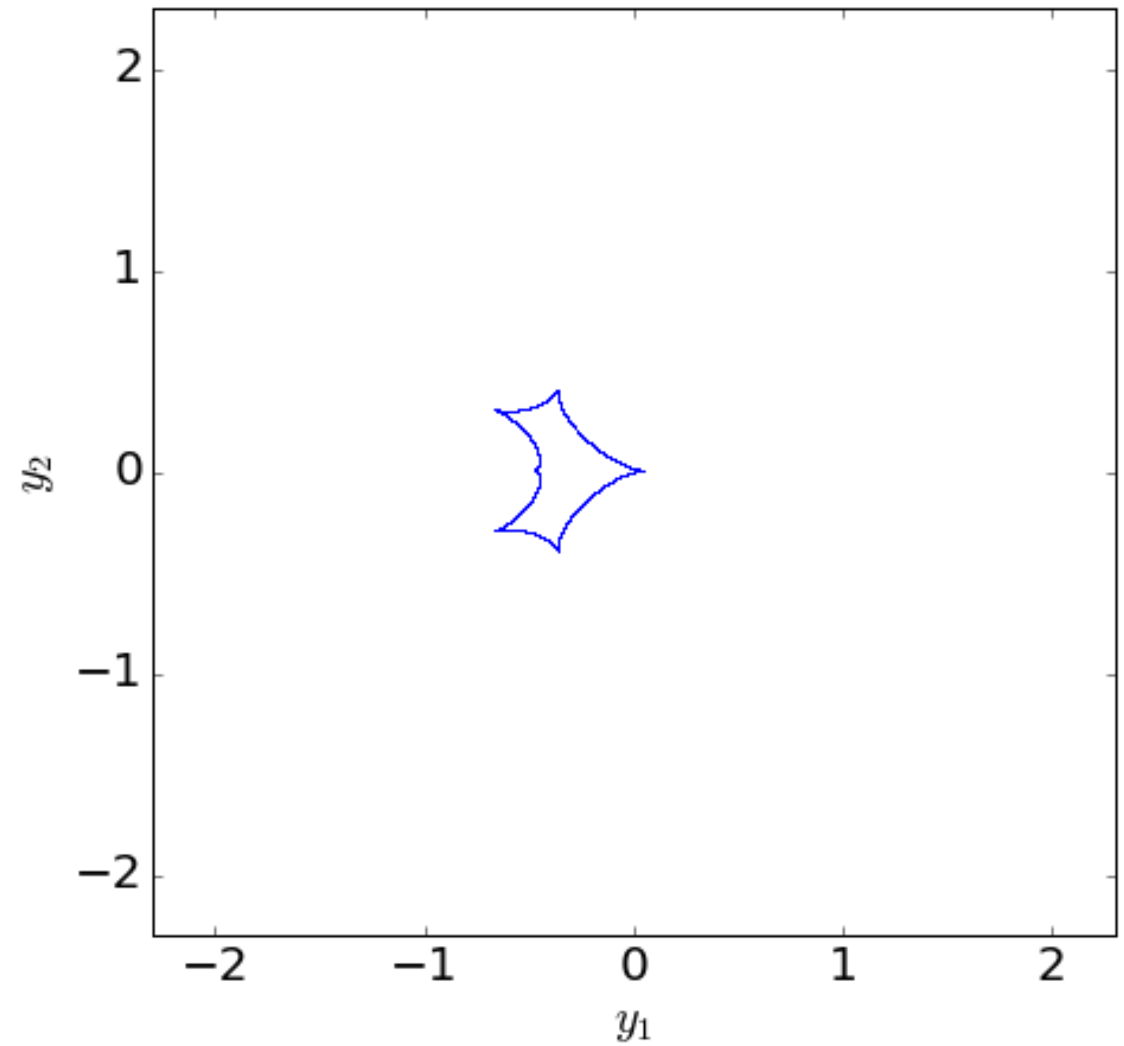

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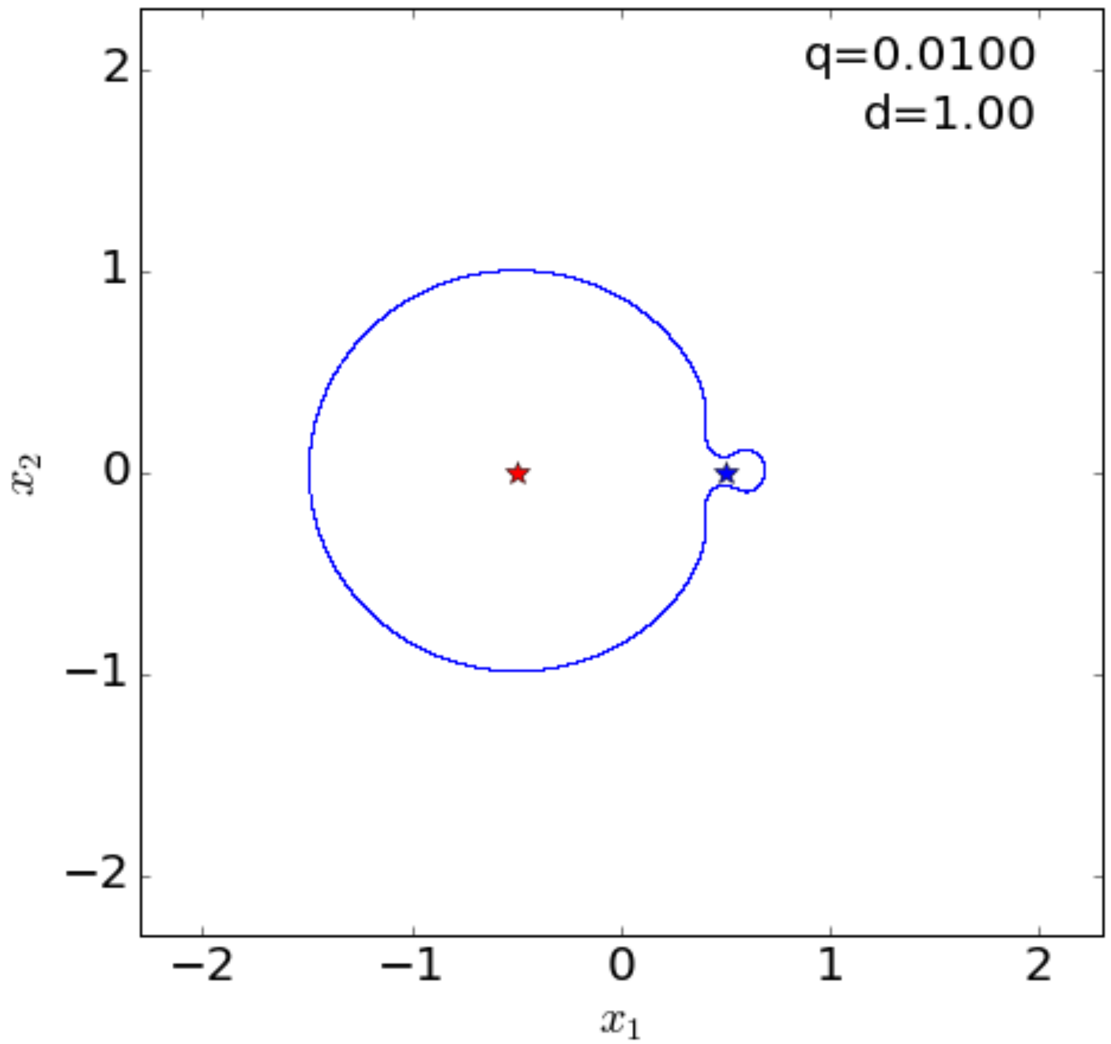

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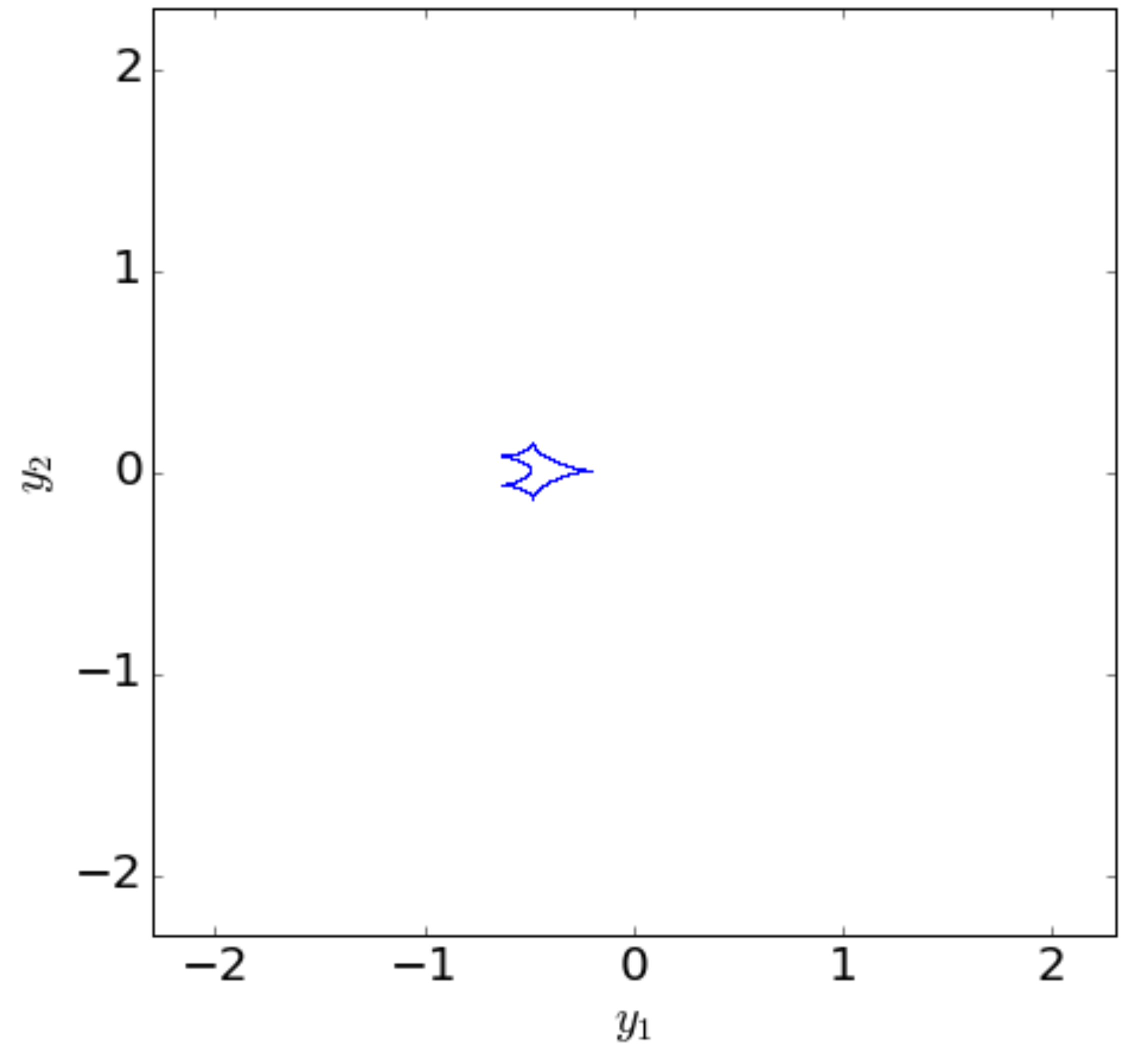

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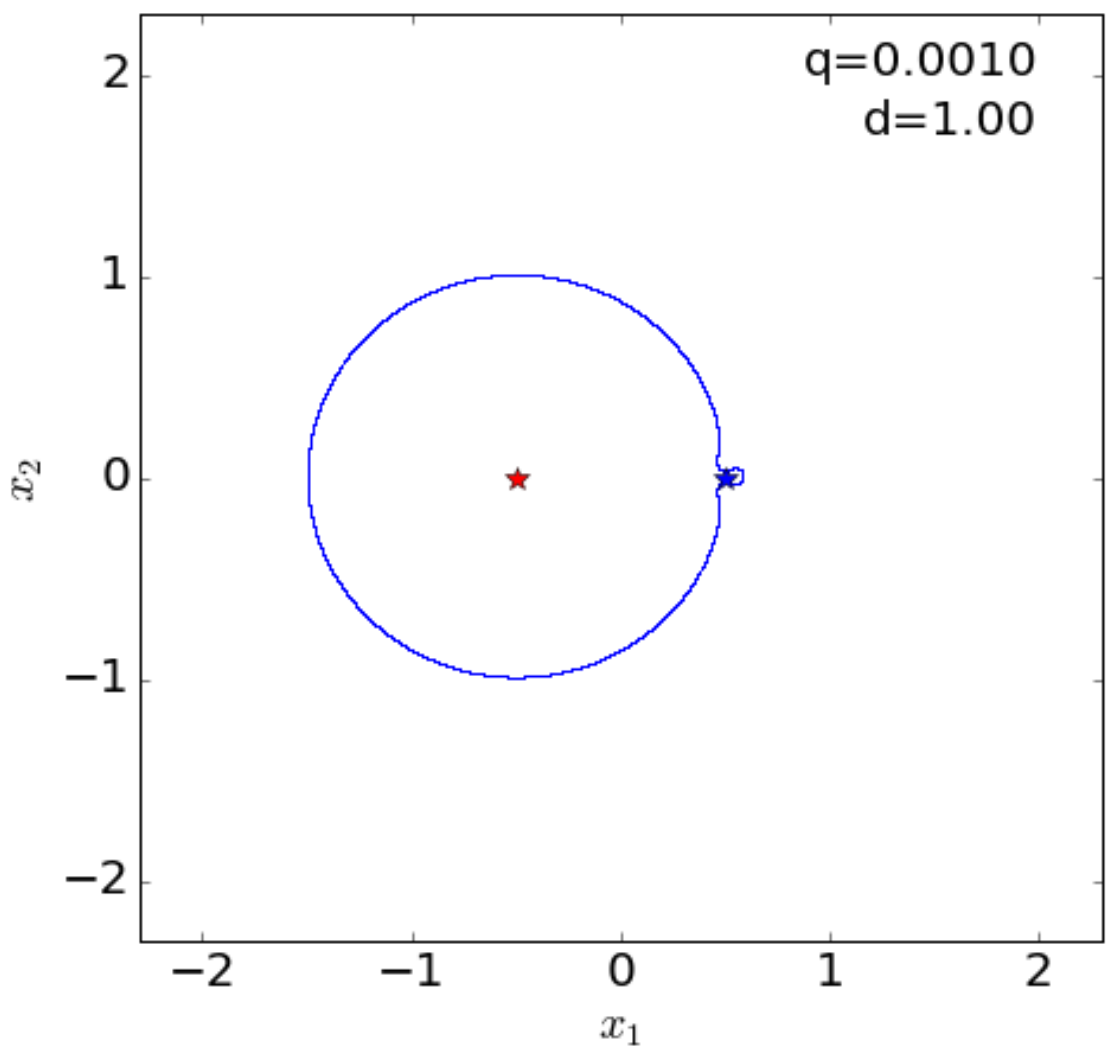

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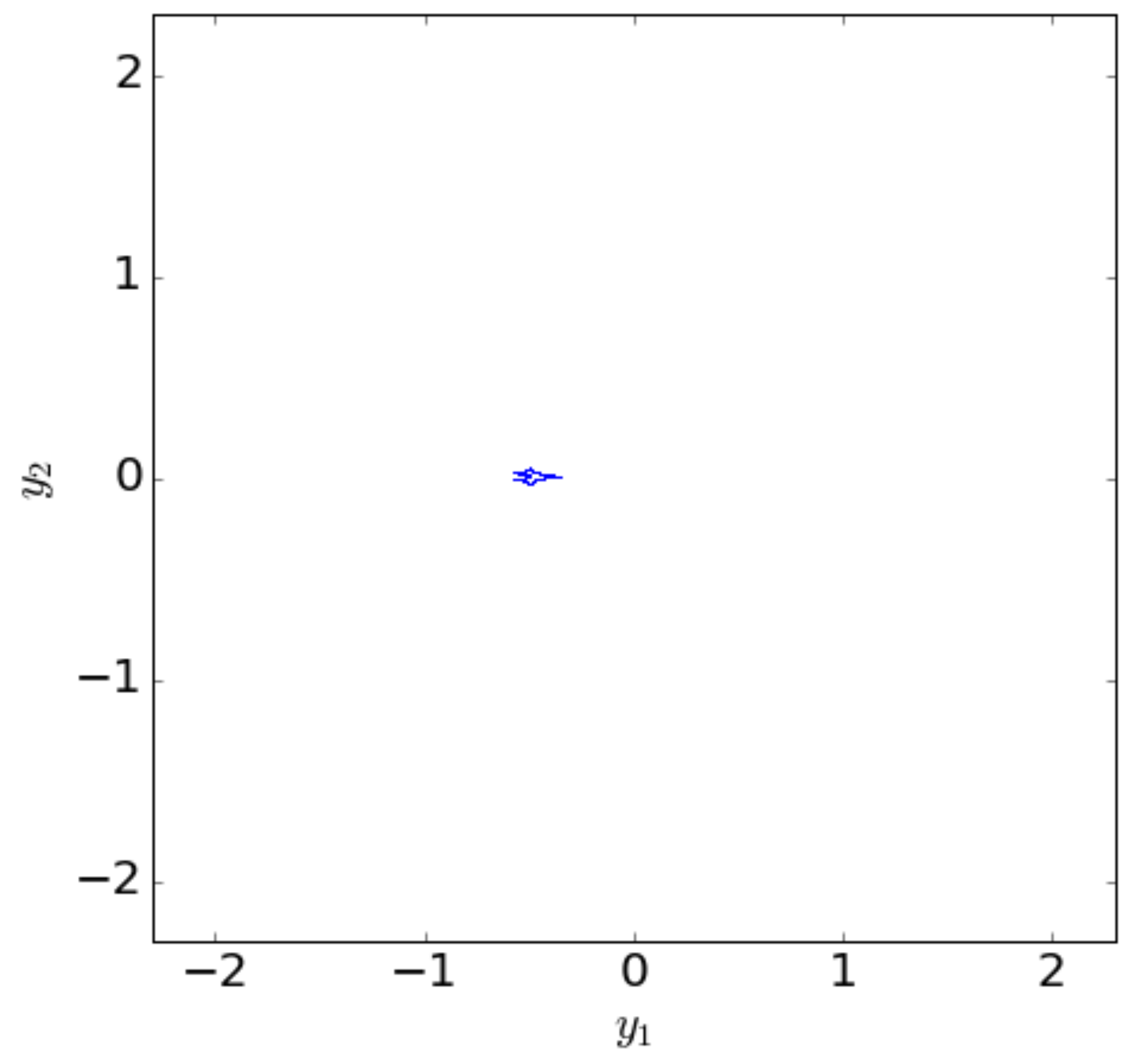

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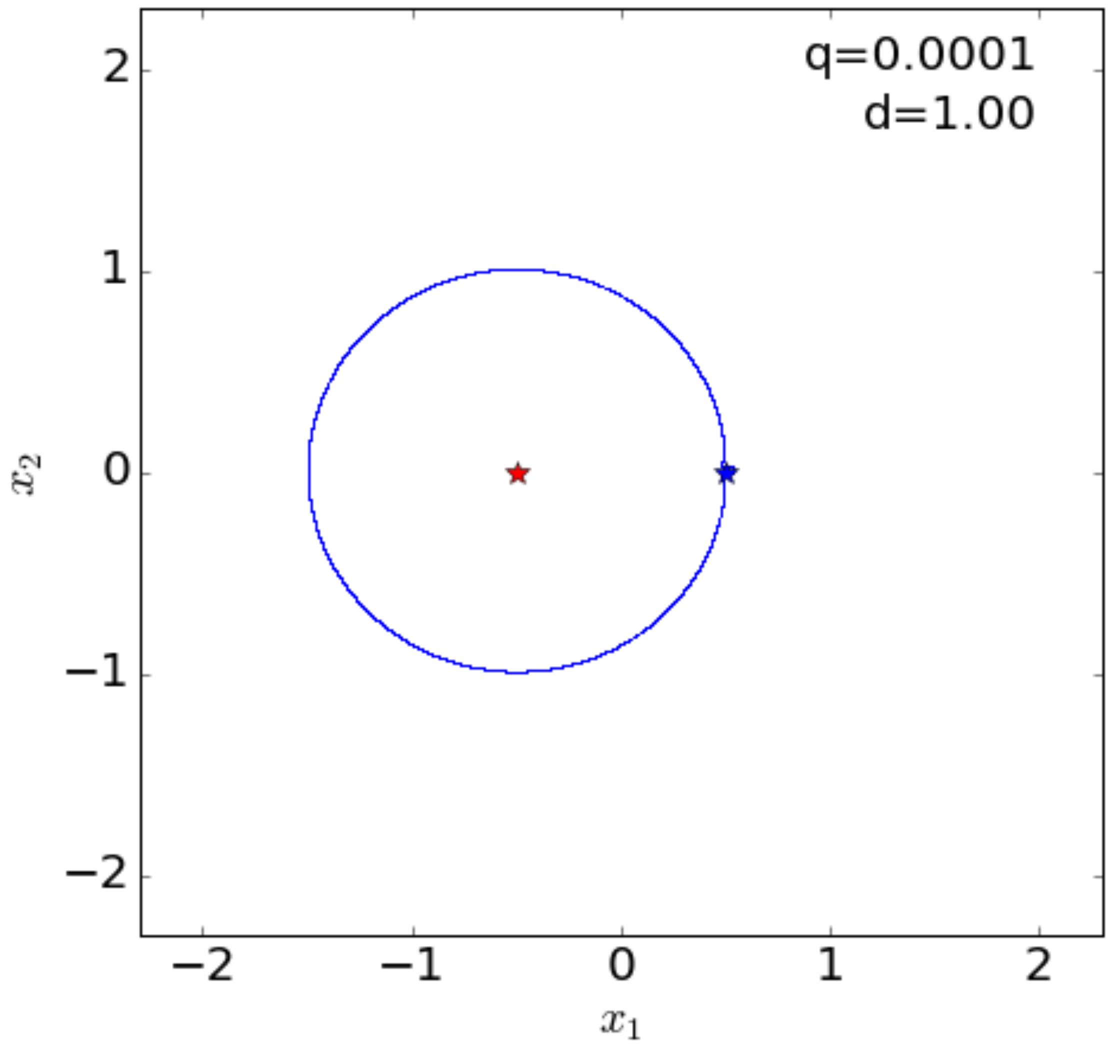

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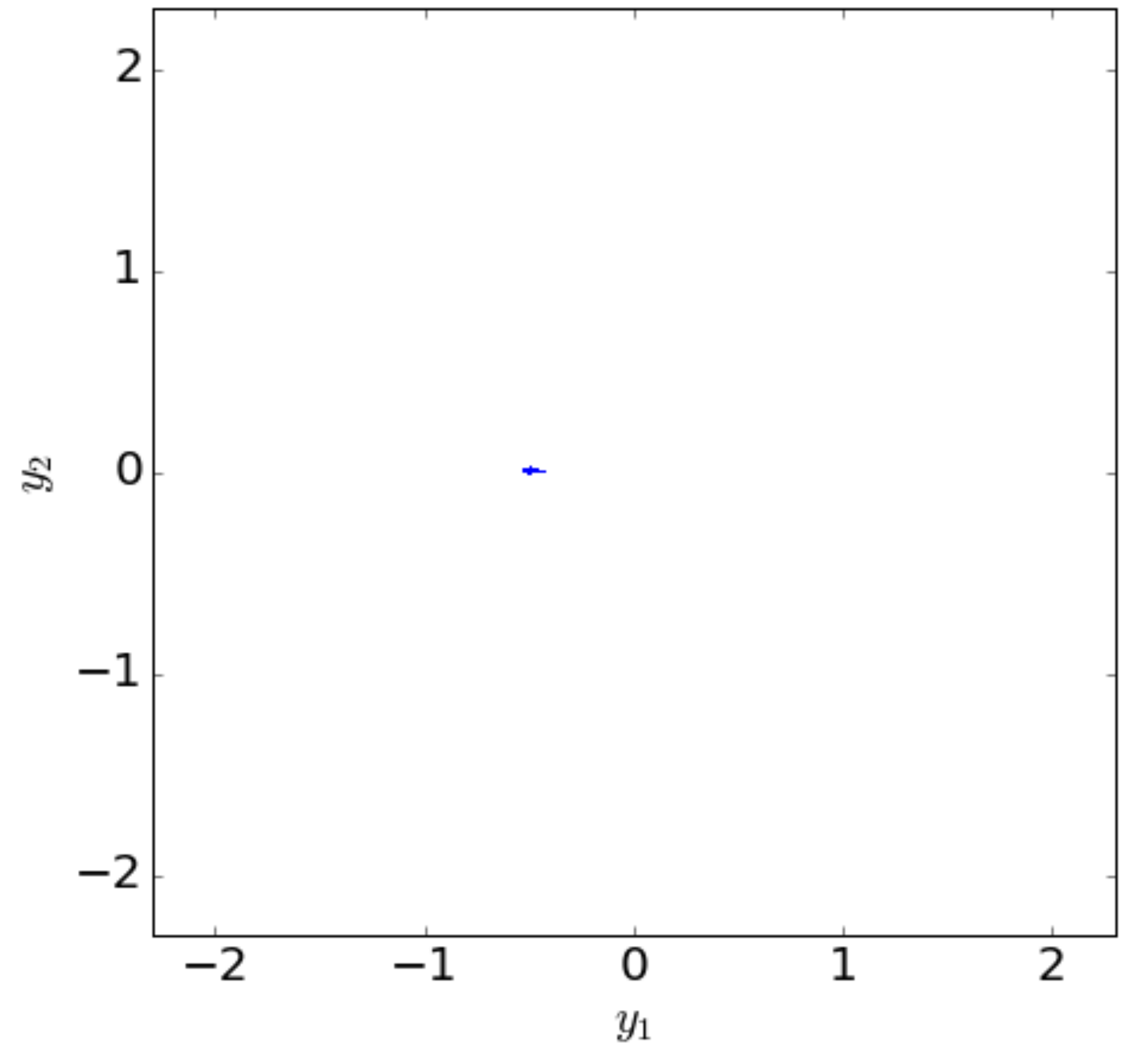

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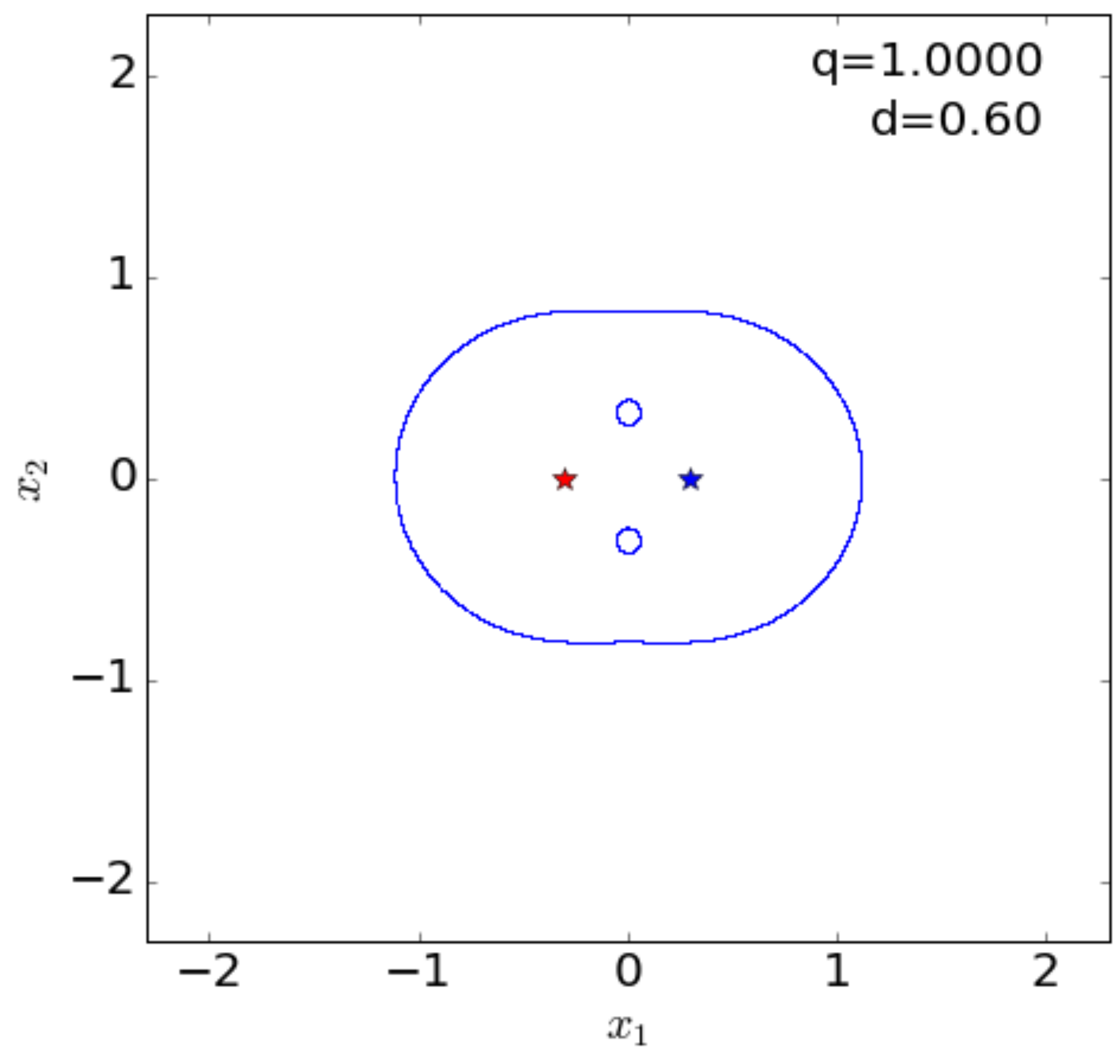

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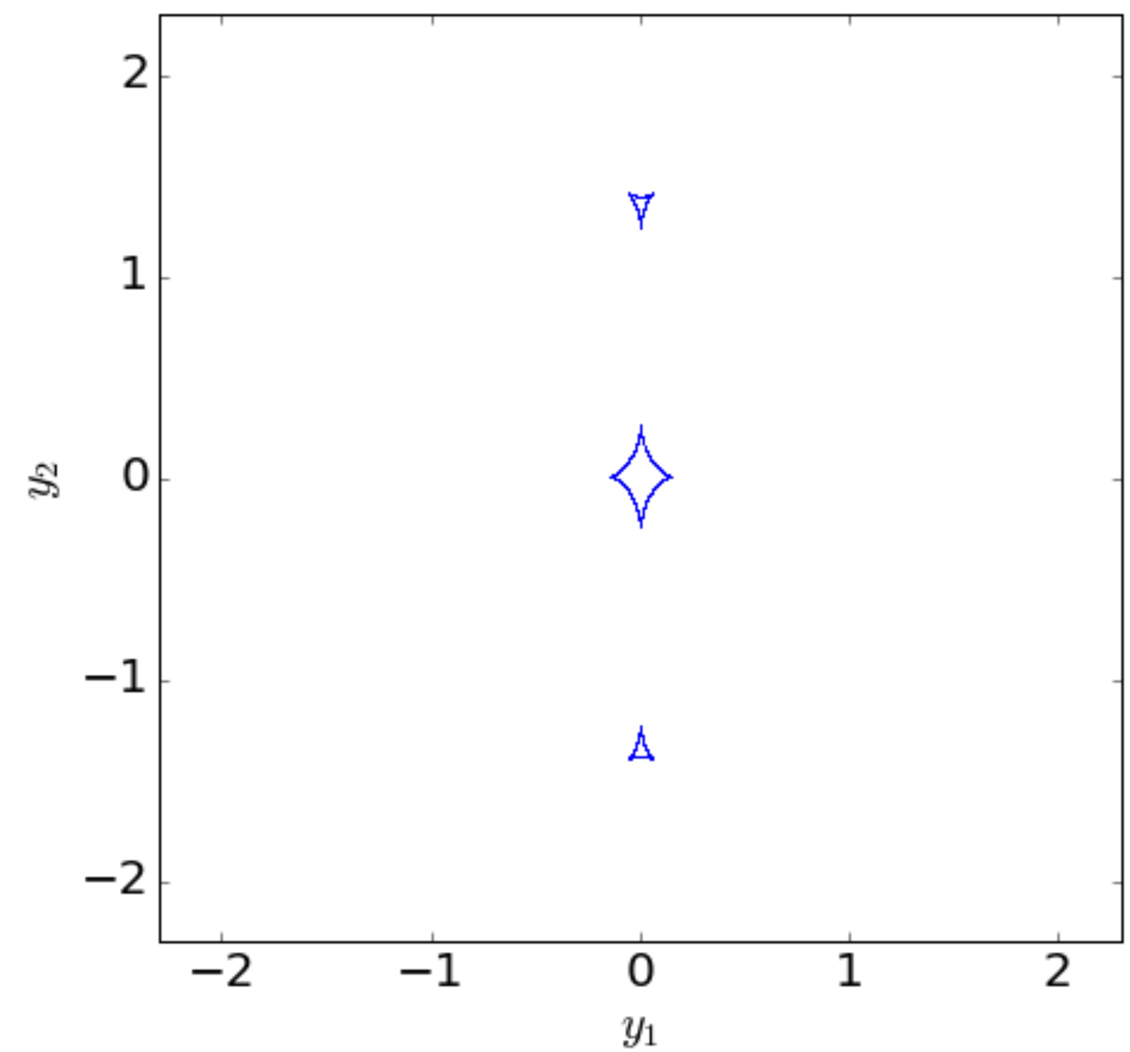

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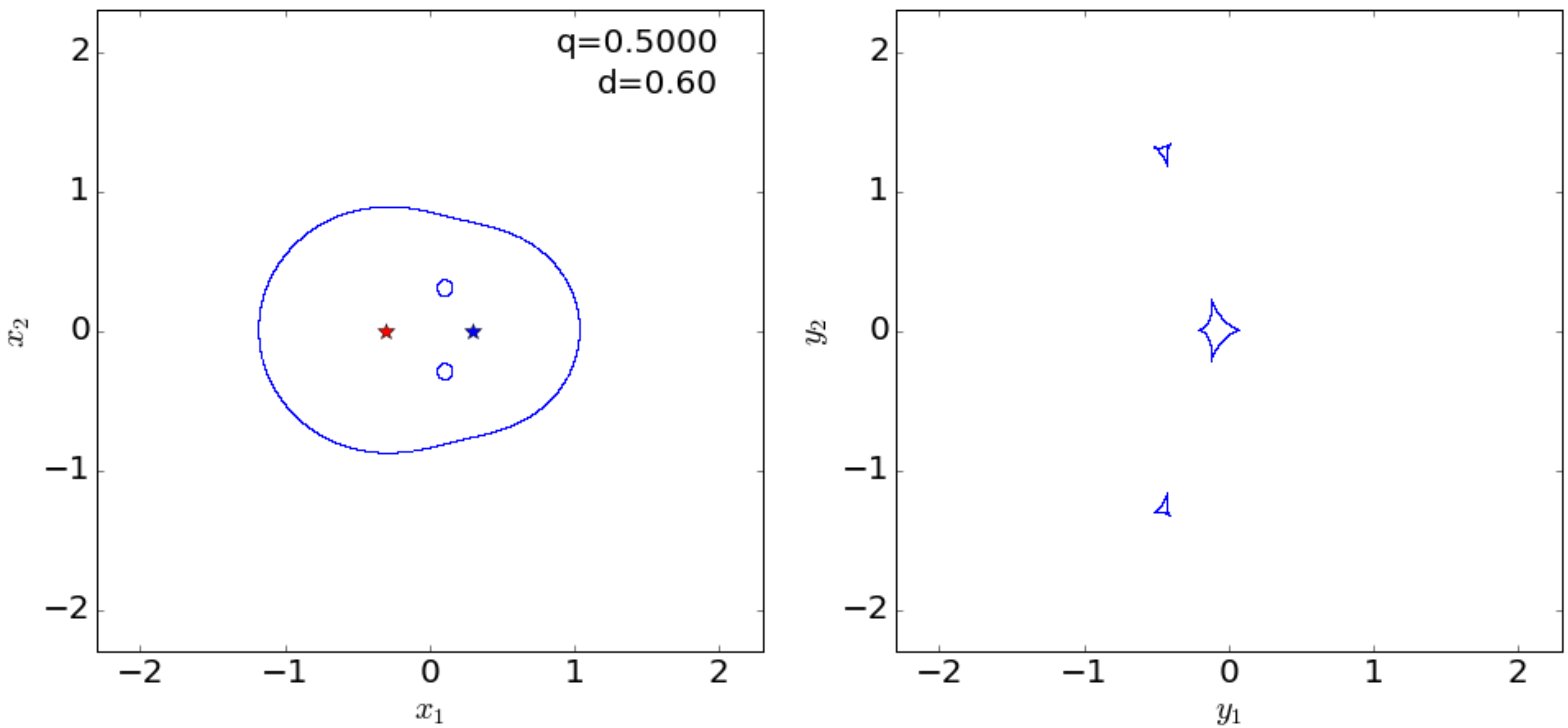

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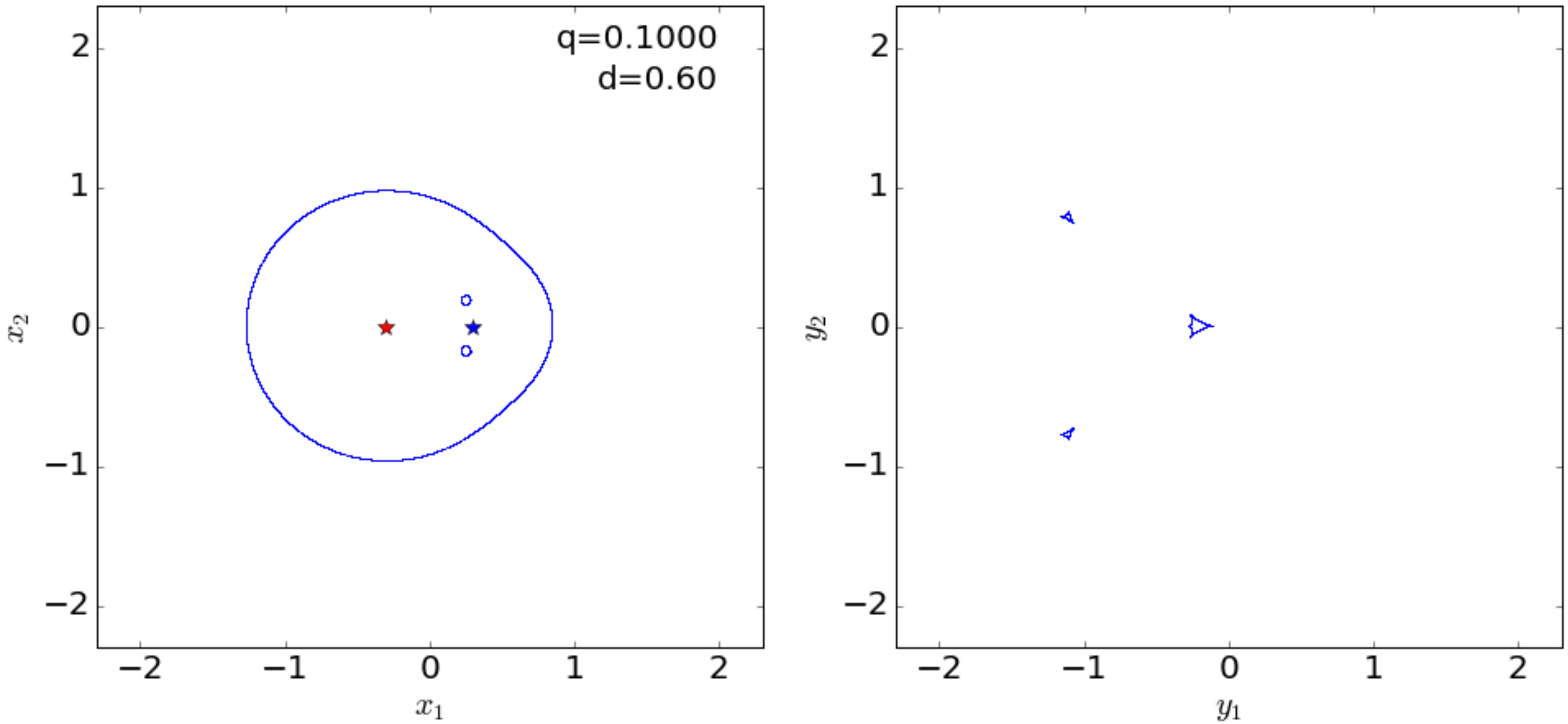

critical lines *caustics*

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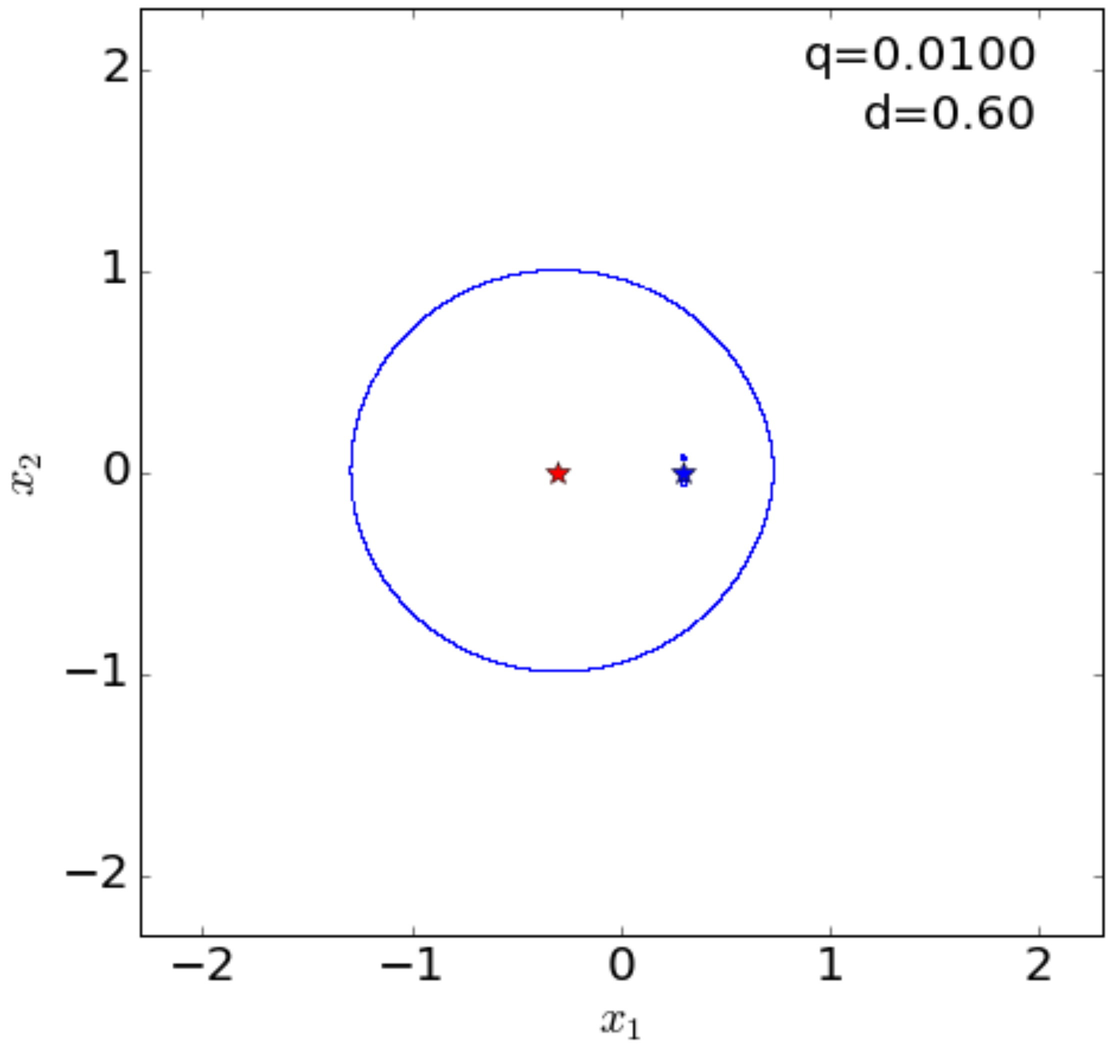

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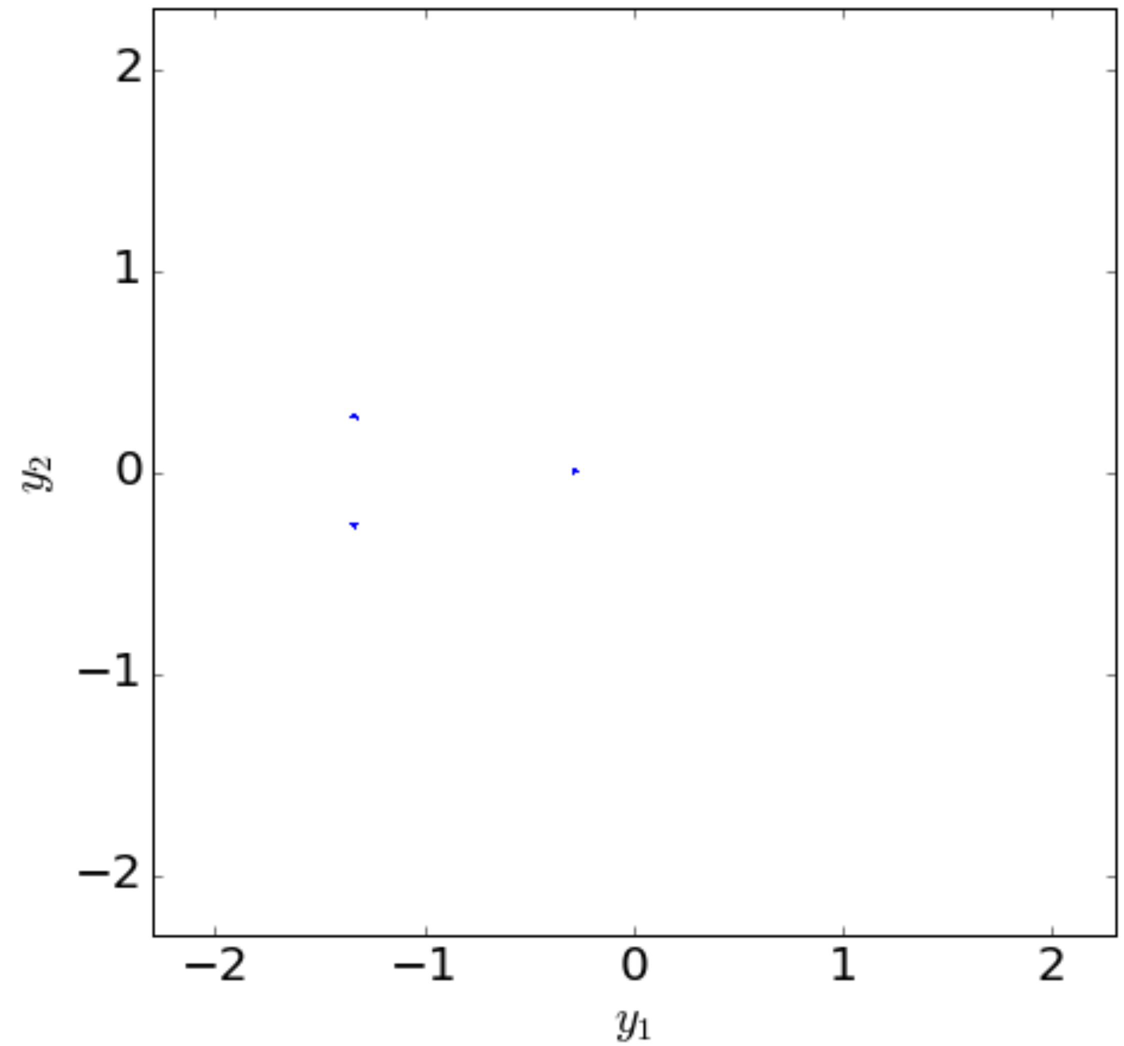

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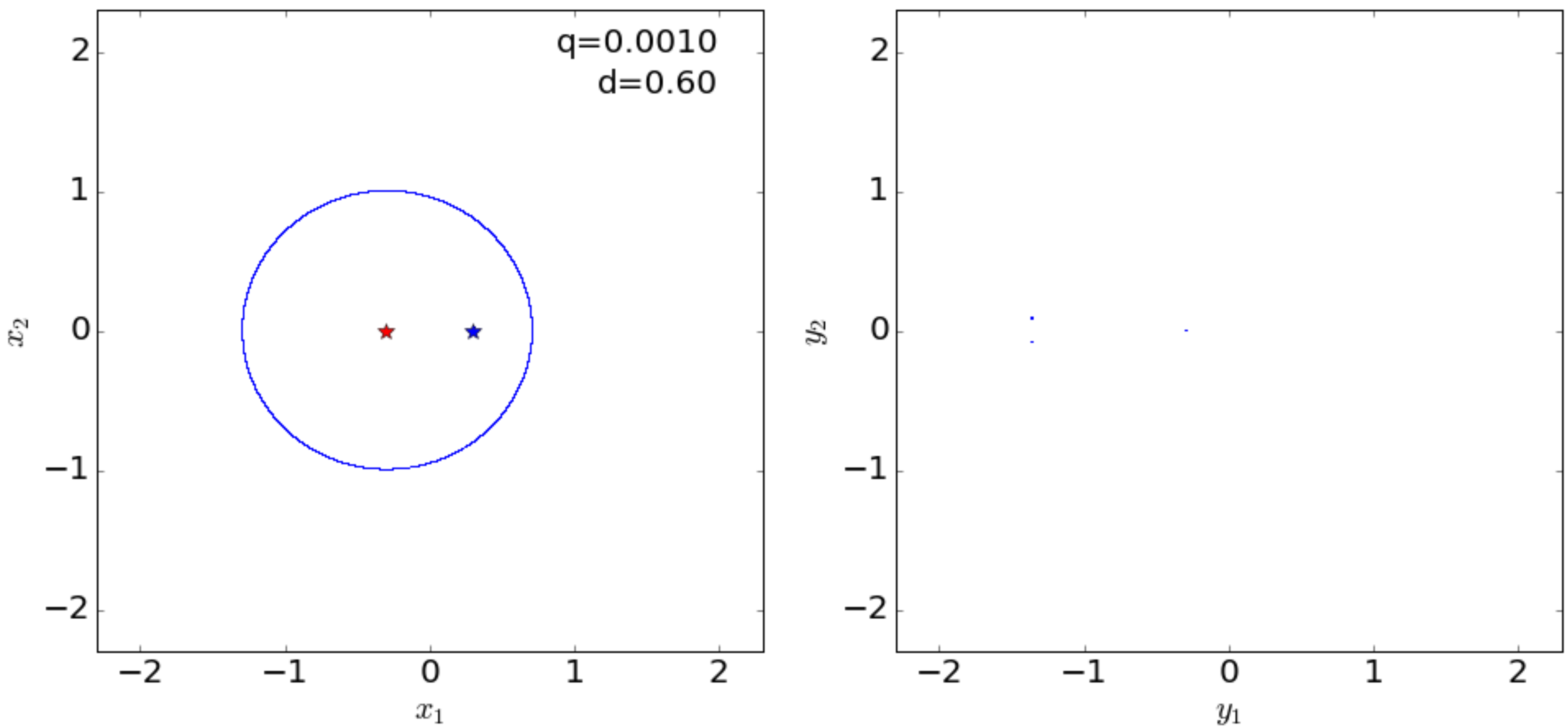

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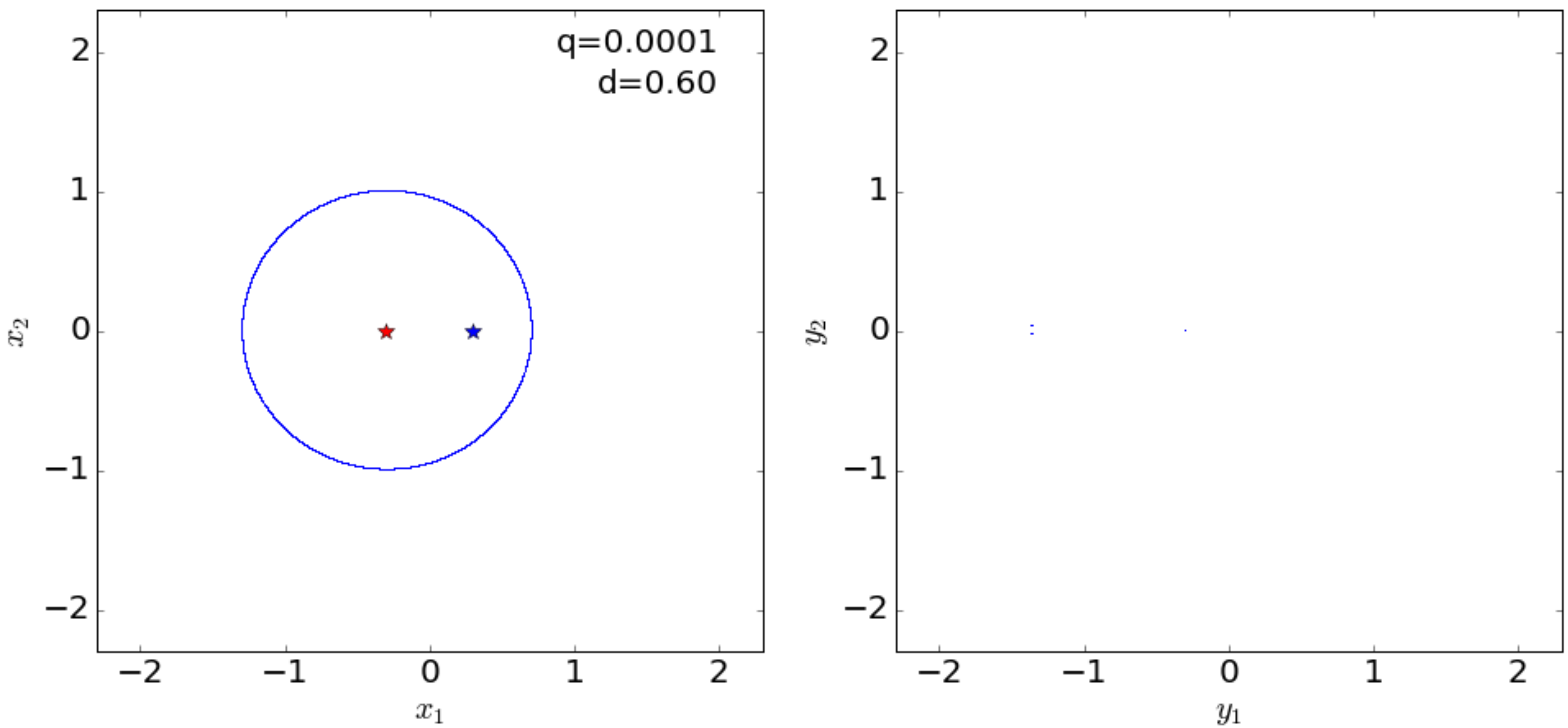
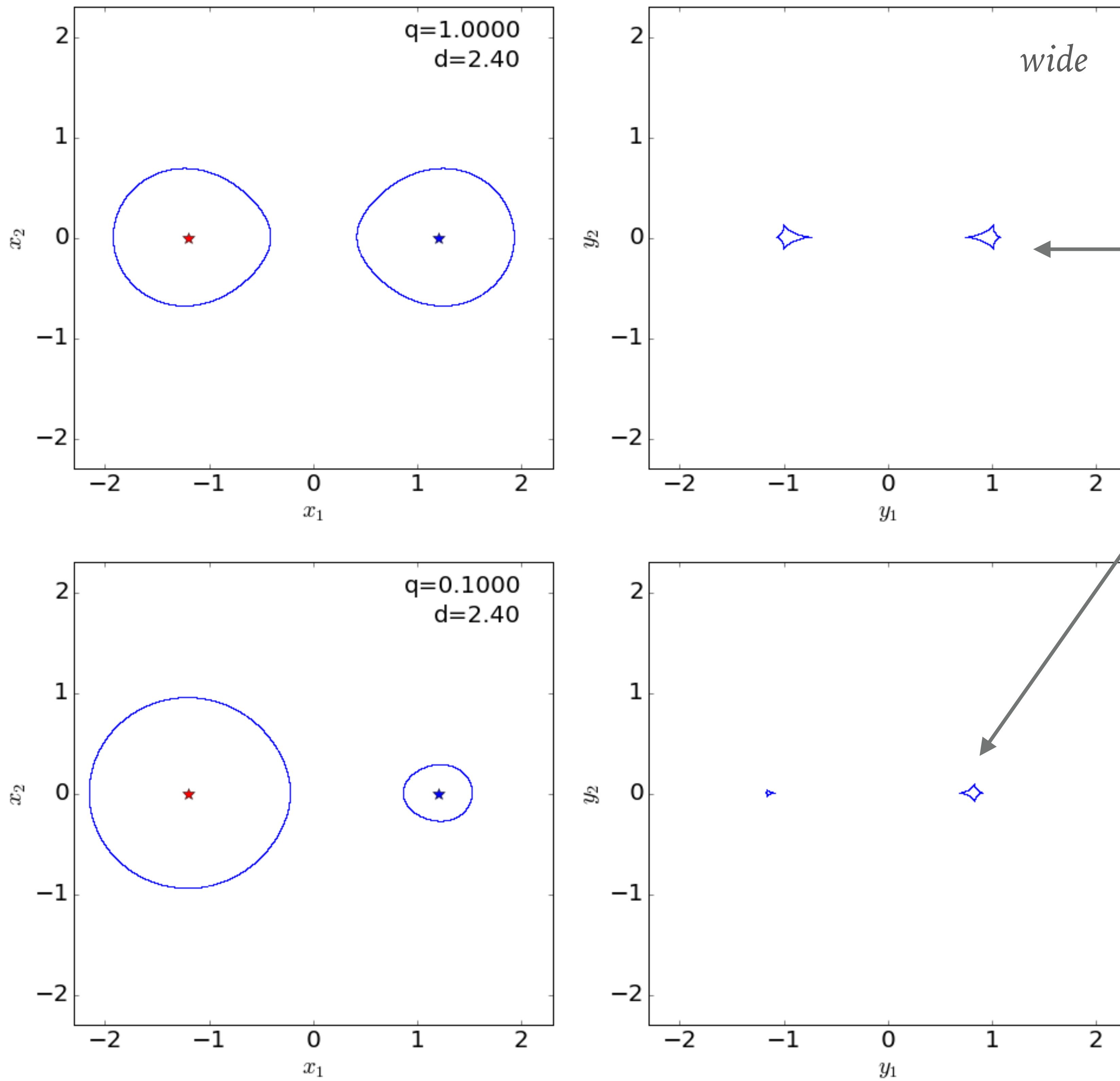


critical lines *caustics*

BINARY LENSES: TOPOLOGY CLASSIFICATION



separate 4-cusp caustics

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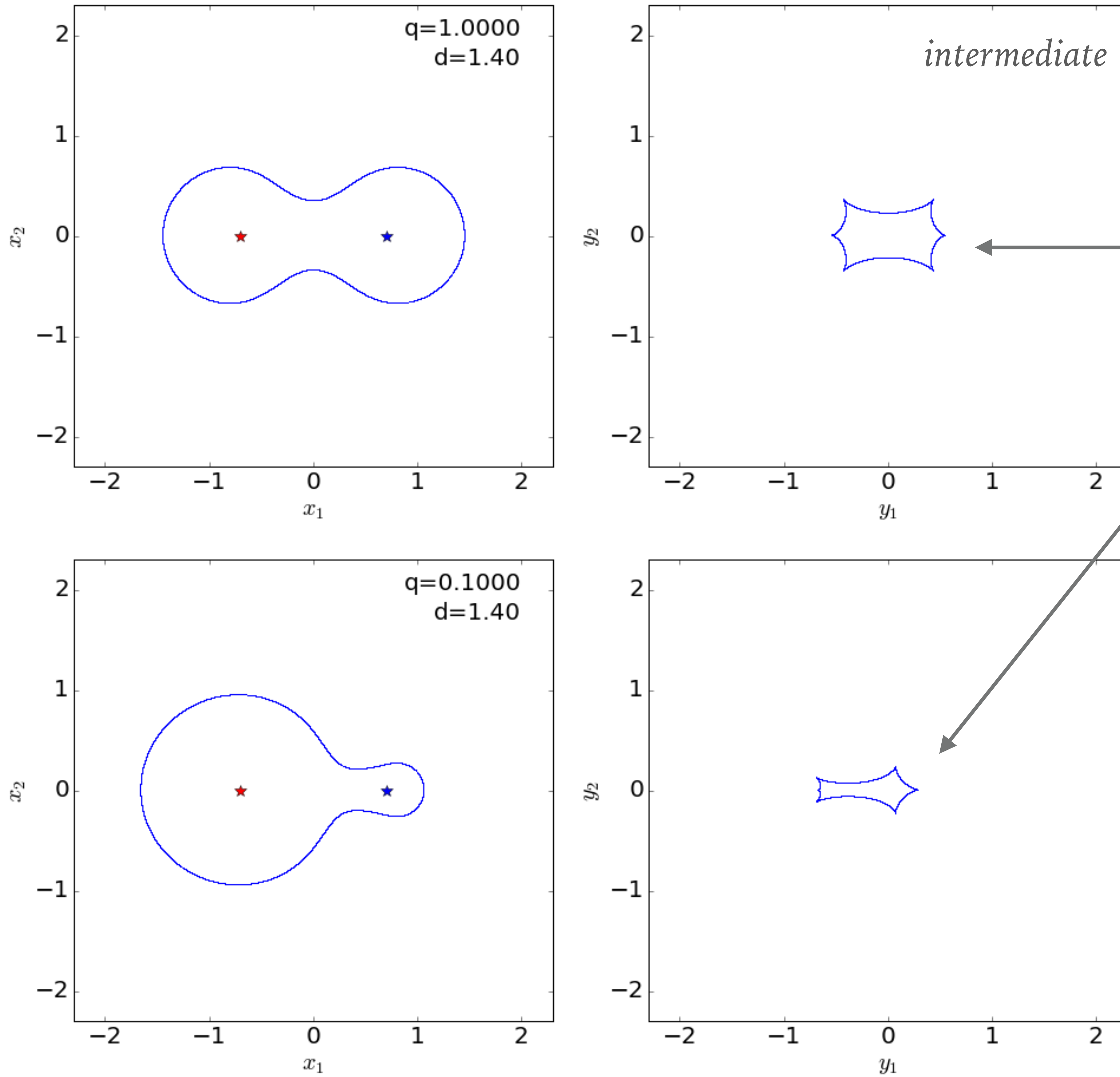

single 6-cusp caustic

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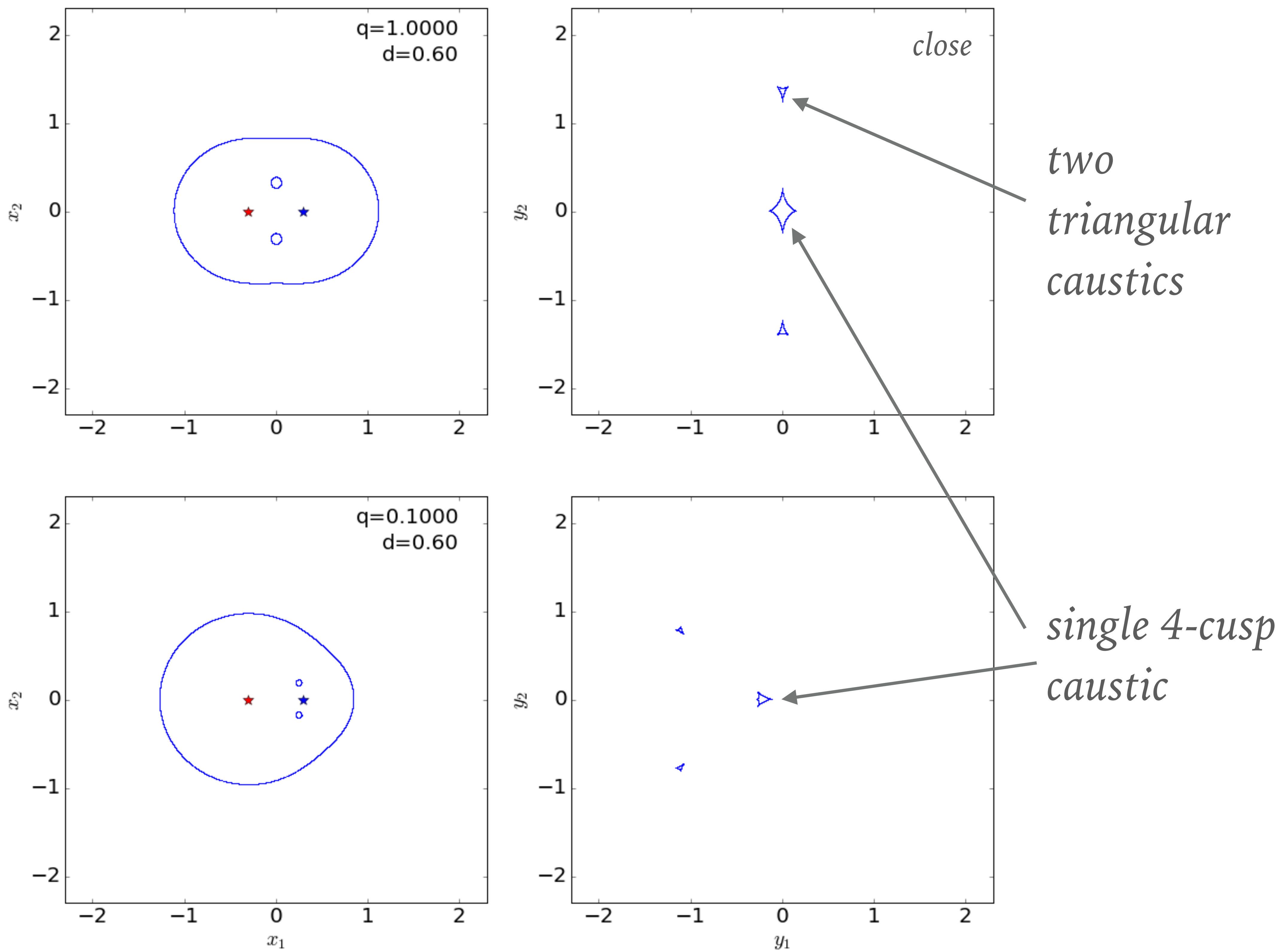
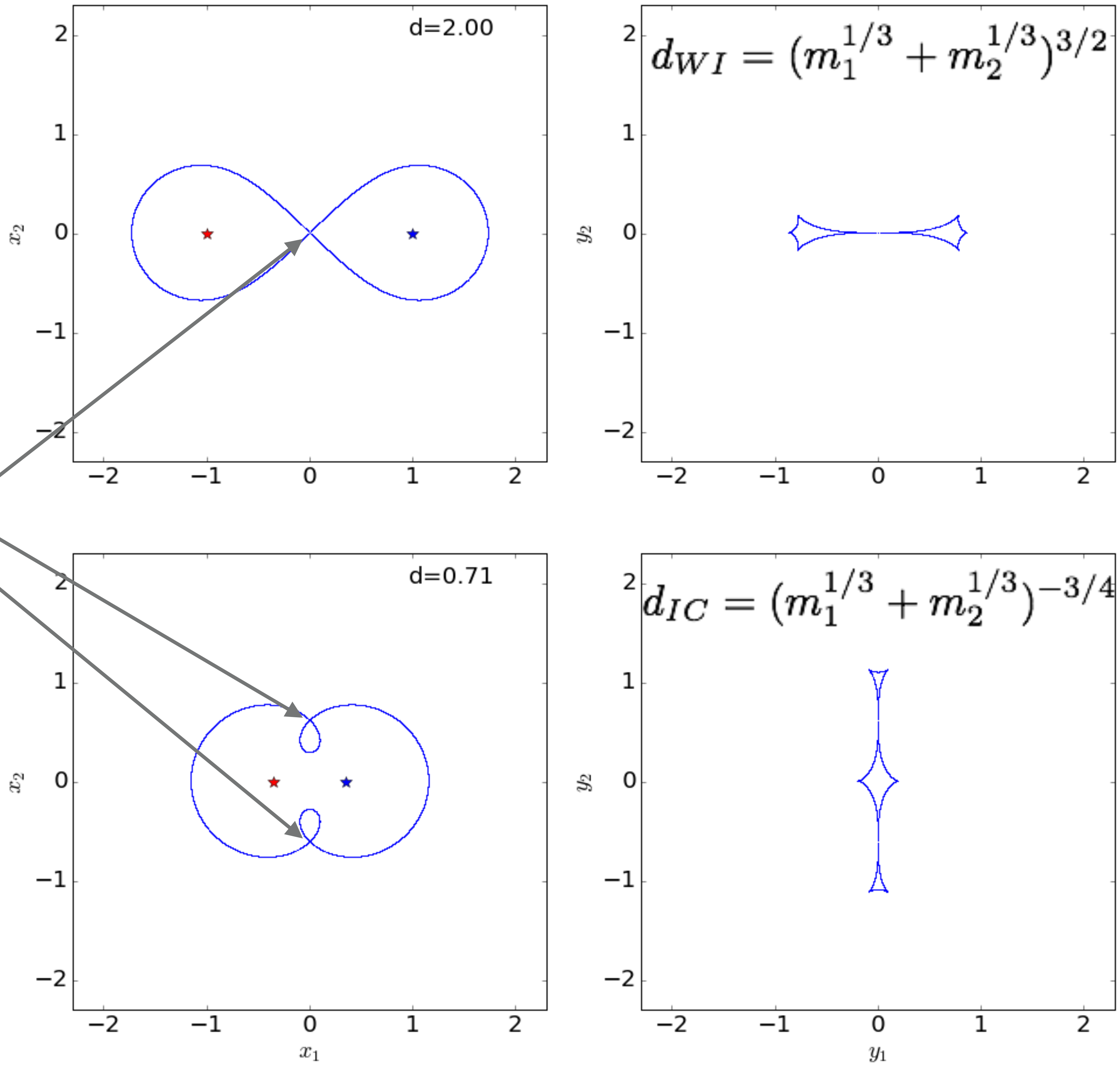


TRANSITIONS

Touching critical lines

$$\det A = 0$$

$$\frac{\partial \det A}{\partial z^*} = 0$$



MULTIPLE IMAGES

- Lens equation: $$z_s = z - \frac{m_1}{z^* - z_1^*} - \frac{m_2}{z^* - z_2^*}$$
- complex polynomial: $$p_5(z) = \sum_{i=0}^{5} c_i z^i$$

$$\Delta m = \frac{m_1 - m_2}{2} \qquad m = \frac{m_1 + m_2}{2} \qquad z_2 = -z_1 \qquad z_1 = z_1^*$$

$$c_0 = z_1^2[4(\Delta m)^2 z_s + 4m\Delta m z_1 + 4\Delta m z_s z_s^* z_1 + 2m z_s^* z_1^2 + z_s z_s^{*2} z_1^2 - 2\Delta m z_1^3 - z_s z_1^4]$$

$$c_1 = -8m\Delta m z_s z_1 - 4(\Delta m)^2 z_1^2 - 4m^2 z_1^2 - 4m z_s z_s^* z_1^2 - 4\Delta m z_s^* z_1^3 - z_s^{*2} z_1^4 + z_1^6$$

$$c_2 = 4m^2 z_s + 4m\Delta m z_1 - 4\Delta m z_s z_s^* z_1 - 2z_s z_s^{*2} z_1^2 + 4\Delta m z_1^3 + 2z_s z_1^4$$

$$c_3 = 4m z_s z_s^* + 4\Delta m z_s^* z_1 + 2z_s^{*2} z_1^2 - 2z_1^4$$

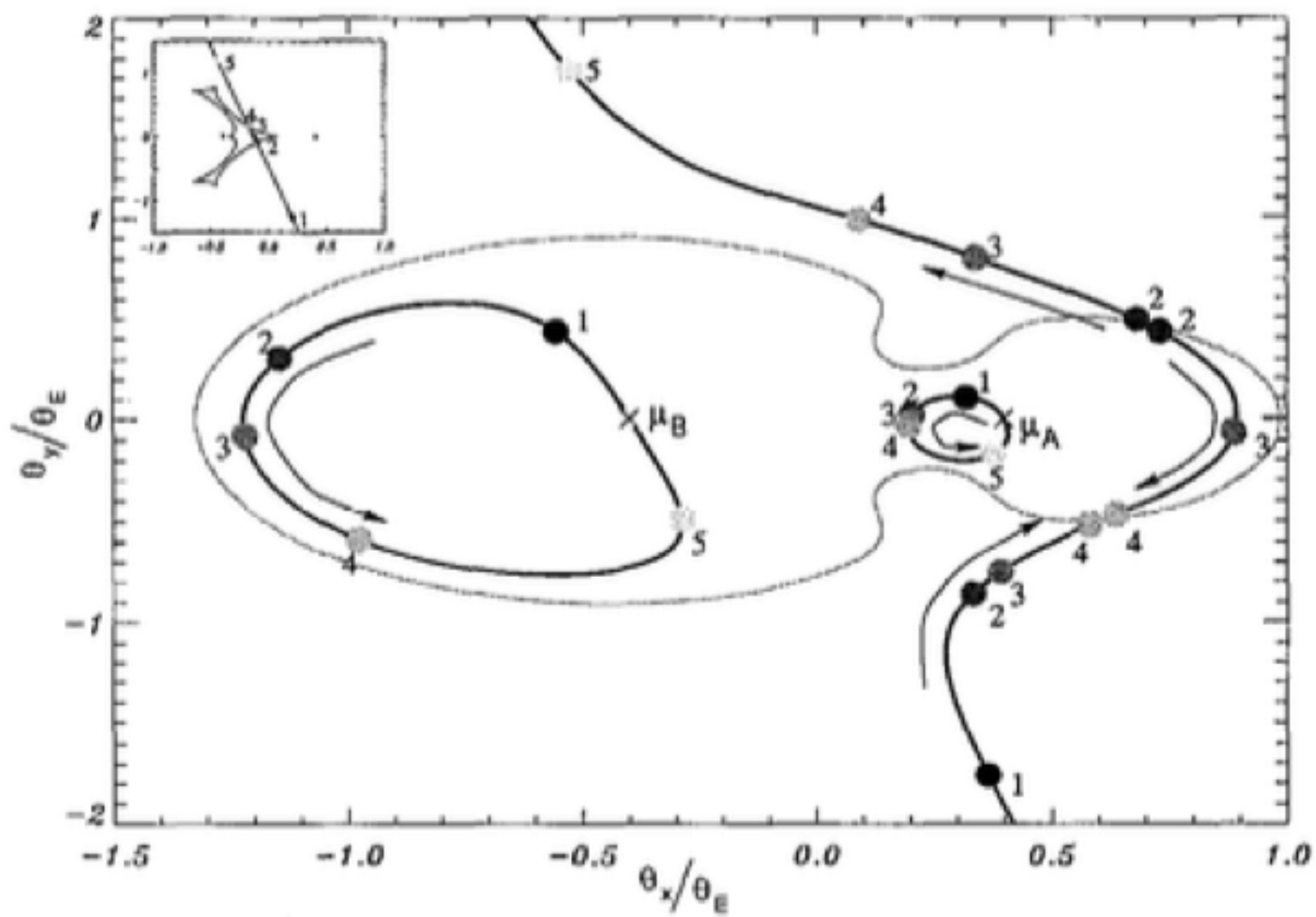
$$c_4 = -2m z_s^* + z_s z_s^{*2} - 2\Delta m z_1 - z_s z_1^2$$

$$c_5 = z_1^2 - z_s^{*2}$$

Witt & Mao, 1995, ApJ, 447, L105

- **3 or 5 images**

MULTIPLE IMAGES



Mollerach & Roulet, "Gravitational Lensing and Microlensing"

IMAGE MAGNIFICATION

- magnification at the image position:

$$\mu = \det A^{-1} = \left[1 - \left|\frac{m_1}{(z^* - z_1^*)^2} + \frac{m_2}{(z^* - z_2^*)^2}\right|\right]^{-1}$$

- total magnification:

$$\mu_{tot} = \sum_{i=1}^{n_i} |\mu_i|$$

- of course, the magnification varies as a function of *z*…