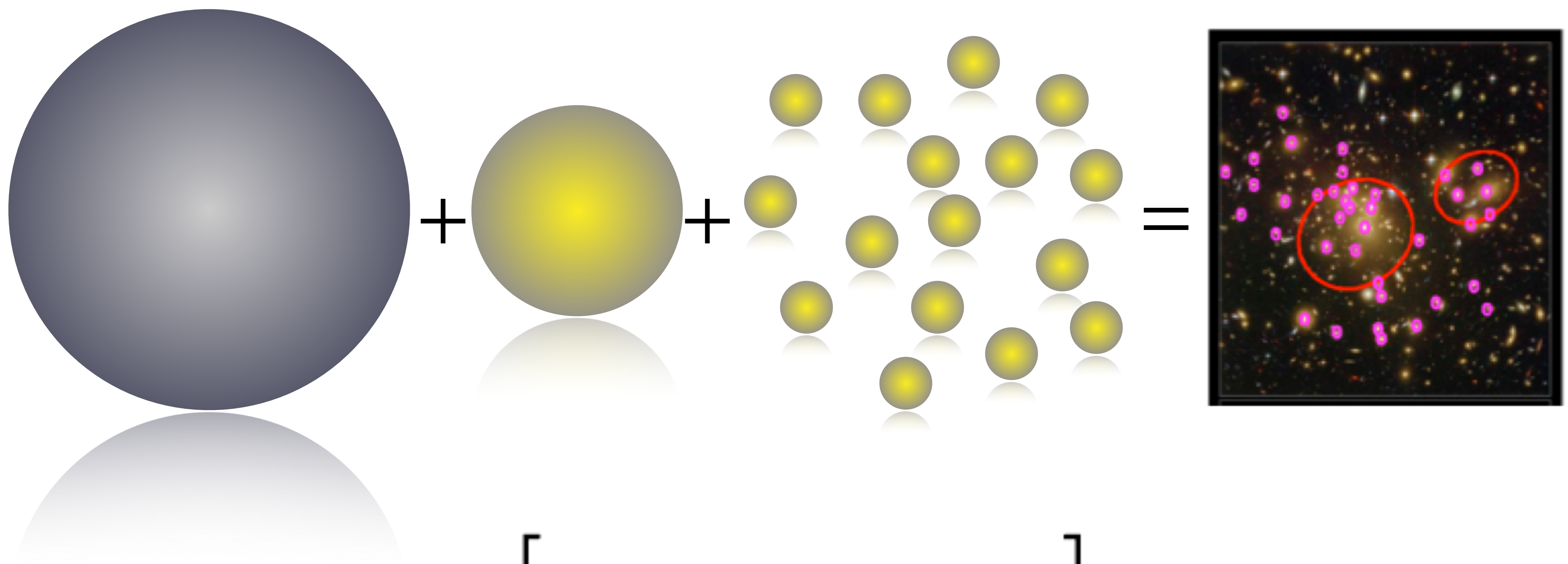
GRAVITATIONAL LENSING

LECTURE 23

Docente: Massimo Meneghetti

AA 2015-2016

THE MODEL



$$\mathbf{p} = [\mathbf{q}, \mathbf{m}, \mathbf{s}, \mathbf{x_c}]$$

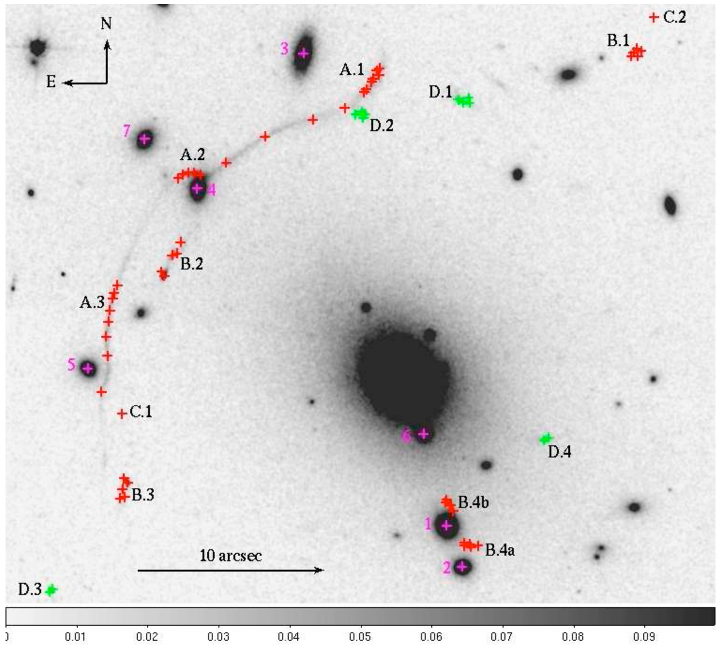
OBSERVABLES

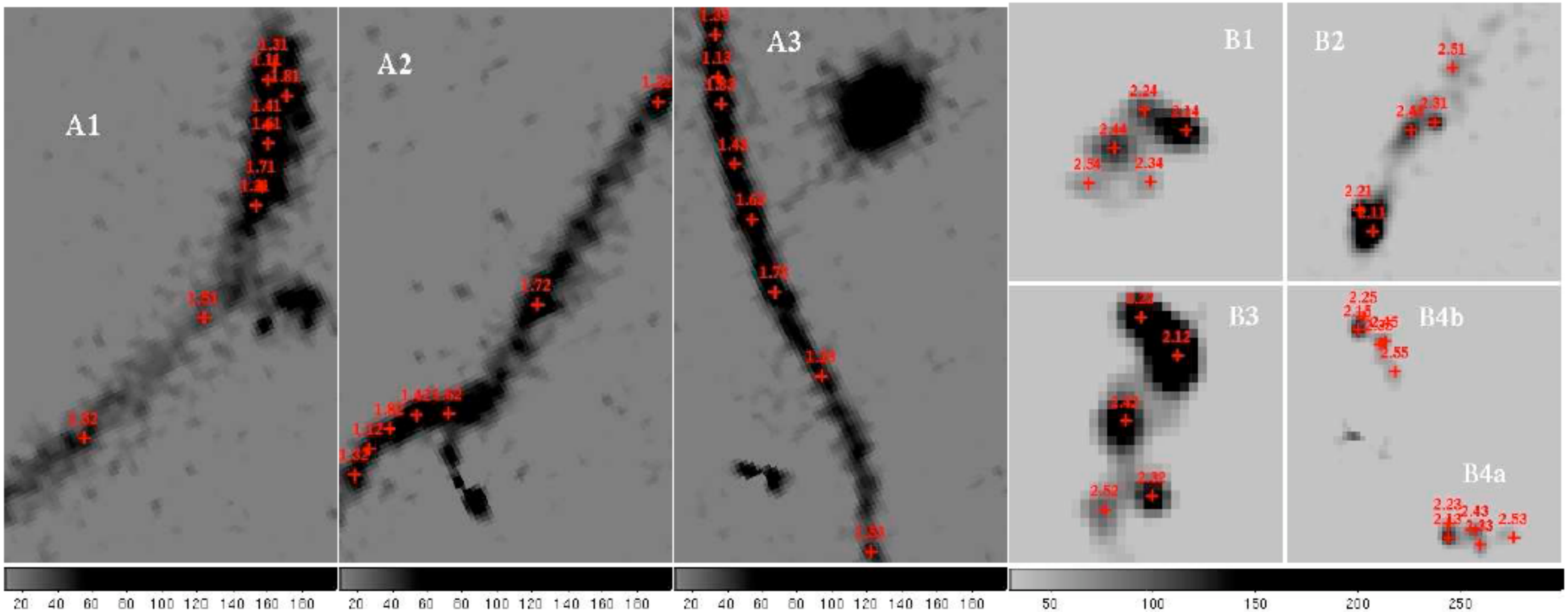
- multiple images (astrometric constraints)
- image distortions
- flux ratios
- time delays
- spectra

- in addition: complementary mass measurements (stellar kinematics, X-ray emission via assumption of hydrostatic equilibrium)

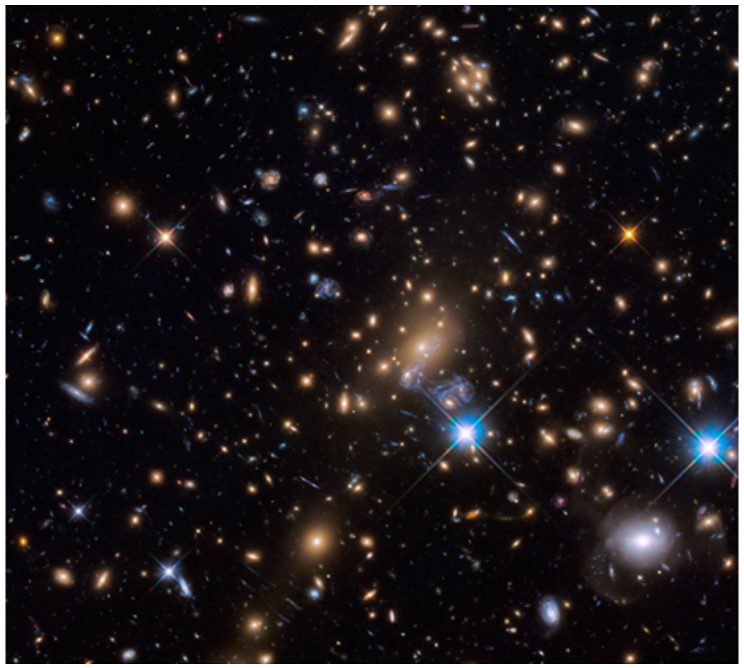
ASTROMETRIC CONSTRAINTS AND IMAGE DISTORTIONS

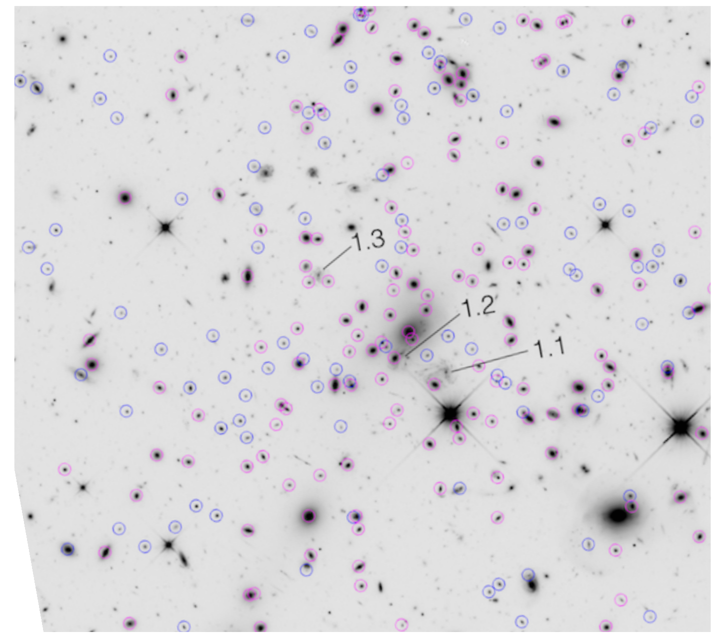
ASTROMETRIC CONSTRAINTS AND IMAGE DISTORTIONS

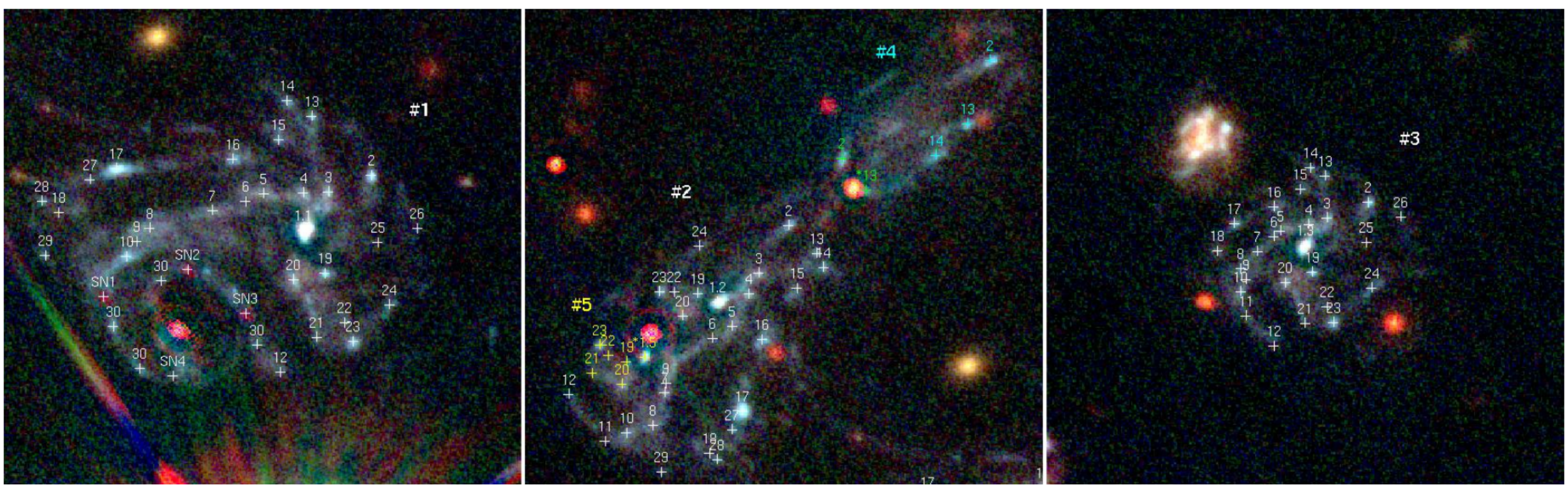




ASTROMETRIC CONSTRAINTS AND IMAGE DISTORTIONS







LENS OPTIMIZATION

- lensing likelihood:

$$\mathcal{L} = \Pr(D|\mathbf{p}) = \prod_{i=1}^{N} \frac{1}{\prod_{j=1}^{n_i} \sigma_{ij}\sqrt{2\pi}} \exp -\frac{\chi_i^2}{2}$$

- minimization of χ^2 to find the best **p** fitting the data
- for example: using astrometric constraints

$$\chi_i^2 = \sum_{j=1}^{n_i} \frac{[\vec{\theta}^j_{obs} - \vec{\theta}^j_{\mathbf{p}}]^2}{\sigma_{ij}^2}$$

- iterate between image and source plane

$$\vec{\beta}^j_{\mathbf{p}} = \vec{\theta}^j_{obs} - \vec{\alpha}(\vec{\theta}^j_{obs}, \mathbf{p})$$

$$\vec{\beta}^j_{\mathbf{p}} = \vec{\theta}^j_{\mathbf{p}} - \vec{\alpha}(\vec{\theta}^j_{\mathbf{p}}, \mathbf{p})$$

- or optimization in the source plane

$$\chi_{S,i}^2 = \sum_{j=1}^{n_i} \frac{[\beta^j_{\mathbf{p}} - \langle\beta^j_{\mathbf{p}}\rangle]}{\mu_j^{-2}\sigma_{ij}^2}$$

BAYESIAN APPROACH

In the case of SL, there are usually few multiple images available.

The model is not well constrained by the data.

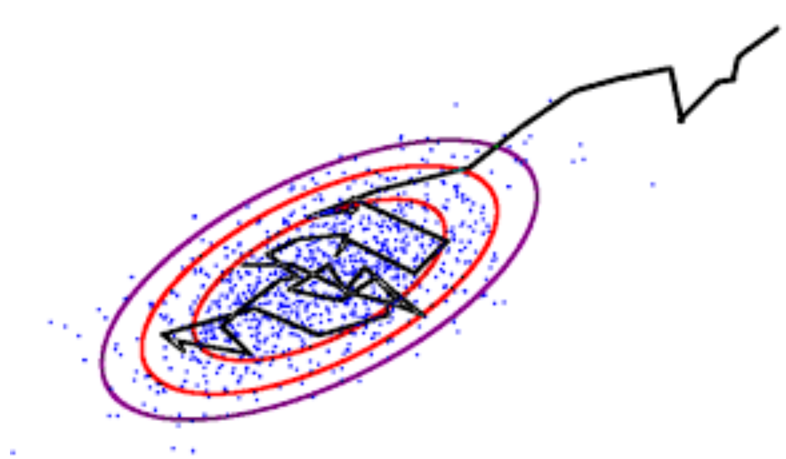
These are the conditions where Bayesian statistics is particularly useful.

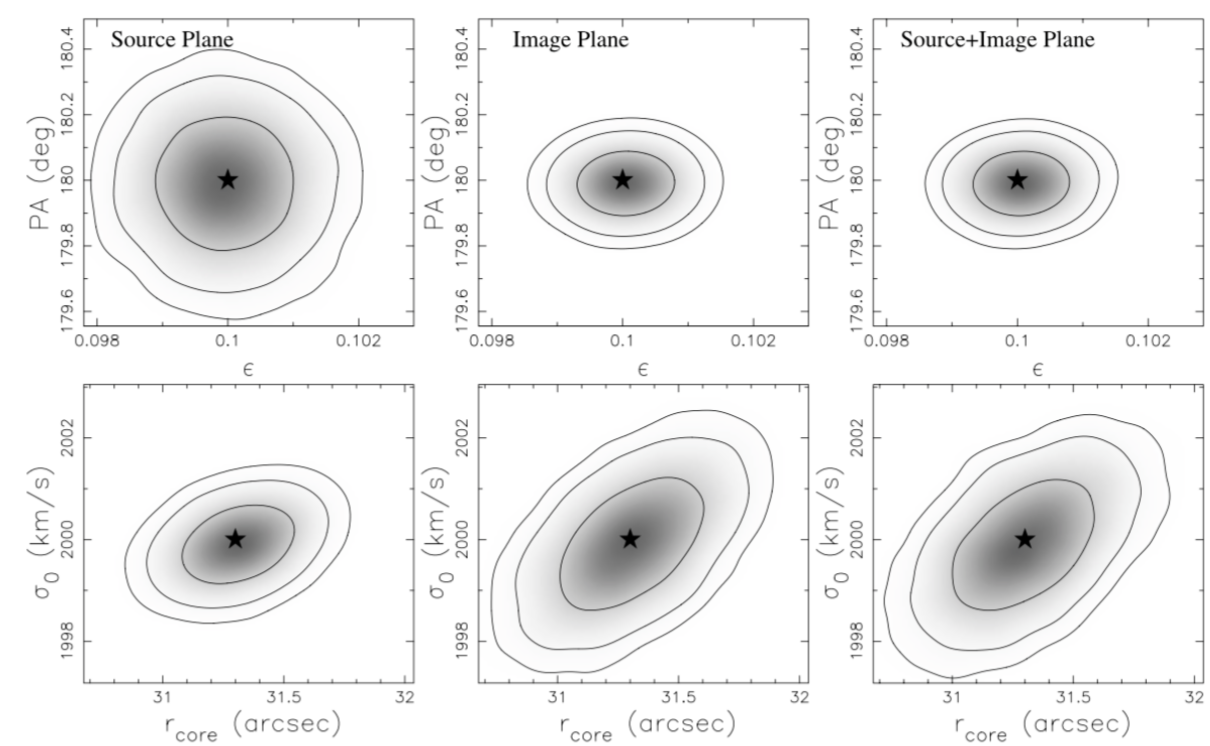
prior PDF

$$\Pr(\mathbf{p}|D, M) = \frac{\Pr(D|\mathbf{p}, M)\Pr(\mathbf{p}|M)}{\Pr(D|M)}$$

posterior PDF

evidence



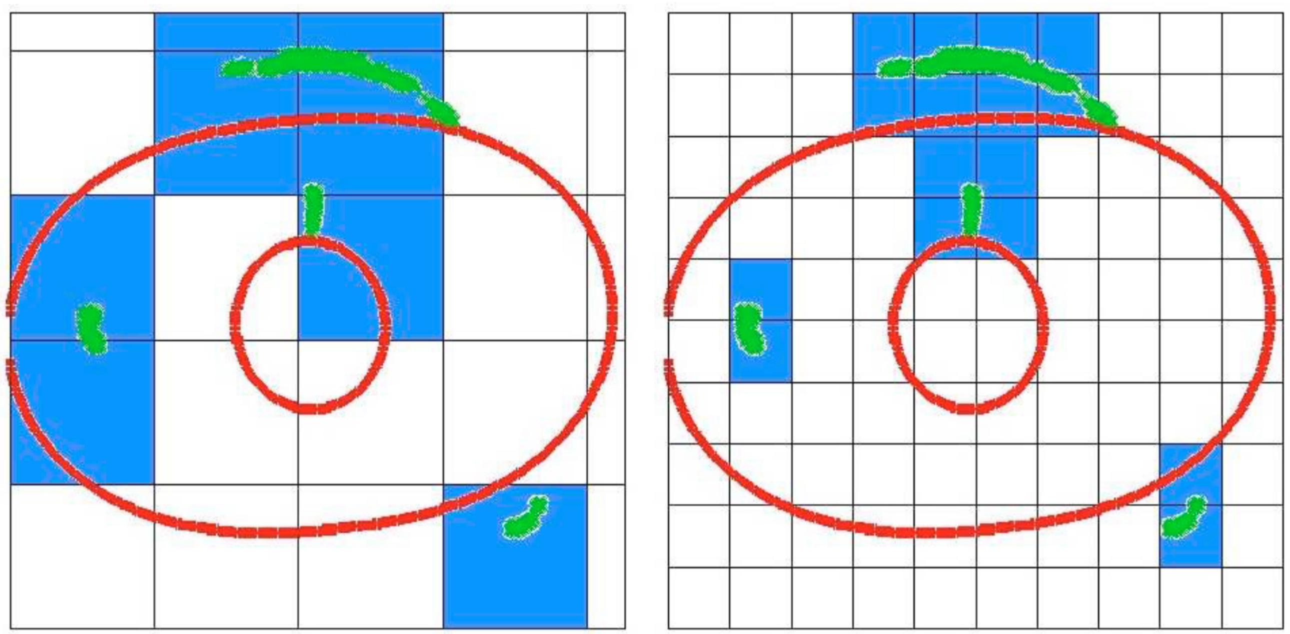


ANOTHER LENS MODELING APPROACH: FREE FORM METHODS

- Free form methods represent an alternative to parametric models
- The mass distribution (or the potential) is not described through parametrized mass clumps but in terms of either
  - pixels or
  - radial-basis functions
- Each pixel or RBF is in fact a parameter, thus these methods are actually parametric as well (and the number of free parameters is even bigger than in parametric modeling)
- Given the extreme flexibility of these techniques, it is mandatory to use regularization terms
- Some advantages:
  - no need to assume that mass follows light
  - relatively easy to combine different probes of the lens potential

AN EXAMPLE: FIT THE CRITICAL LINES WITH A FREE-FORM METHOD

Arc and image positions let us guess where the critical lines are



Working on a grid, derivatives can be approximated to finite differences. This enables to relate the lensing observables to the potential values via linear equations.

$$\kappa_i = K_{ij}\psi_j$$
$$\gamma_i^1 = G_{ij}^1\psi_j$$
$$\gamma_i^2 = G_{ij}^2\psi_j$$

Define: $\chi^2 = \sum_i \frac{(\det A_i)^2}{\sigma_i^2}$

The potential is found by solving

$$\frac{\partial}{\partial\psi}[\chi^2(\psi) + R(\psi)] = 0$$

See e.g. Bradac et al. 2005; Cacciato et al. 2006; Merten et al. 2008

A FUNDAMENTAL LIMIT OF MASS MODELING: THE MASS-SHEET DEGENERACY

$$\psi(\vec{x}) \to \psi'(\vec{x}) = \frac{1-\lambda}{2}x^2 + \lambda\psi$$

This transformation leaves many observables unchanged!

This is called the "mass-sheet degeneracy" (Falco et al. 1985), because it corresponds to transformations of the convergence of the kind:

$$\kappa \to \kappa' = (1-\lambda) + \lambda\kappa$$

Provided that the source plane is isotropically scaled as $y \to y' = \lambda y$

the following quantities remain the same:

- critical lines
- image positions
- image shapes
- flux ratios

AFFECTED QUANTITIES: TIME DELAYS AND MAGNIFICATIONS

$$
\begin{aligned}
\Delta t' &\propto \frac{1}{2}(\vec{x}-\vec{y}')^2 - \psi'(\vec{x}) \\
&= \frac{\vec{x}-\lambda\vec{y}}{2} - \lambda\psi(\vec{x}) - \frac{1-\lambda}{2}x^2 \\
&= \lambda\left[\frac{1}{2}(\vec{x}-\vec{y})^2 - \psi(\vec{x})\right] + \frac{\lambda(\lambda-1)}{2}y^2 \\
&= \lambda\Delta t + \mathrm{const}
\end{aligned}
$$

$$
\mu' = (\lambda_t'\lambda_r')^{-1} = (\det A')^{-1} = (\lambda^2 \det A)^{-1} = \frac{\mu}{\lambda^2}
$$

Time delays and magnifications are changed by mass-sheet transformations.

In principle, they can be used to break the degeneracy, but…

BREAKING THE DEGENERACY

- Complementary measurements of the mass profiles
  - Example: using stellar kinematics, in the case of an elliptical galaxy
- Adopting a shape for the mass profile
- Assuming that the convergence goes to zero at large distances from the center of the lens
- Using sources at different redshifts
- Measuring the magnification statistically, or via galaxy number counts

STELLAR KINEMATICS

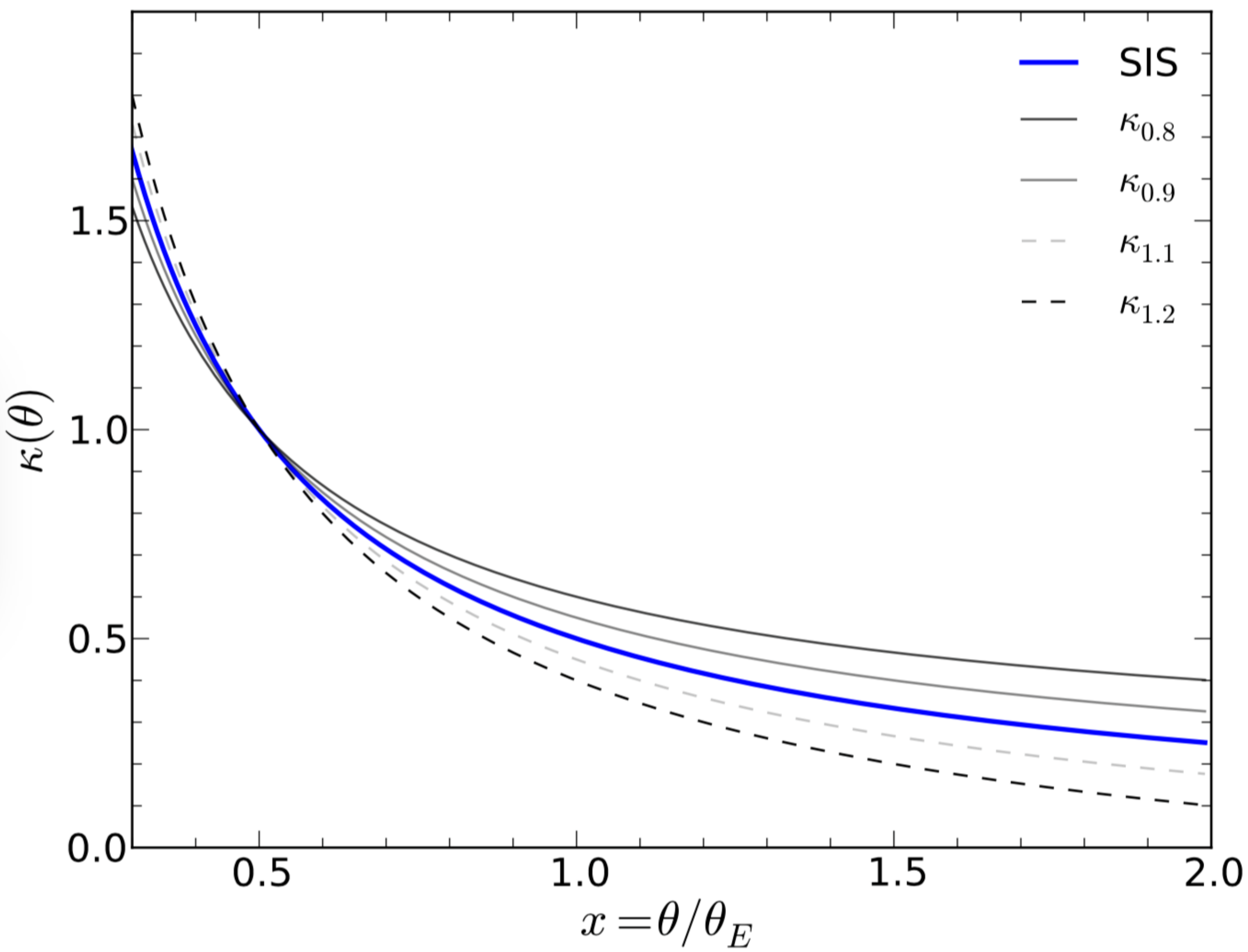
$$\frac{d(\nu\sigma_r^2)}{dr} + 2\frac{\beta}{r}\nu\sigma_r^2 = -\nu\frac{GM(r)}{r}$$

- Measure 3D mass
- Project onto the lens plane
- compute convergence up to a given radius
- break the degeneracy
- probe region of the galaxy well within the Einstein radius

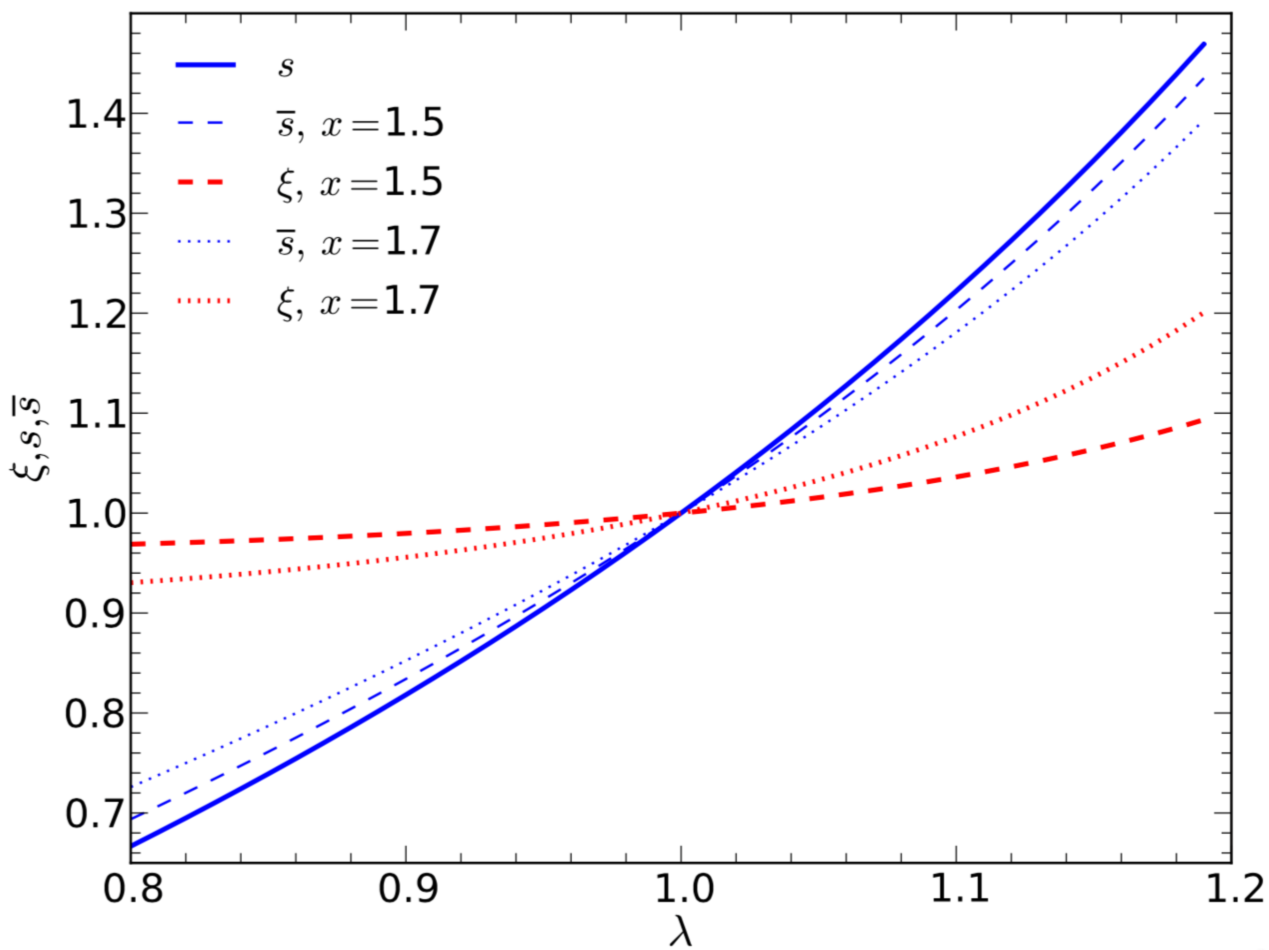
FIXING THE SHAPE OF THE DENSITY PROFILE

Schneider & Sluse, 2013

$$\xi = \frac{\kappa(\sqrt{\theta_1\theta_2})}{\sqrt{\kappa(\theta_1)\kappa(\theta_2)}}$$

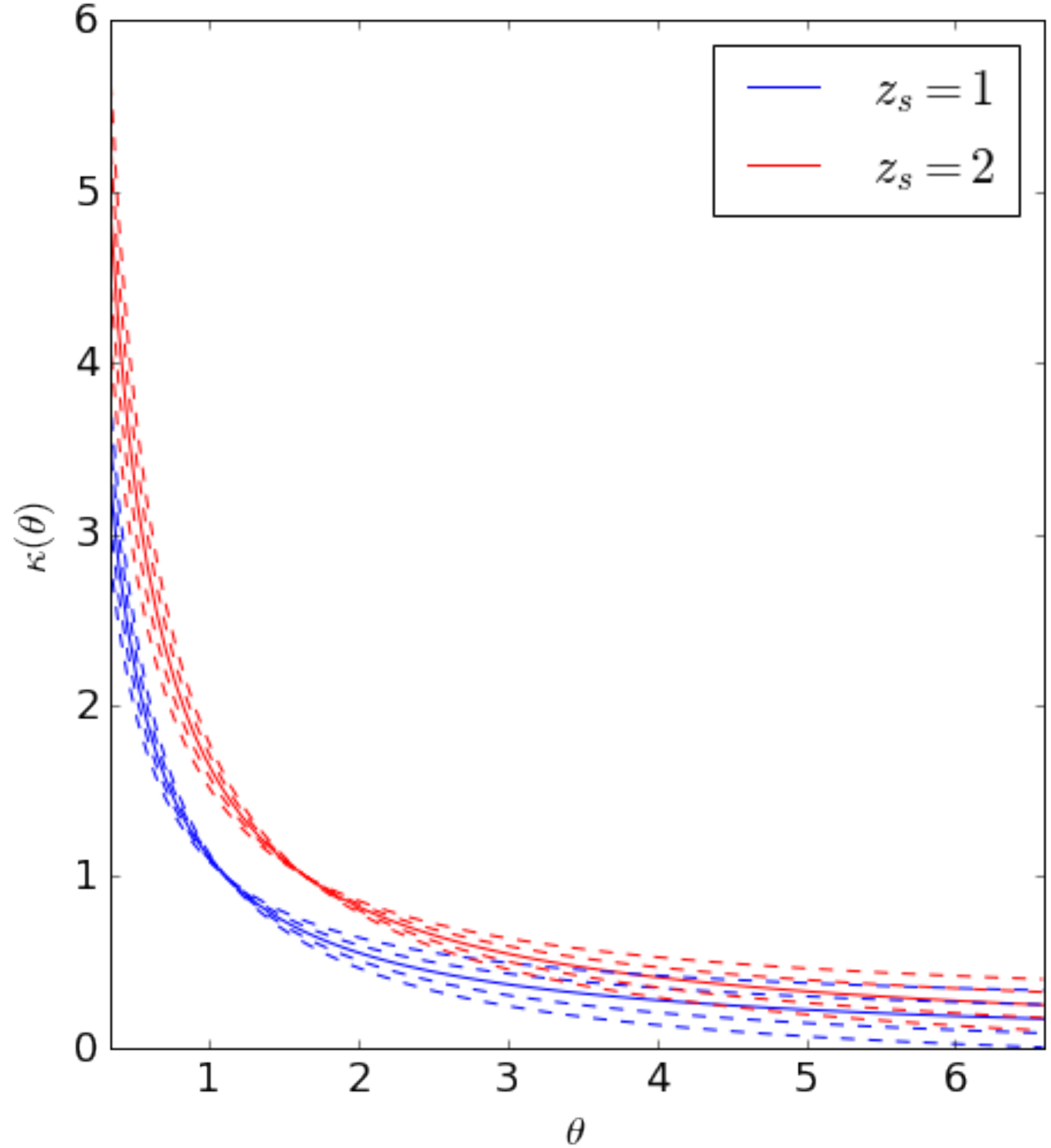


Mass-sheet transformations of a SIS profile

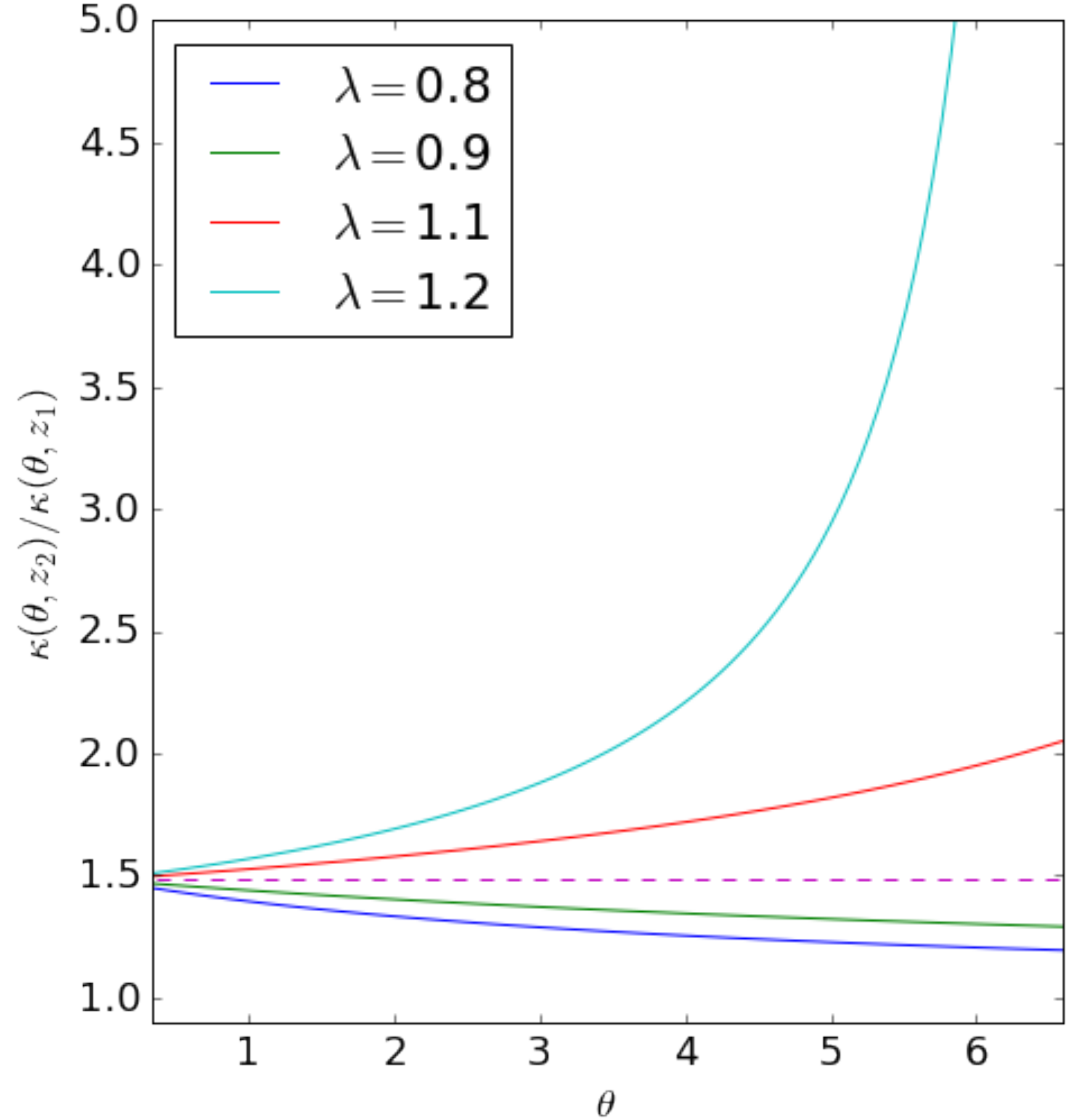


profile slope and "deviation from power-law"

SOURCES AT DIFFERENT REDSHIFTS



MSTs of convergence profiles of a SIS lens for two different source redshifts



ratio of convergence profiles is not preserved

USING GALAXY NUMBER COUNTS

$$n(>S) = n_0(S/\mu)/\mu$$

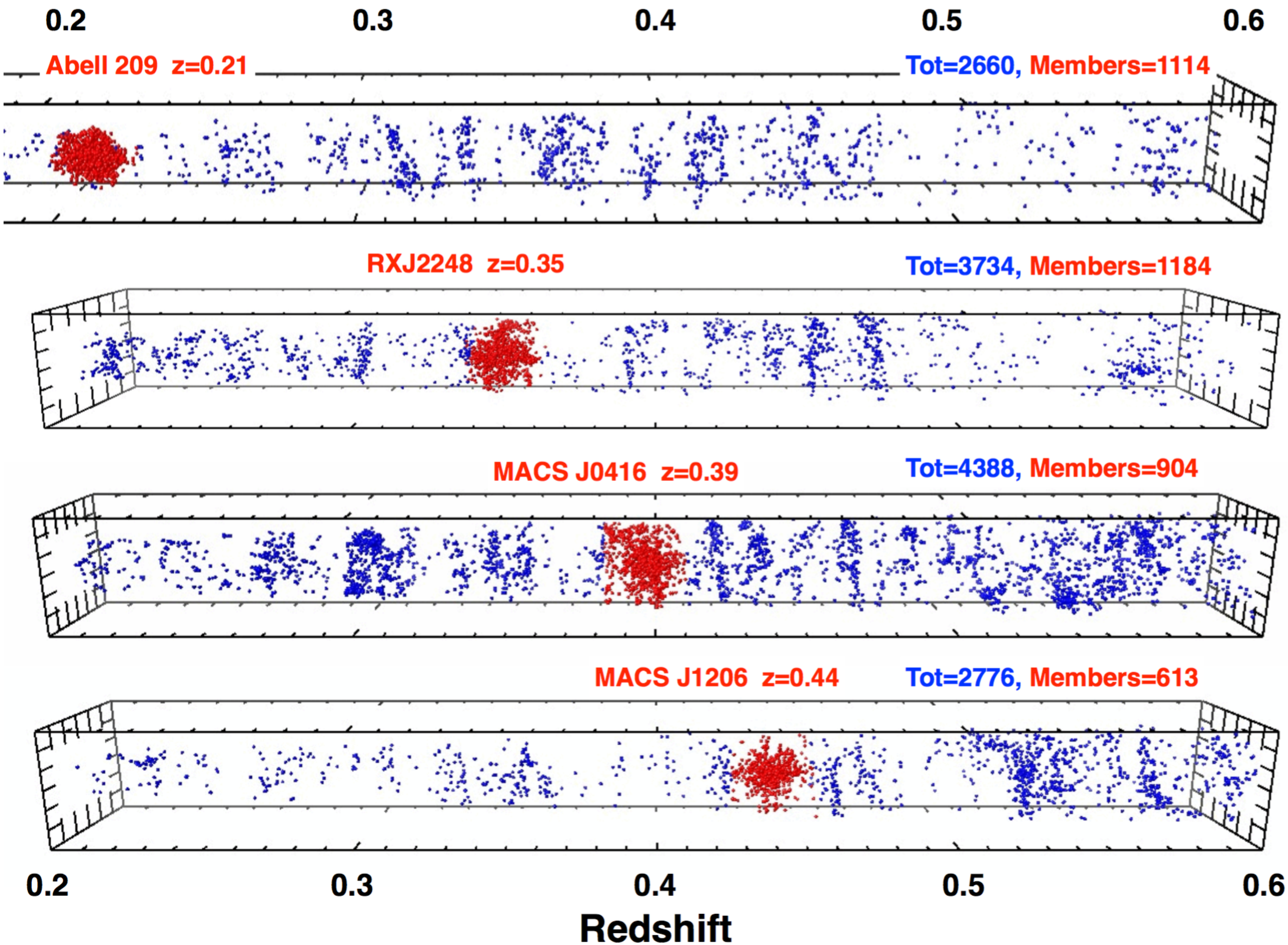
$$n_0(>S) \propto S^{-\alpha}$$

$$n(>S) \propto \frac{S^{-\alpha}}{\mu^{1-\alpha}}$$

$$n(>S)/n_0(>S) = \mu^{\alpha-1}$$

Knowing the unlensed number density of galaxies, and the slope of the number counts, one can estimate the magnification and break the degeneracy

YET ANOTHER LIMIT: PERTURBATIONS ALONG THE LINE OF SIGHT



YET ANOTHER LIMIT: PERTURBATIONS ALONG THE LINE OF SIGHT

- usually modeled using an external convergence perturbation
- relying on results of “ray-tracing” simulations through large cosmological boxes (e.g. Hilbert et al. 2009)
- trying to estimate the PDF of the external convergence given the observed galaxy number counts
- problem: need many realizations of the universe for different values of the cosmological parameters
- correlations with external shear perturbations (Suyu et al. 2013)