GRAVITATIONAL LENSING

LECTURE 26

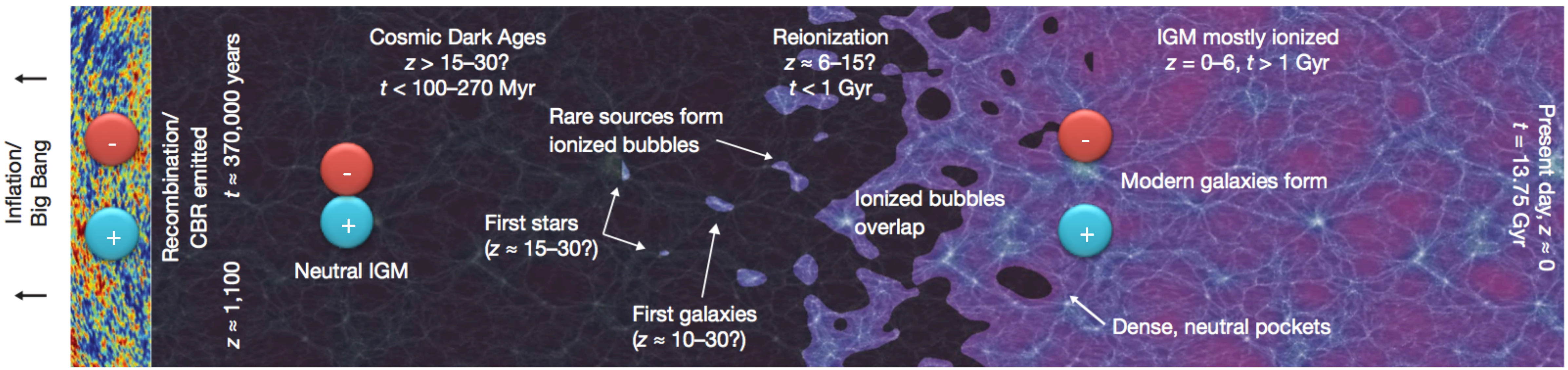
Docente: Massimo Meneghetti

AA 2015-2016

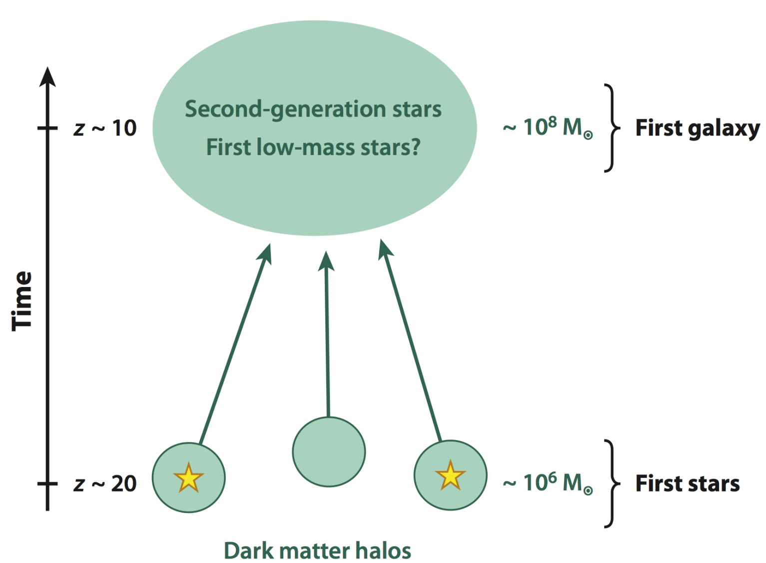
COSMIC RE-IONIZATION

The high redshift universe is still a mystery!

Robertson et al. 2010



CMB polarization (E-modes) → |— **Reionization** —| ← Gunn-Peterson



Bromm & Yoshida al. 2011

How and when did the first galaxies form?

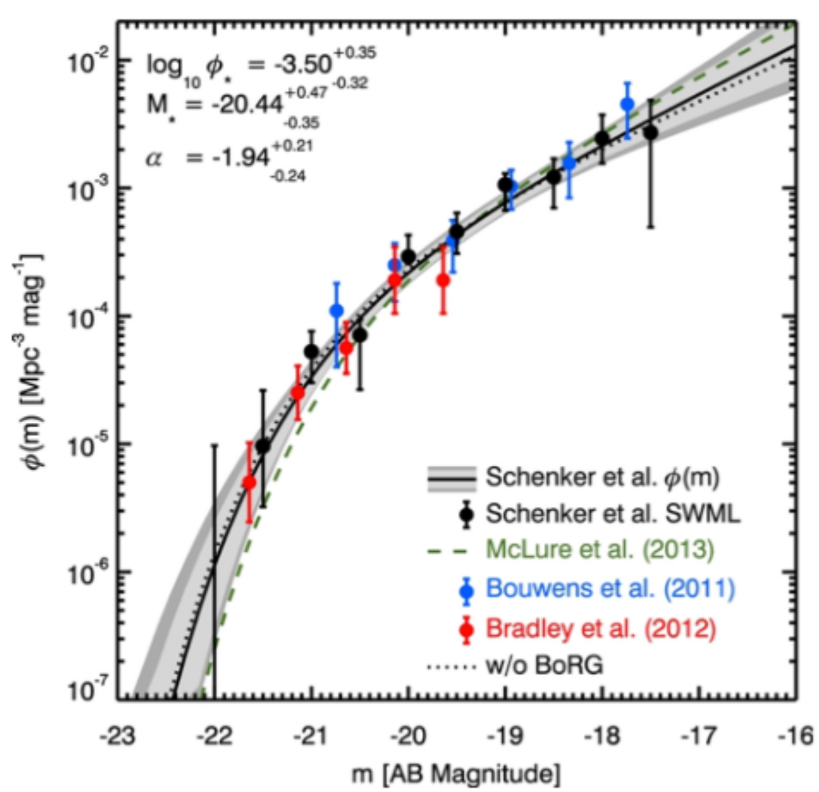
Where they responsible for the re-ionization of the universe?

COSMIC RE-IONIZATION

- galaxies with L>0.5L* at z~3 have very low (<1%) UV escape fraction (e.g. Vanzella et al. 2012; Bridge et al. 2010; Siana et al. 2010)
- sources of ionizing radiation must be low luminosity galaxies (Ferrara & Loeb, 2013; Wise et al. 2014; Kimm & Cen 2014)!
- ***We need to count and characterize faint galaxies!***

Bullock (Yale, 2014)

z=8



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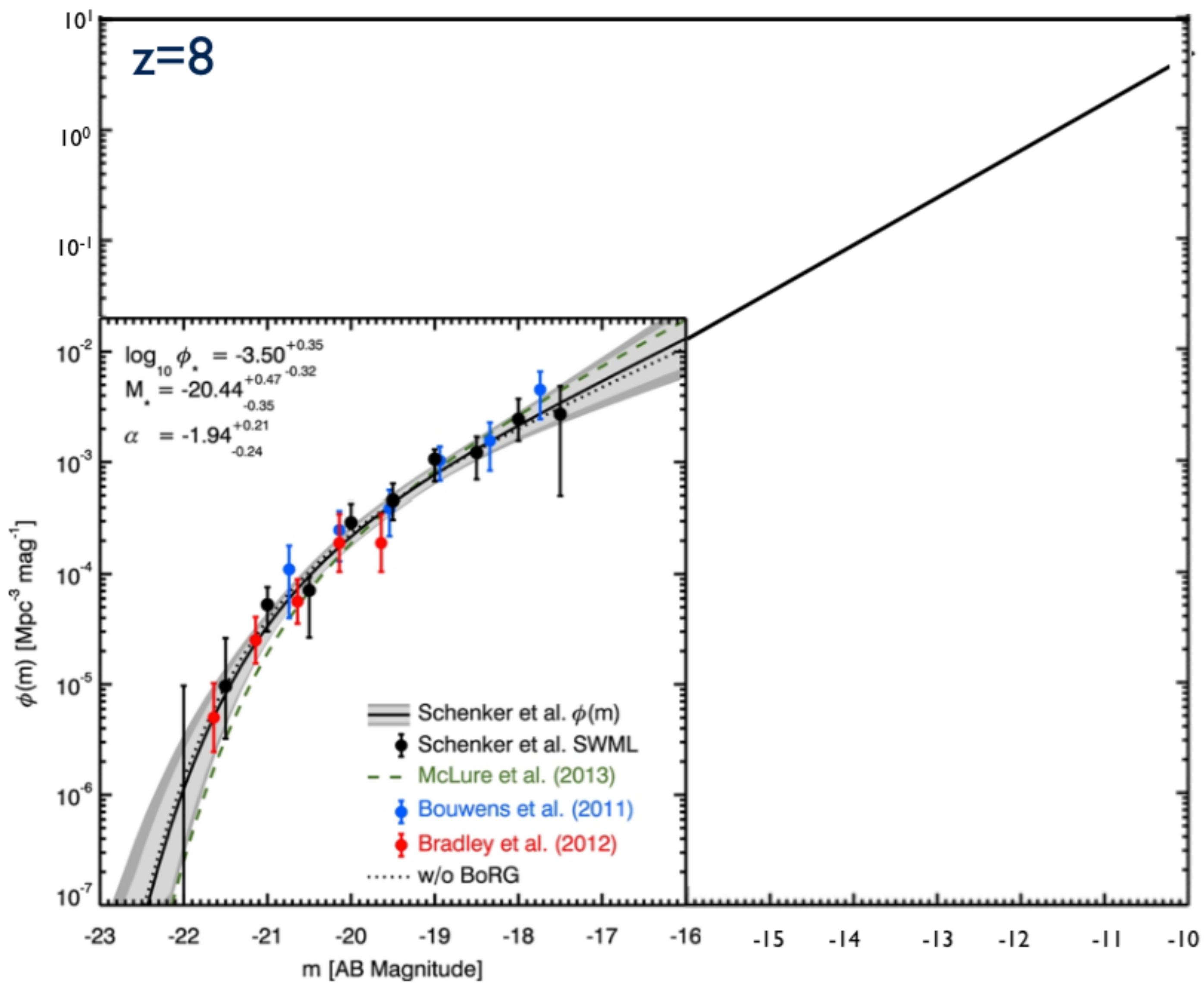

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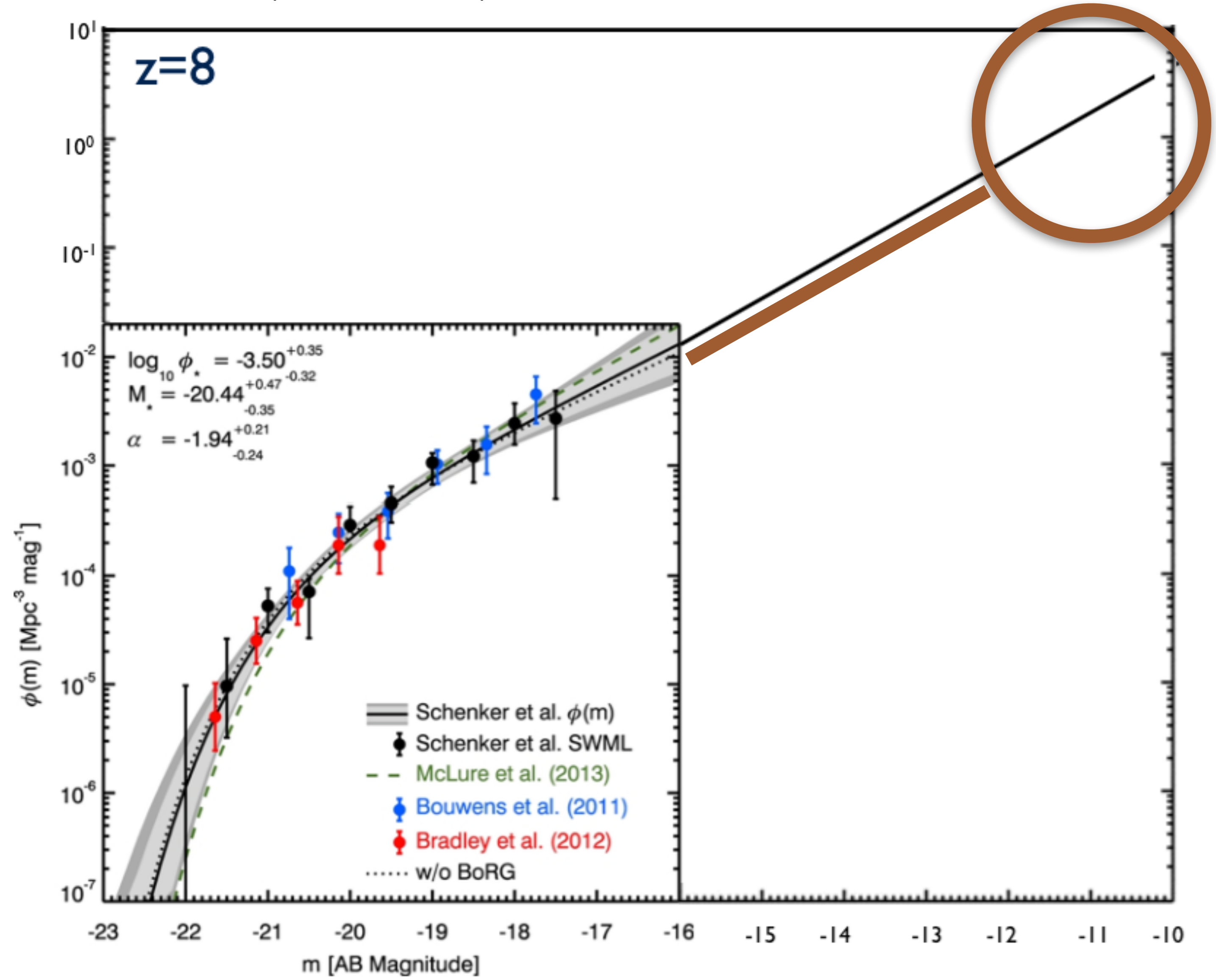
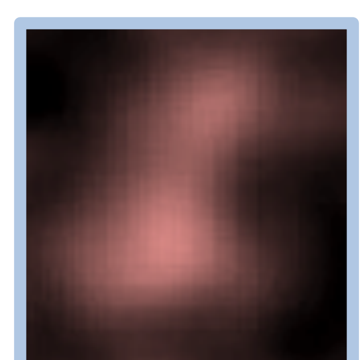


Stellar mass?

Star formation rate?

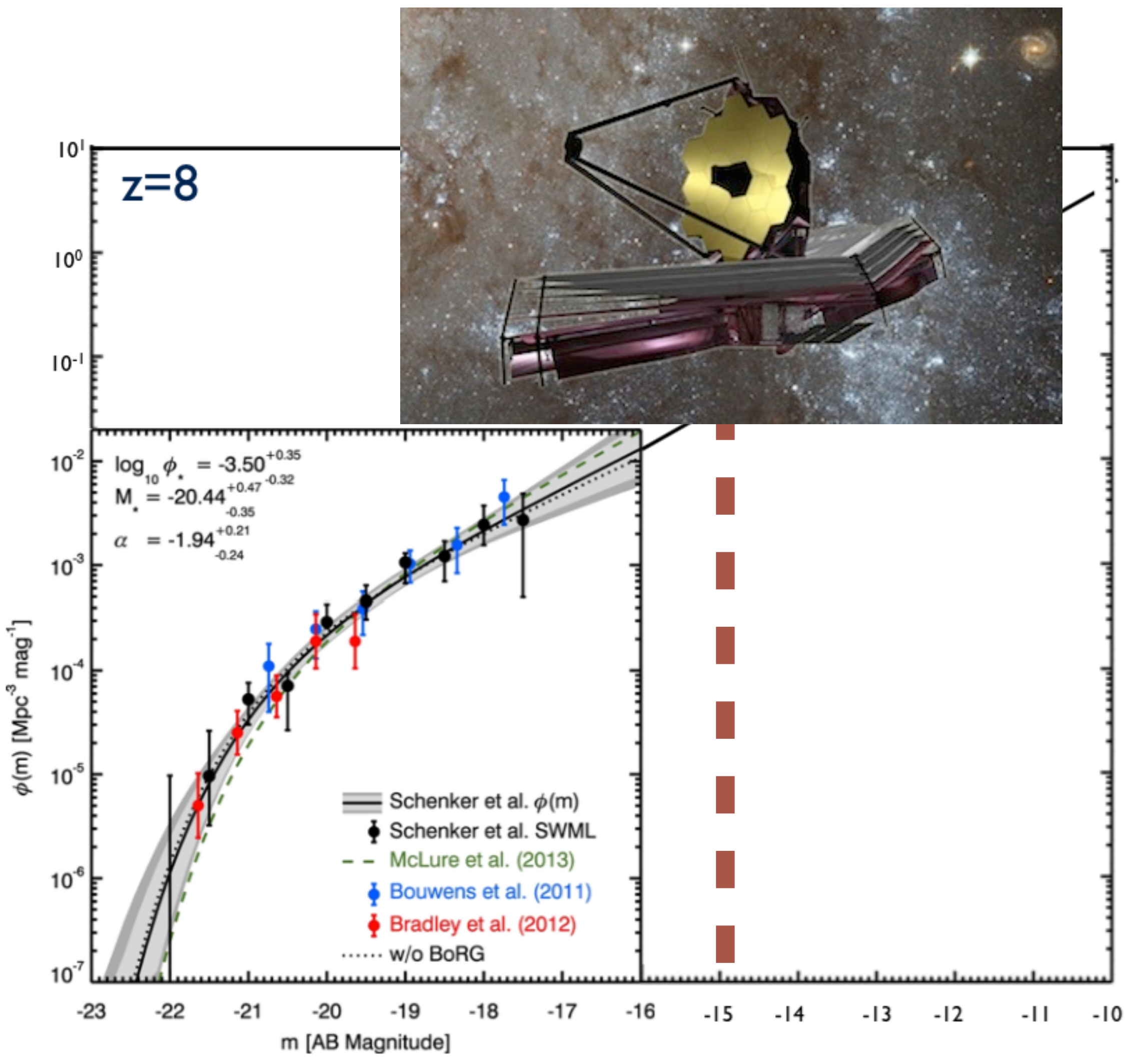


Morphology?

Escape fraction?

COSMIC TELESCOPES

Answering this questions is among the scientific goals of JWST, but…



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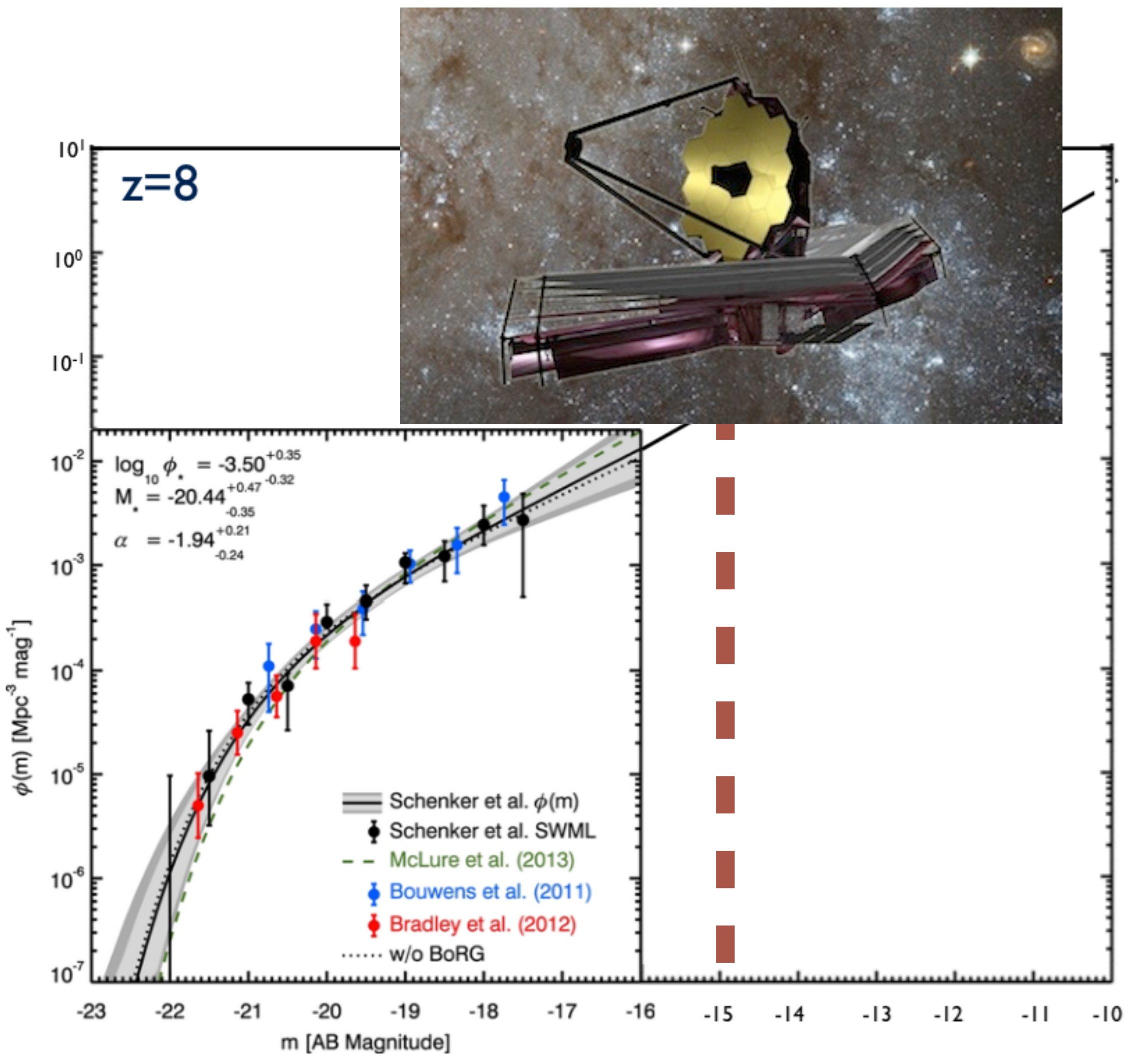
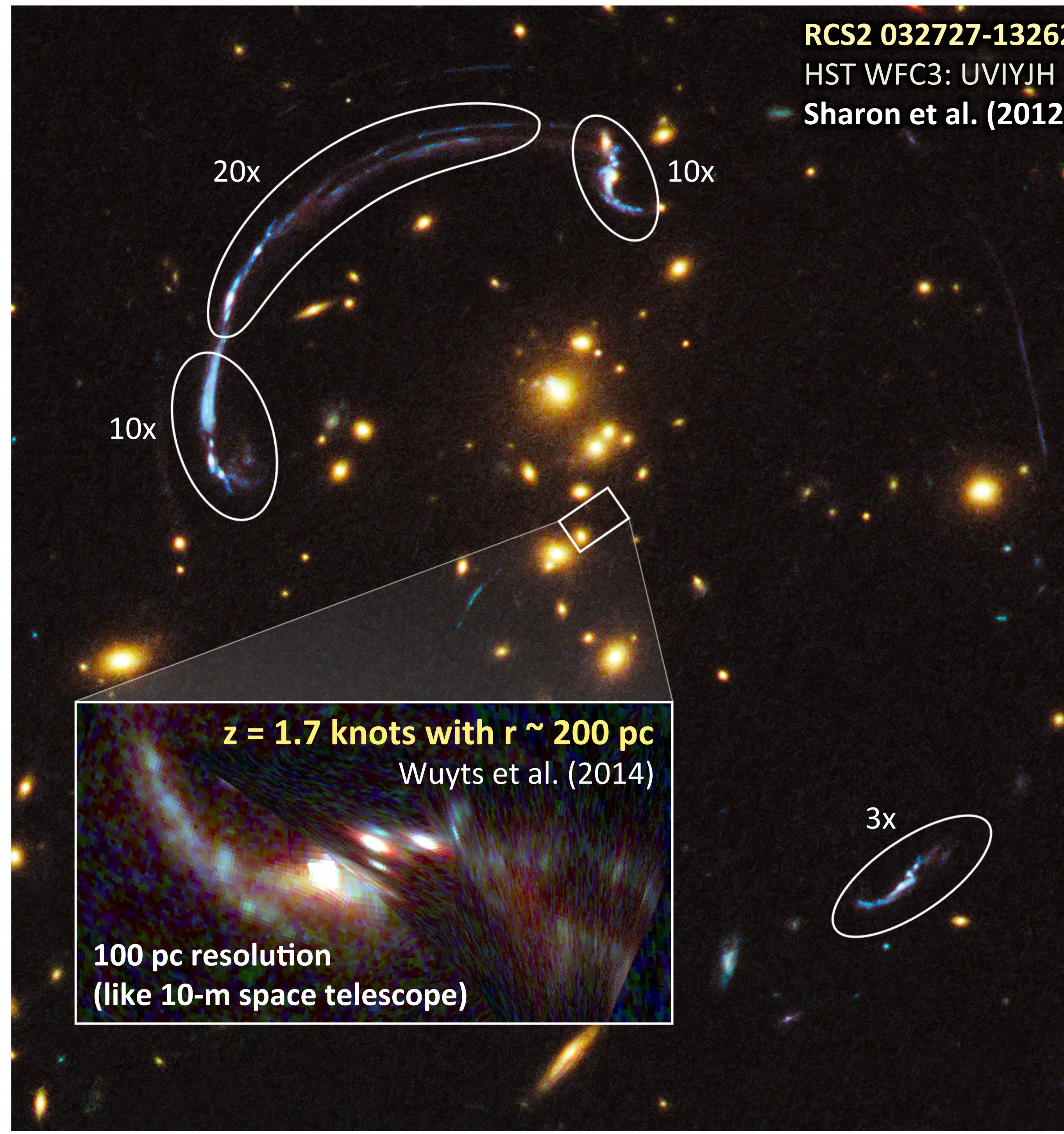
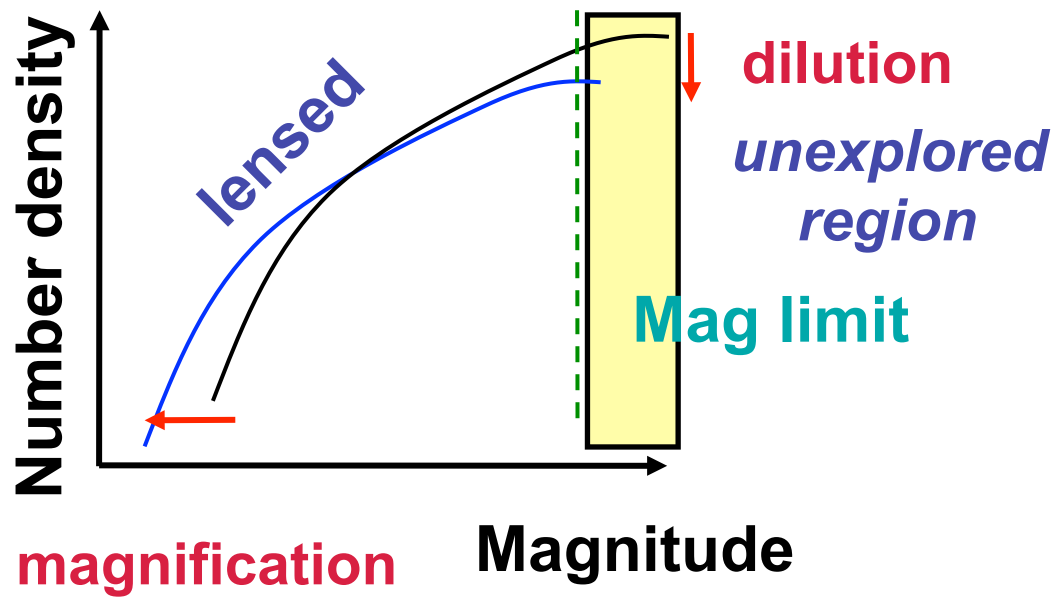


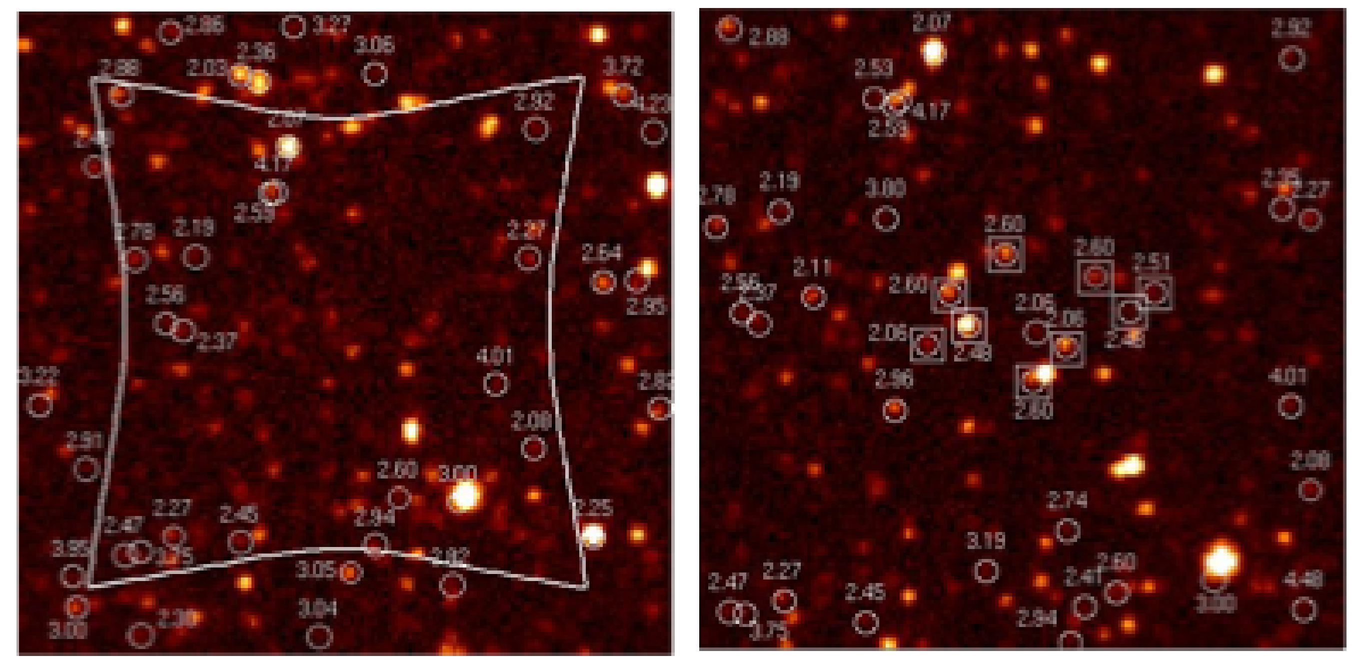


…exploting lensing magnification by galaxy clusters, HST can reach high-z galaxies as faint as those that JWST will detect in blank fields!

COSMIC TELESCOPES

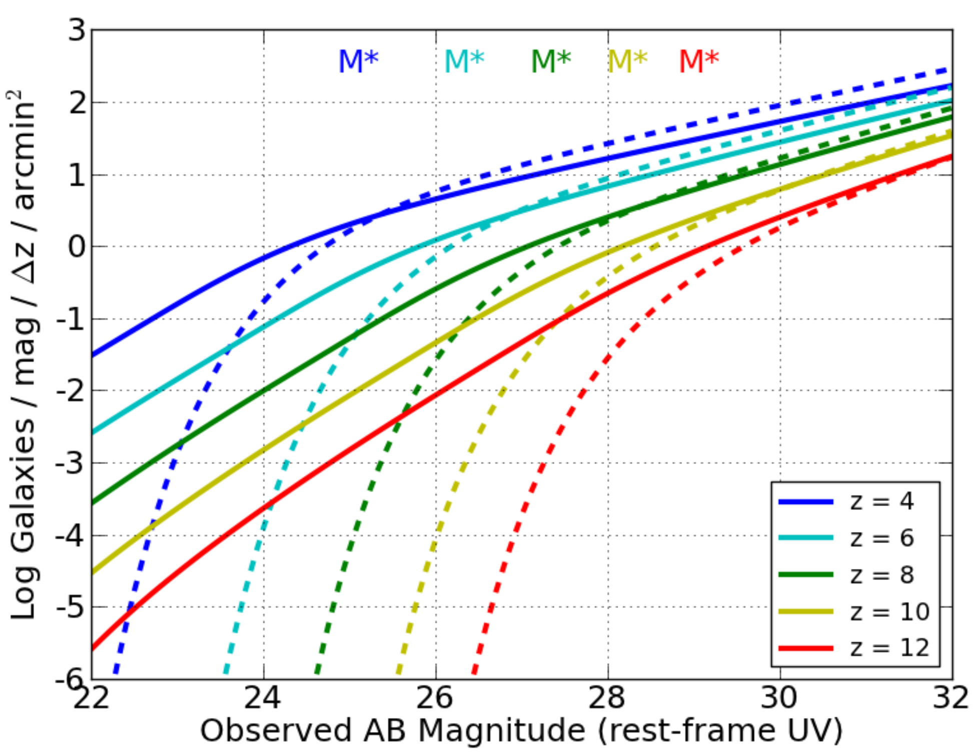


7x7 arcmin^2 Herschel simulation



Unlensed field   Lensed field

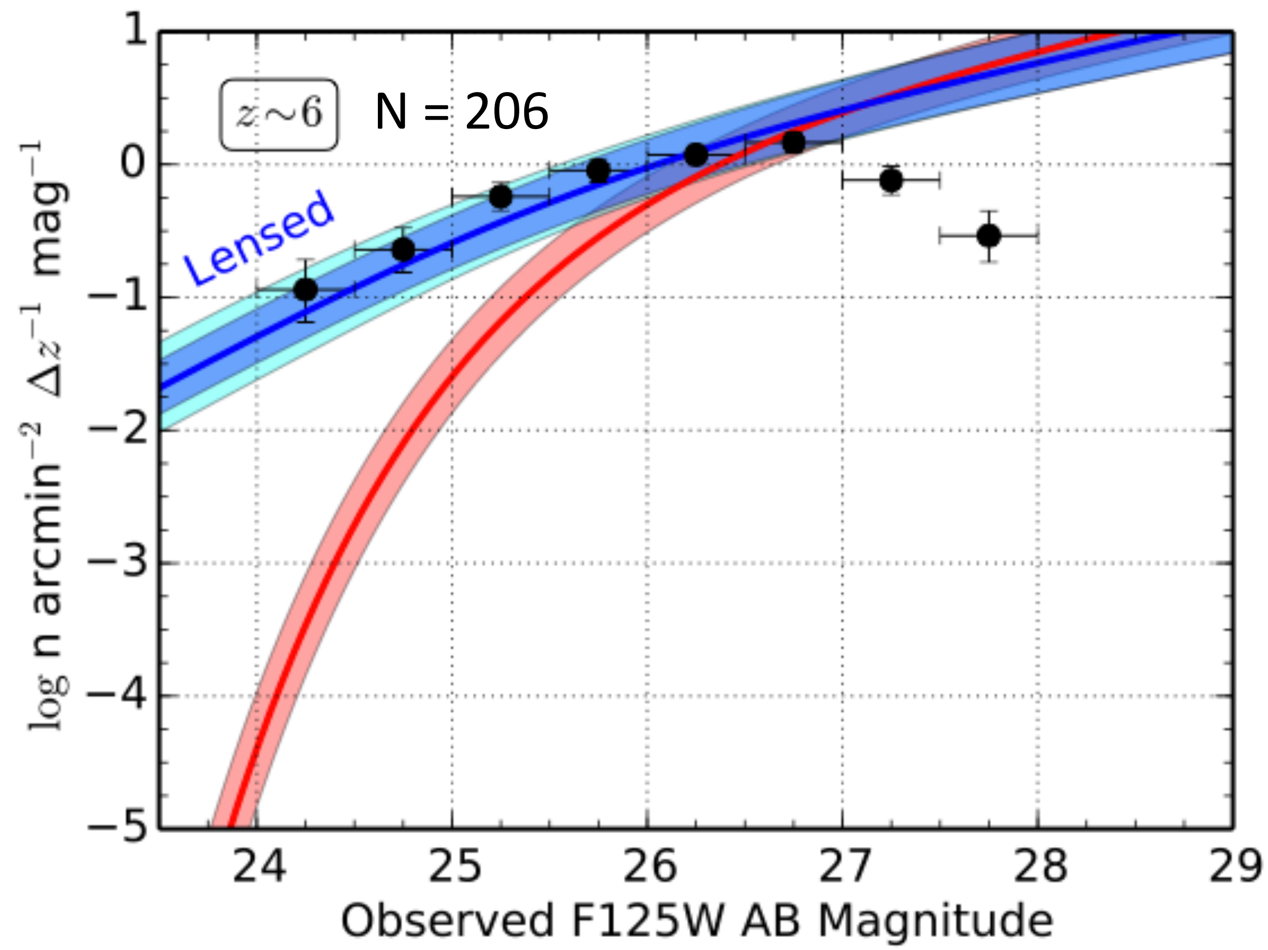
Courtesy of J-P. Kneib



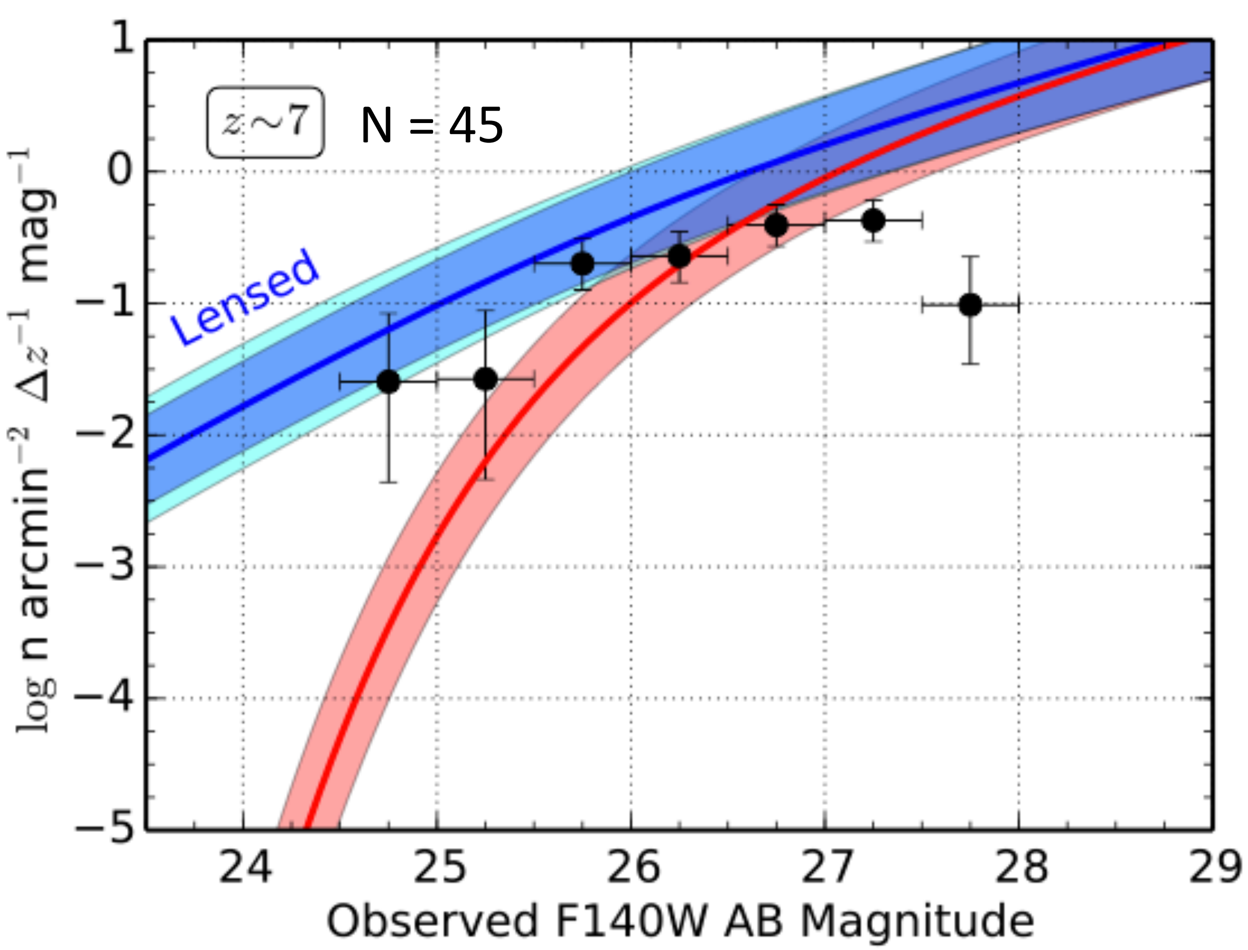
Courtesy of D. Coe

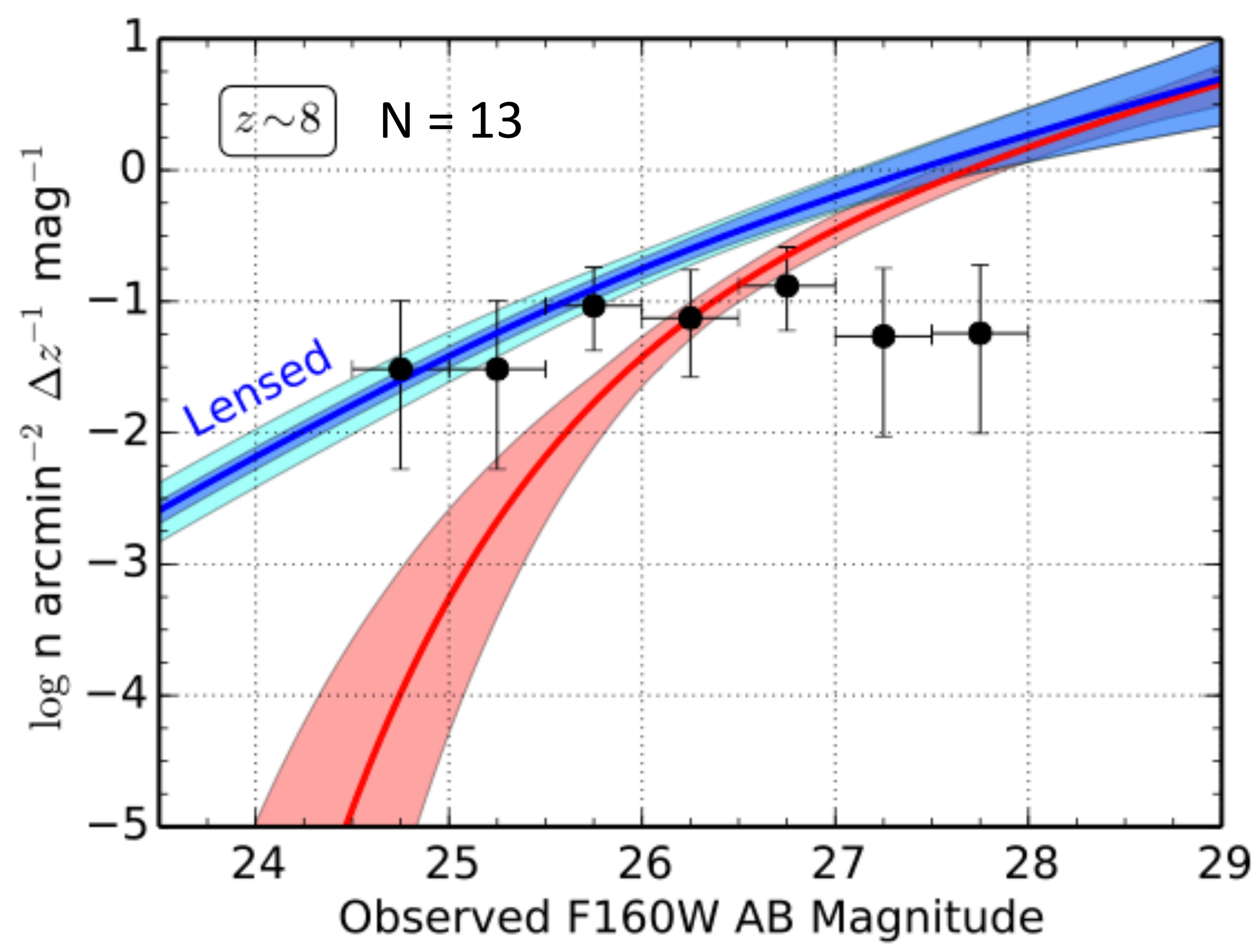
Lensing wins at the brightest magnitudes and at the highest redshifts

HIGH-Z GALAXIES FROM CLASH



Bradley et al., 2014

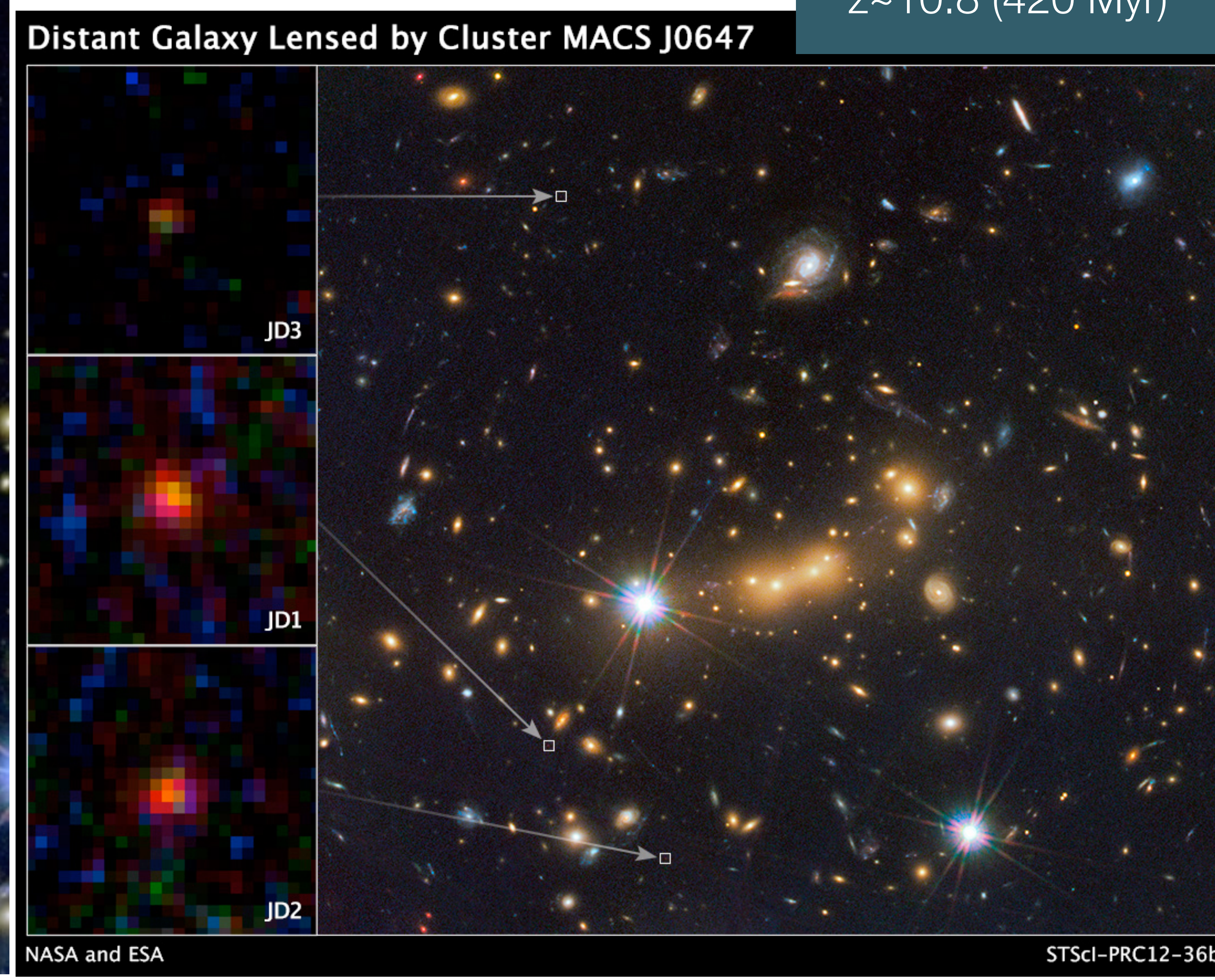




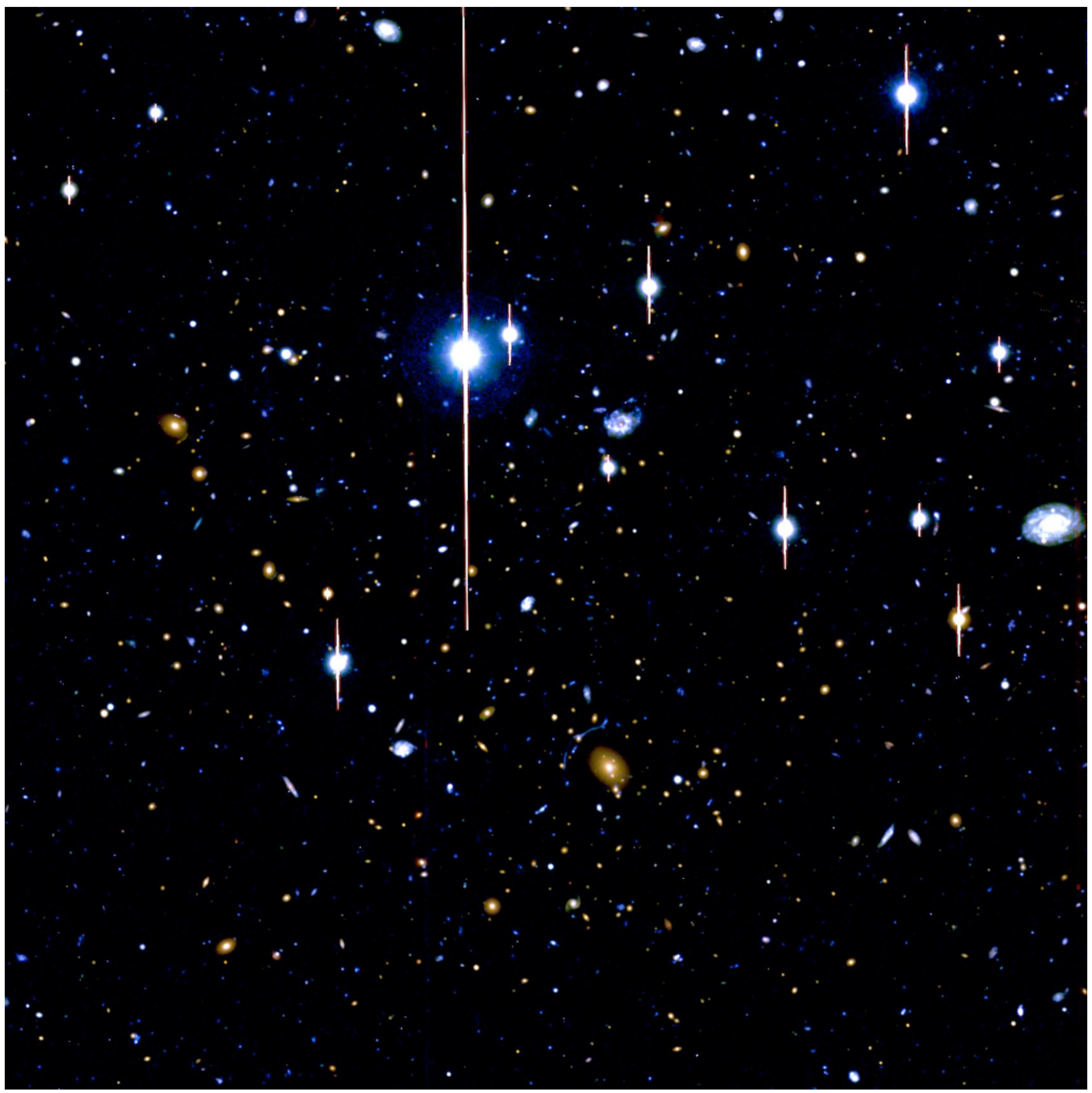
HIGH-Z GALAXIES FROM CLASH

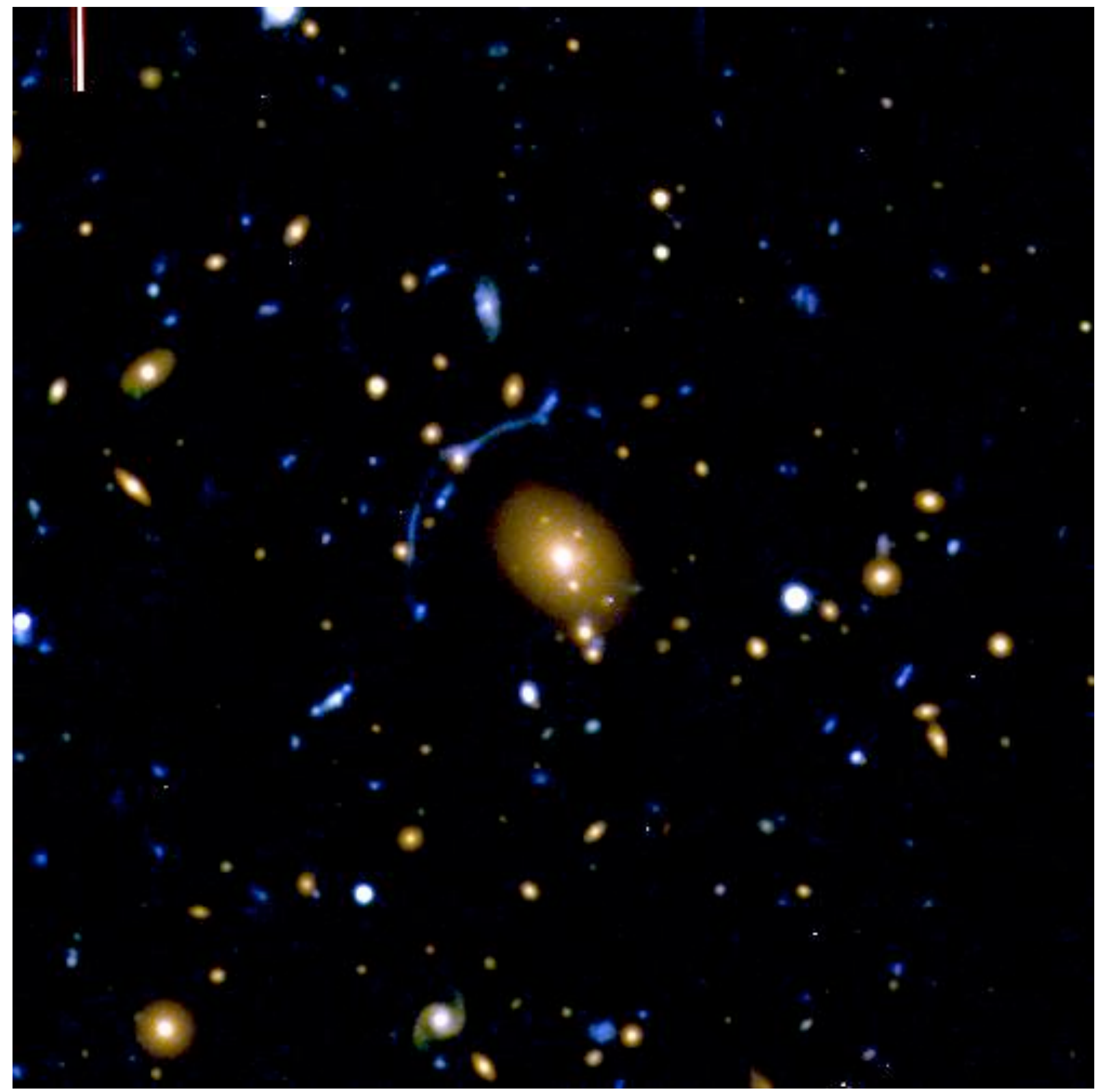
MACSJ1149-JD
Zheng et al. (2012)
z~9.6 (490 Myr)

MACSJ0647-JD
Coe et al. (2013)
z~10.8 (420 Myr)



GALAXY CLUSTERS AS WEAK GRAVITATIONAL LENSES





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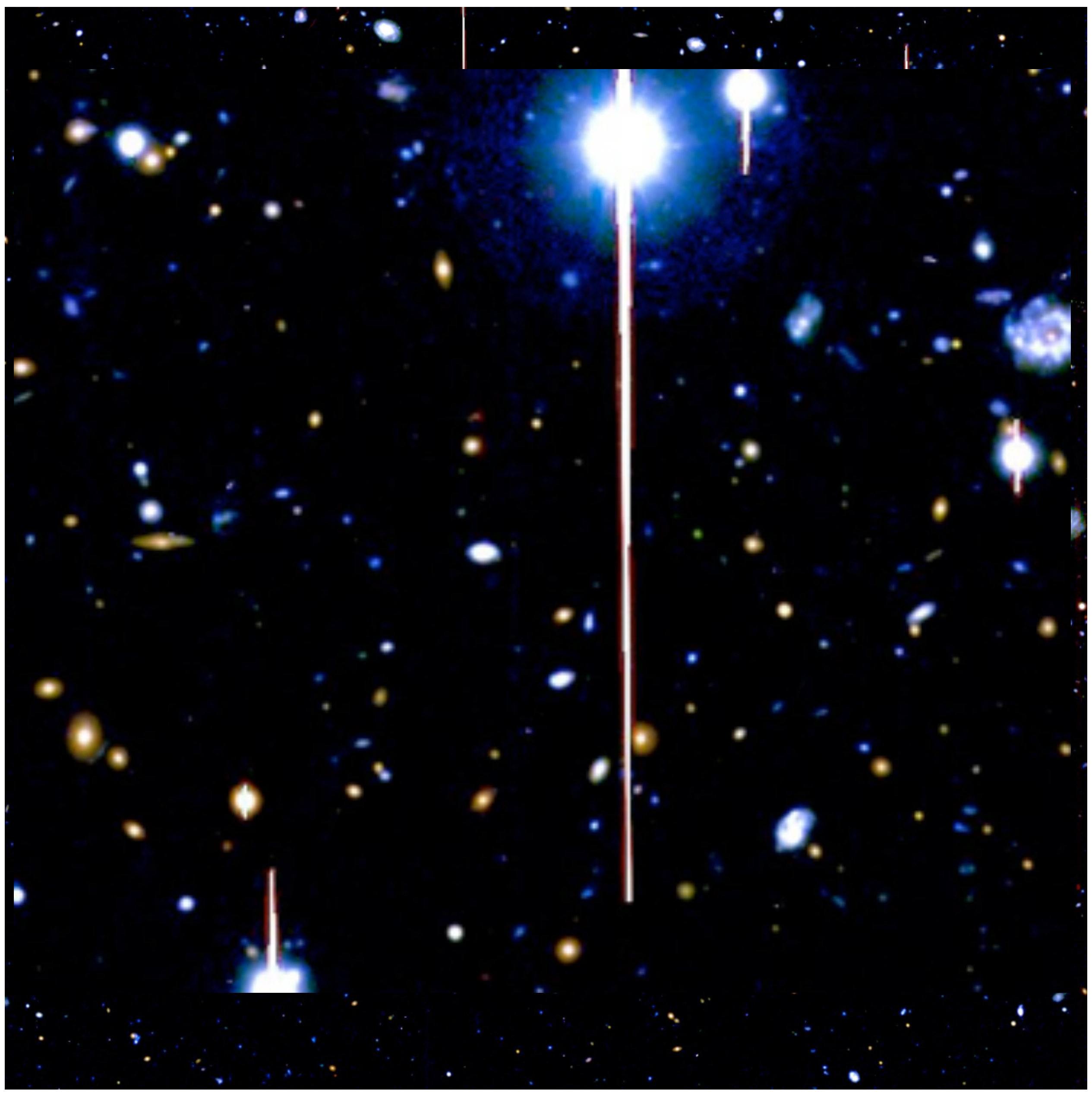
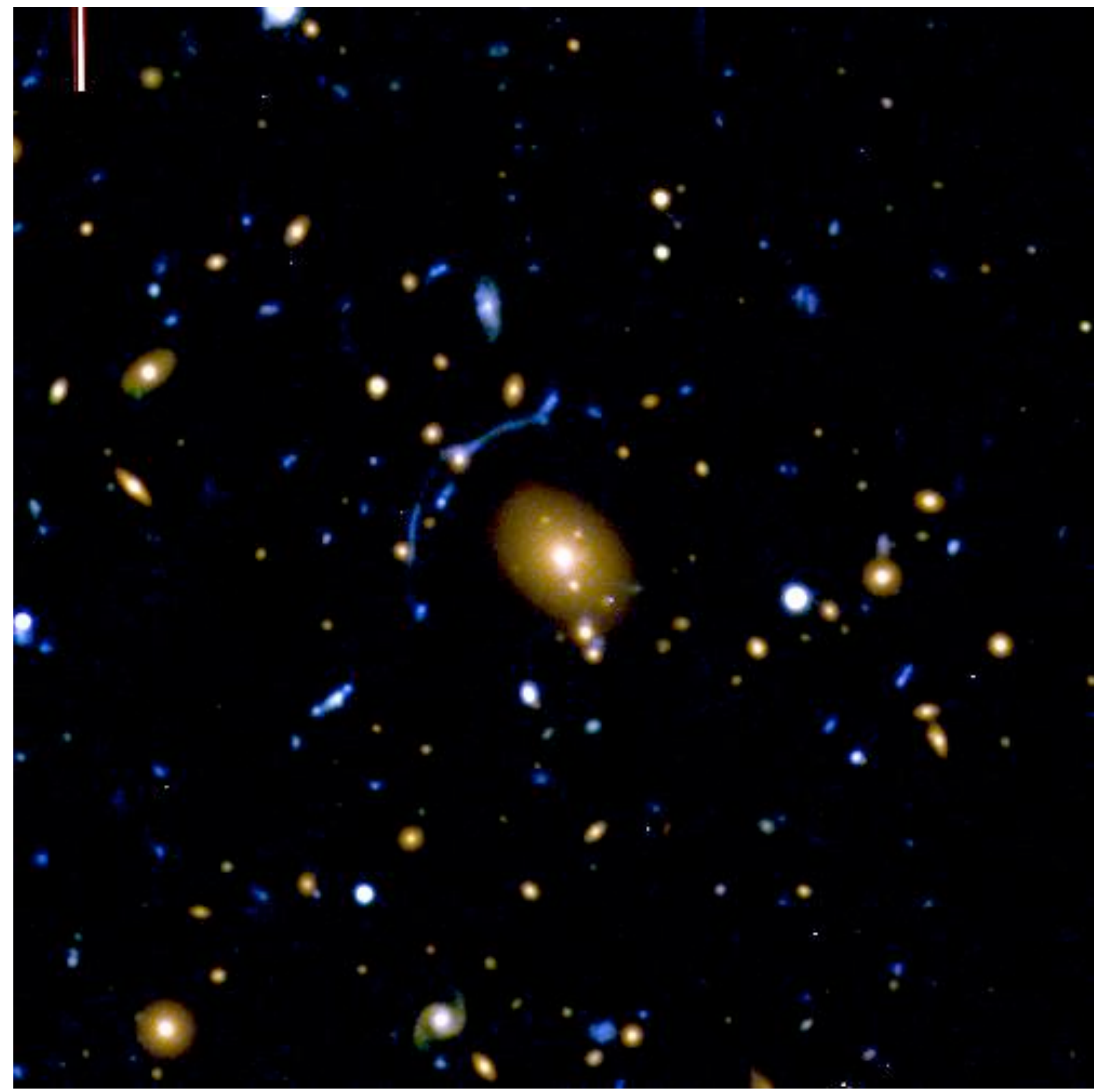
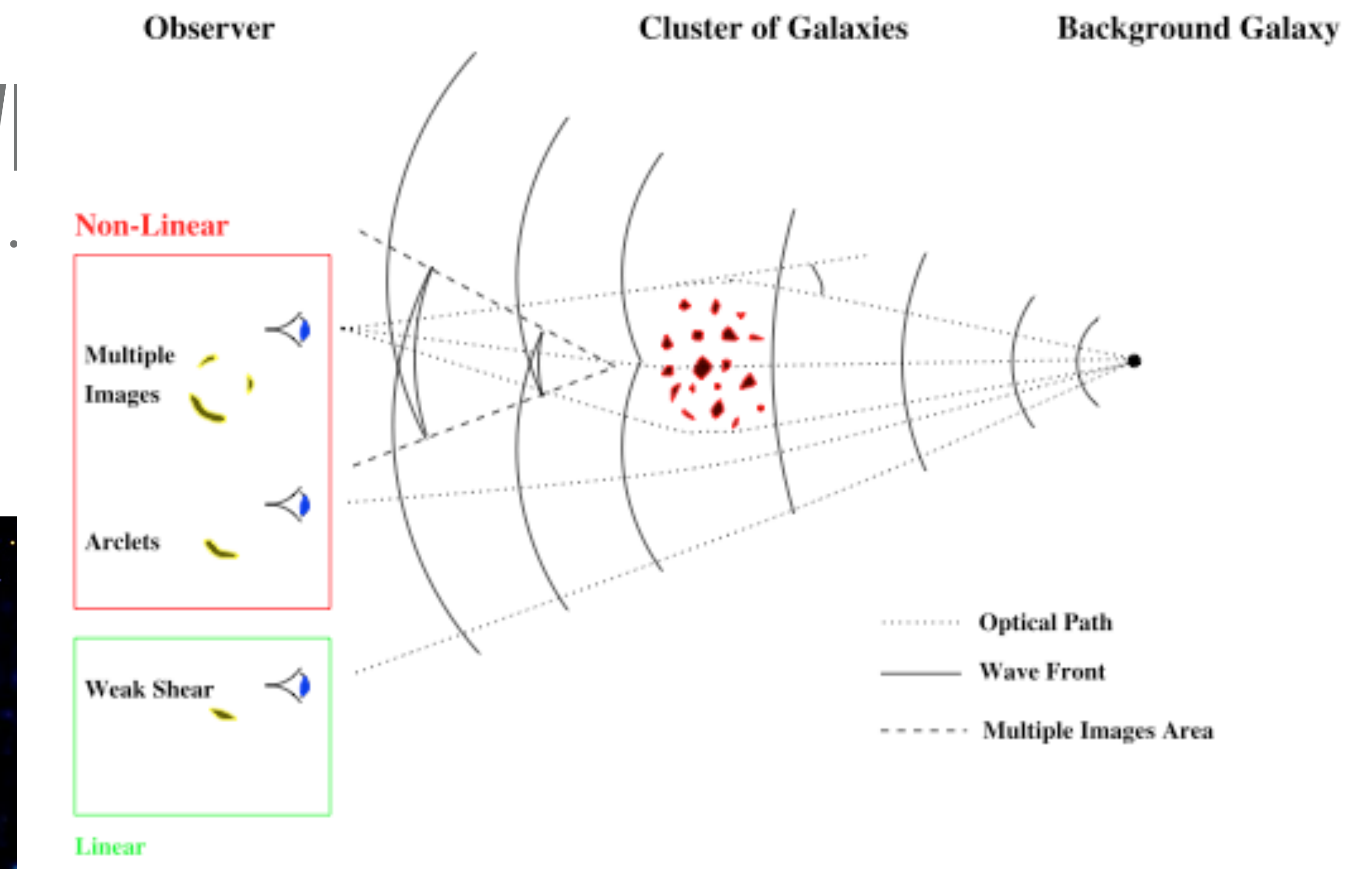
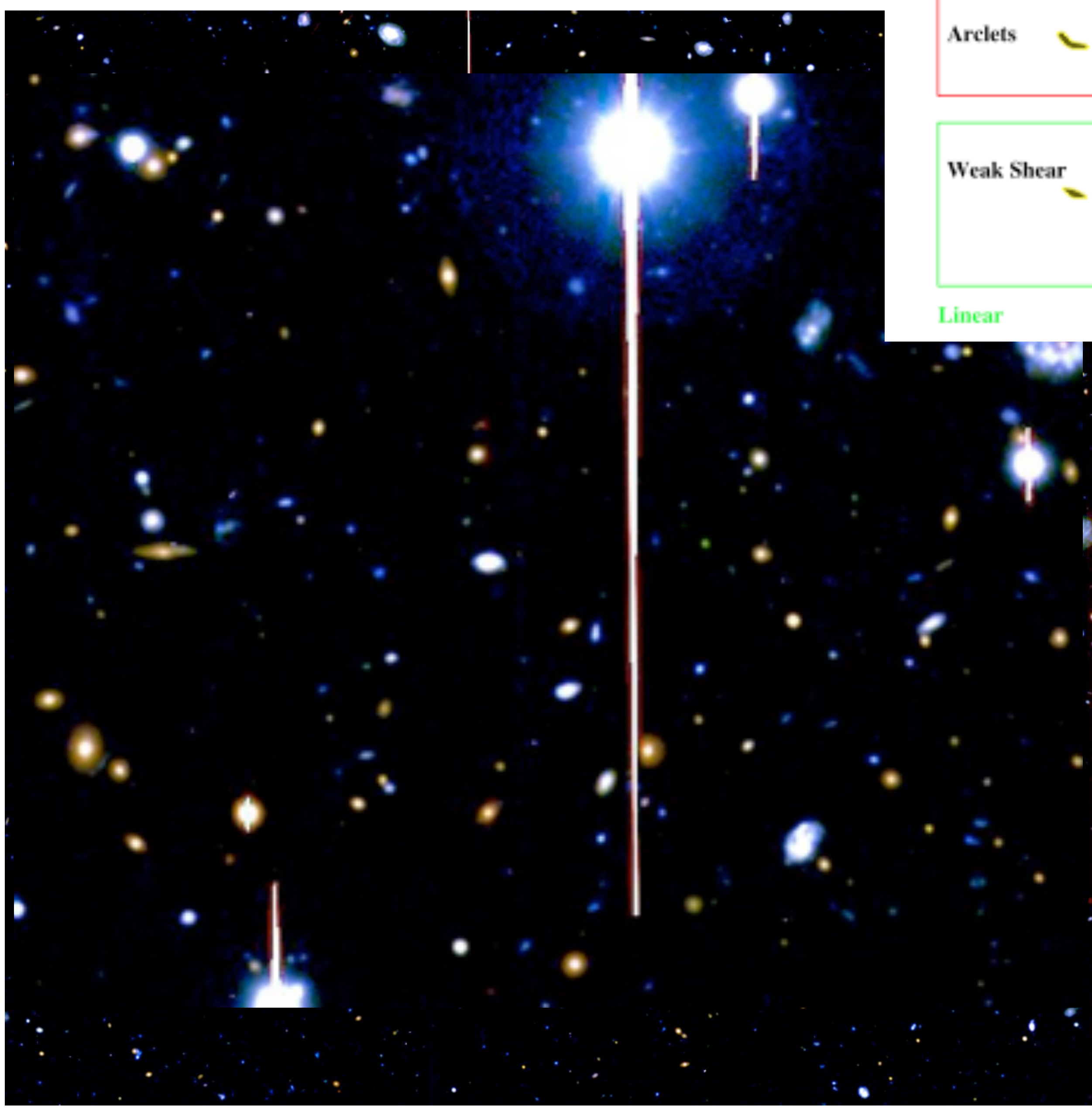
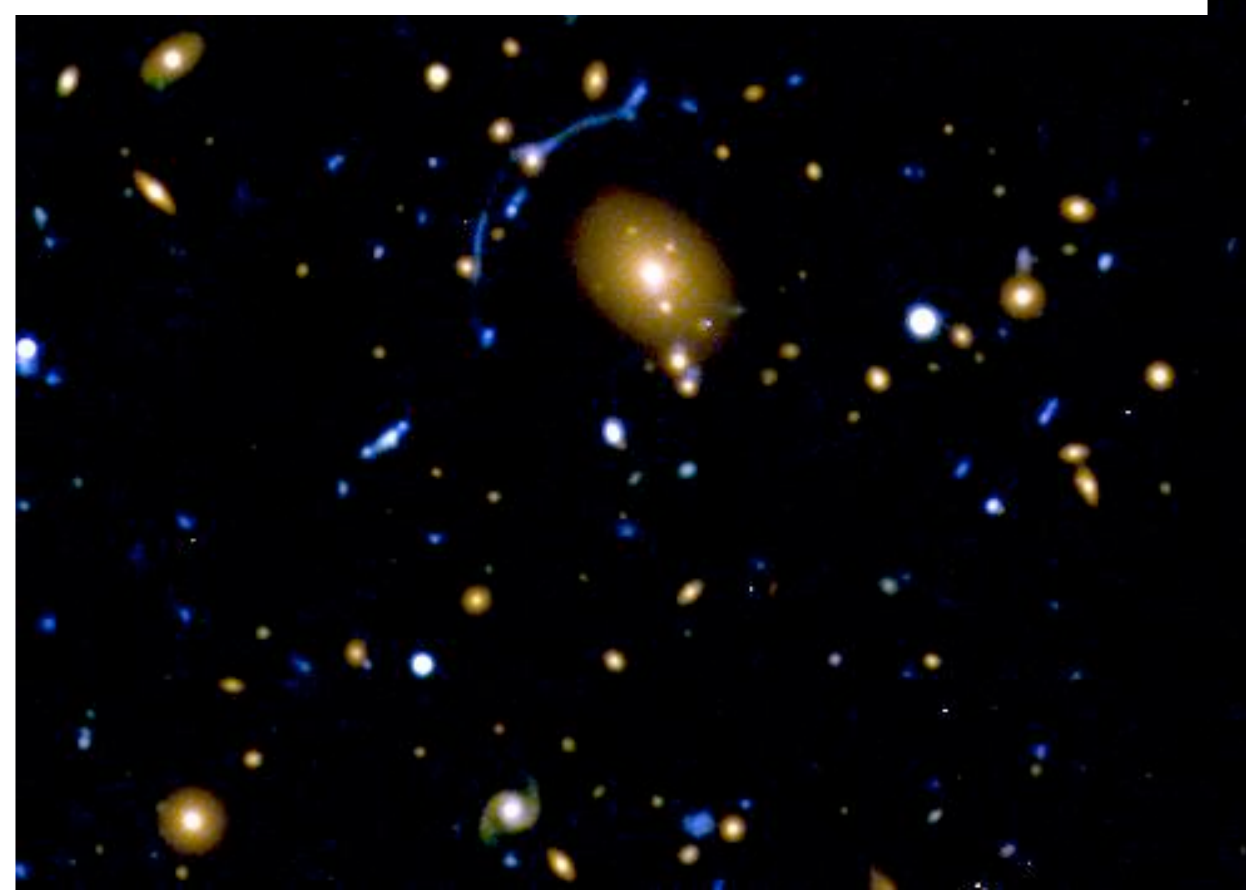




GALAXY CLUSTERS AS W







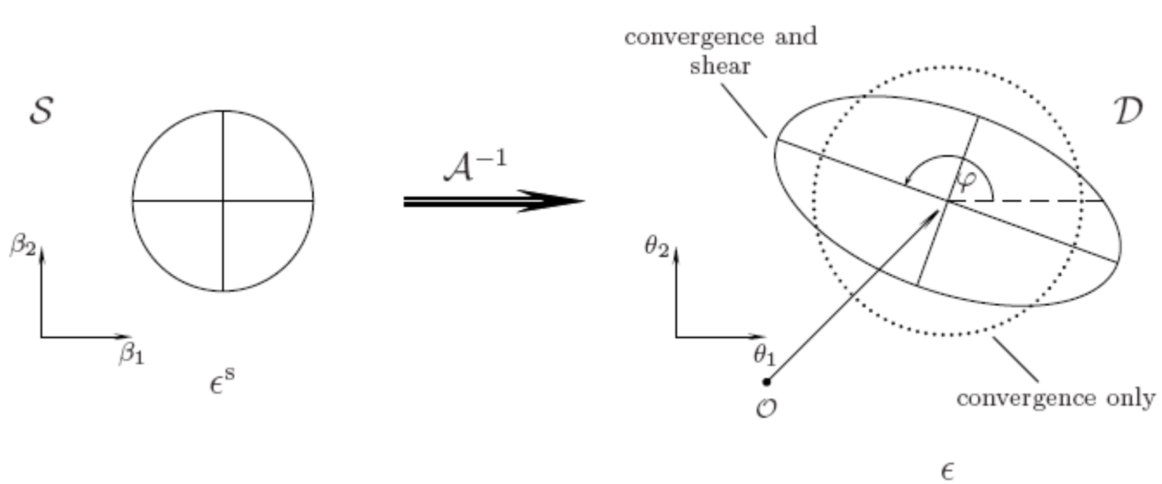
FIRST ORDER LENSING

- Let remind how lensing works in the limit of small deflections
- As we have seen, in this regime, the lens equation can be linearized and the lens mapping is described by the Jacobian matrix
- Circular sources are mapped on elliptical images

$$\boldsymbol{\beta} - \boldsymbol{\beta}_0 = \mathcal{A}(\boldsymbol{\theta}_0) \cdot (\boldsymbol{\theta} - \boldsymbol{\theta}_0)$$

$$\mathcal{A}(\boldsymbol{\theta}) = (1-\kappa) \begin{pmatrix} 1-g_1 & -g_2 \\ -g_2 & 1+g_1 \end{pmatrix}$$

$$g(\boldsymbol{\theta}) = \frac{\gamma(\boldsymbol{\theta})}{[1-\kappa(\boldsymbol{\theta})]}$$



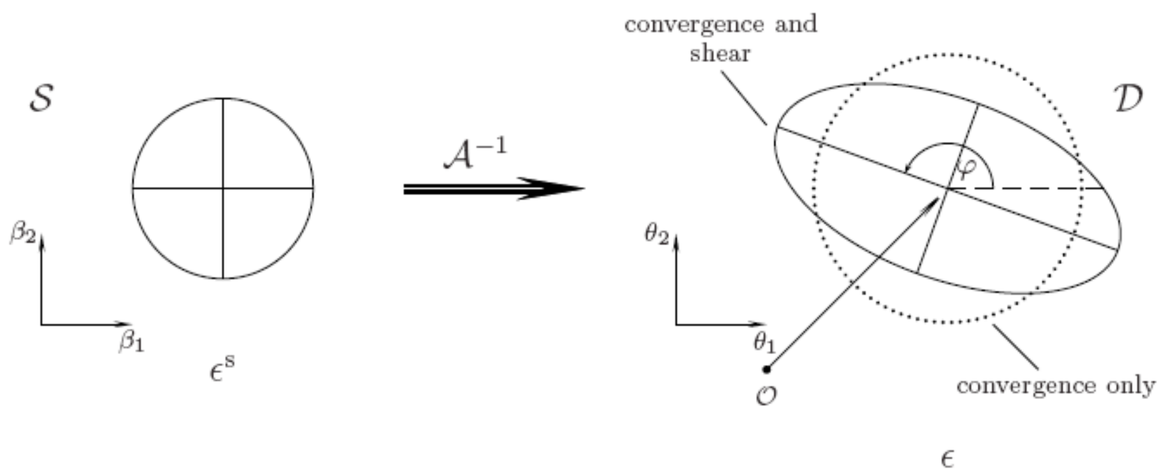
$$a = \frac{r}{1-\kappa-\gamma} \quad , \quad b = \frac{r}{1-\kappa+\gamma}$$

$$\epsilon = \frac{a-b}{a+b} = \frac{2\gamma}{2(1-\kappa)} = \frac{\gamma}{1-\kappa} \approx \gamma$$

$$|g| = \frac{1-b/a}{1+b/a} \quad \Leftrightarrow \quad \frac{b}{a} = \frac{1-|g|}{1+|g|}$$

FIRST ORDER LENSING

Thus, measuring the image ellipticities, we could measure a combination of shear and convergence, i.e. the second derivatives of the potential



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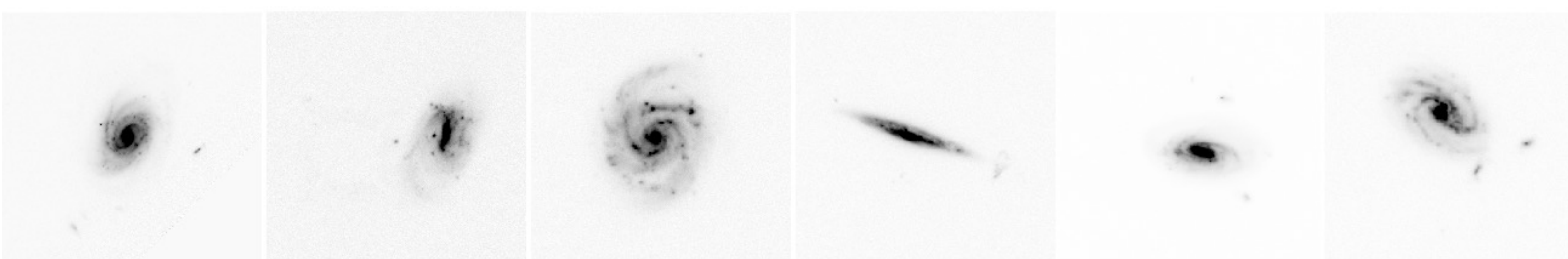
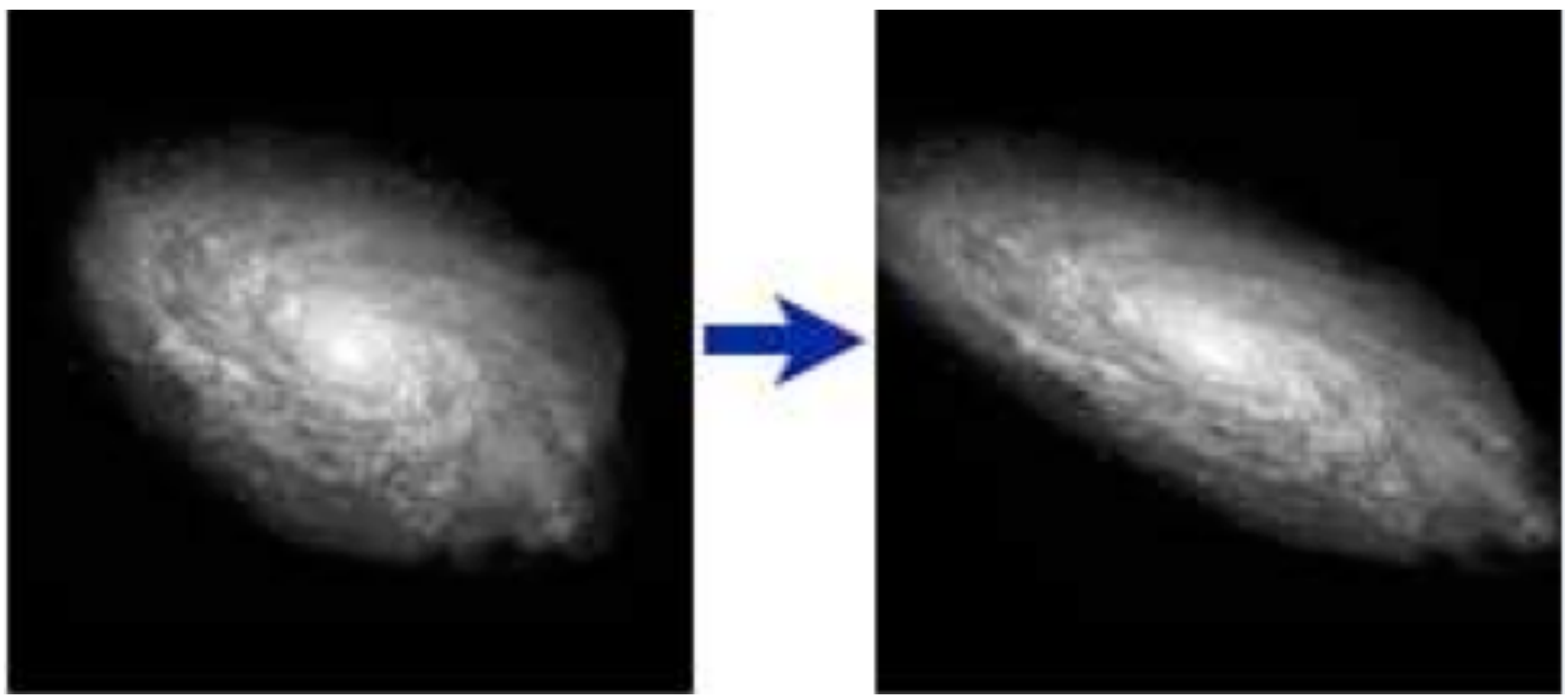
WHAT IS THE IMAGE ELLIPTICITY?



HOW TO SEPARATE THE LENSING SIGNAL FROM THE INTRINSIC ELLIPTICITY AND OTHER NOISES?



ONCE THE (REDUCED) SHEAR IS MEASURED, HOW DO WE CONVERT IT INTO A MASS (OR POTENTIAL) MEASUREMENT?

$$\kappa = \frac{1}{2}(\psi_{11} + \psi_{22})$$

$$\gamma_1 = \frac{1}{2}(\psi_{11} - \psi_{22})$$

$$\gamma_2 = \psi_{12}$$

MEASUREMENTS OF GALAXY SHAPES

Observable: brightness distribution

First moment: image centroid
$$\bar{\boldsymbol{\theta}} \equiv \frac{\int d^2\theta \, I(\boldsymbol{\theta}) \, q_I[I(\boldsymbol{\theta})] \, \boldsymbol{\theta}}{\int d^2\theta \, I(\boldsymbol{\theta}) \, q_I[I(\boldsymbol{\theta})]}$$

$$q_I(I) = \mathrm{H}(I - I_{\mathrm{th}}).$$

Define a tensor of second order brightness moments:

$$Q_{ij} = \frac{\int d^2\theta \, I(\boldsymbol{\theta}) \, q_I[I(\boldsymbol{\theta})] \, (\theta_i - \bar{\theta}_i) \, (\theta_j - \bar{\theta}_j)}{\int d^2\theta \, I(\boldsymbol{\theta}) \, q_I[I(\boldsymbol{\theta})]} \, , \quad i, j \in \{1, 2\}$$

Diagonalizing the tensor, we find the principal axes of the ellipse. The eigenvalues are:

$$\lambda_\pm = \frac{1}{2}\left[Q_{11} + Q_{22} \pm \sqrt{(Q_{11} - Q_{22})^2 - 4Q_{12}}\right]$$

giving the squares of the ellipse semi-axes.

The position angle is: $\tan(2\phi) = \dfrac{2Q_{12}}{Q_{11} - Q_{22}}$

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COMPLEX ELLIPTICITY AND SHEAR

For the shear, we defined two components: $\gamma = (\gamma_1, \gamma_2)$

$$\gamma_1 = \gamma \cos(2\phi)$$
$$\gamma_2 = \gamma \sin(2\phi)$$

It is very common to use a complex notation to write the shear as:

$$\gamma = \gamma_1 + i\gamma_2 = |\gamma| e^{2i\phi}$$

Similarly, we can define the complex reduced shear and ellipticity:

$$g = \frac{\gamma}{1-\kappa} = g_1 + ig_2 = |g| e^{2i\phi}$$

$$\epsilon = \epsilon_1 + i\epsilon_2 = |\epsilon| e^{2i\phi}$$

Using the previous formulae:

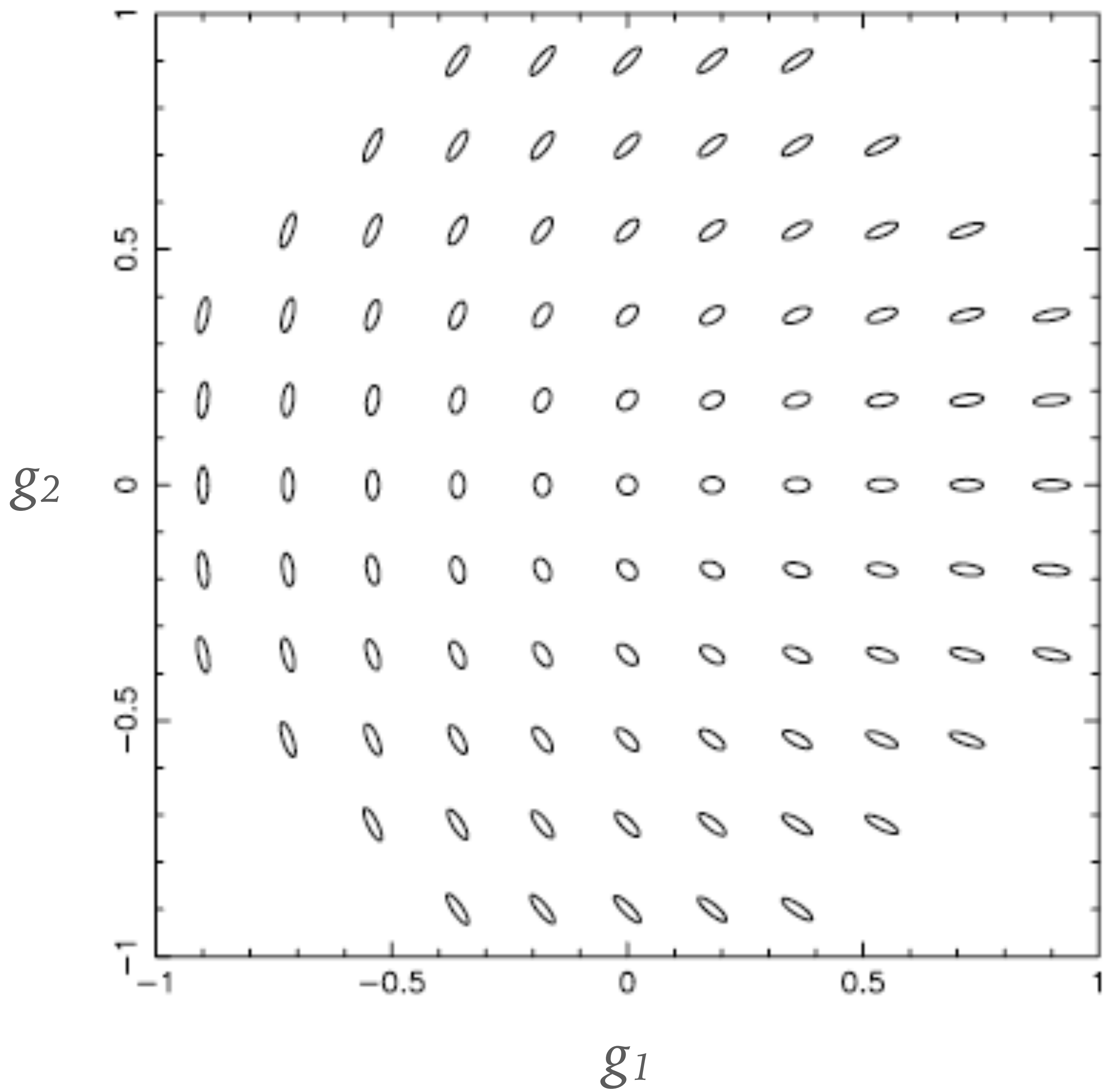
$$\epsilon \equiv \frac{Q_{11} - Q_{22} + 2\mathrm{i}Q_{12}}{Q_{11} + Q_{22} + 2(Q_{11}Q_{22} - Q_{12}^2)^{1/2}}$$

COMPLEX ELLIPTICITY AND SHEAR

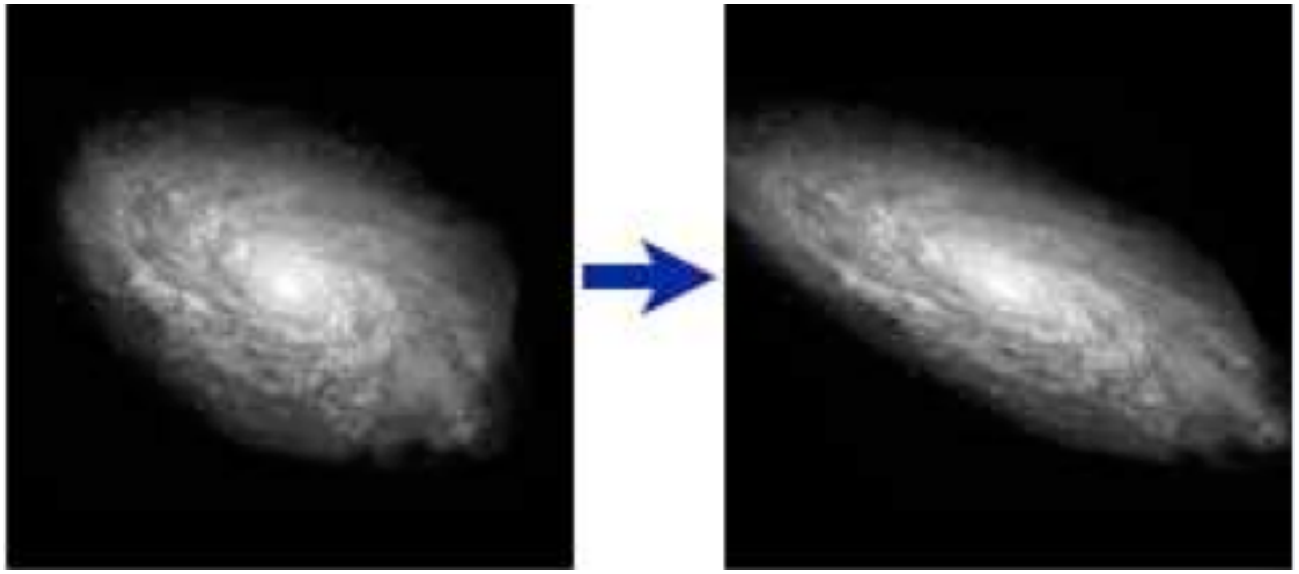
g_2

g_1

COMPLEX ELLIPTICITY AND SHEAR



FROM SOURCE TO IMAGE ELLIPTICITY

$I_s(\vec{\beta})$  $I(\vec{\theta})$

How are the two brightness distributions linked?

FROM SOURCE TO IMAGE ELLIPTICITY

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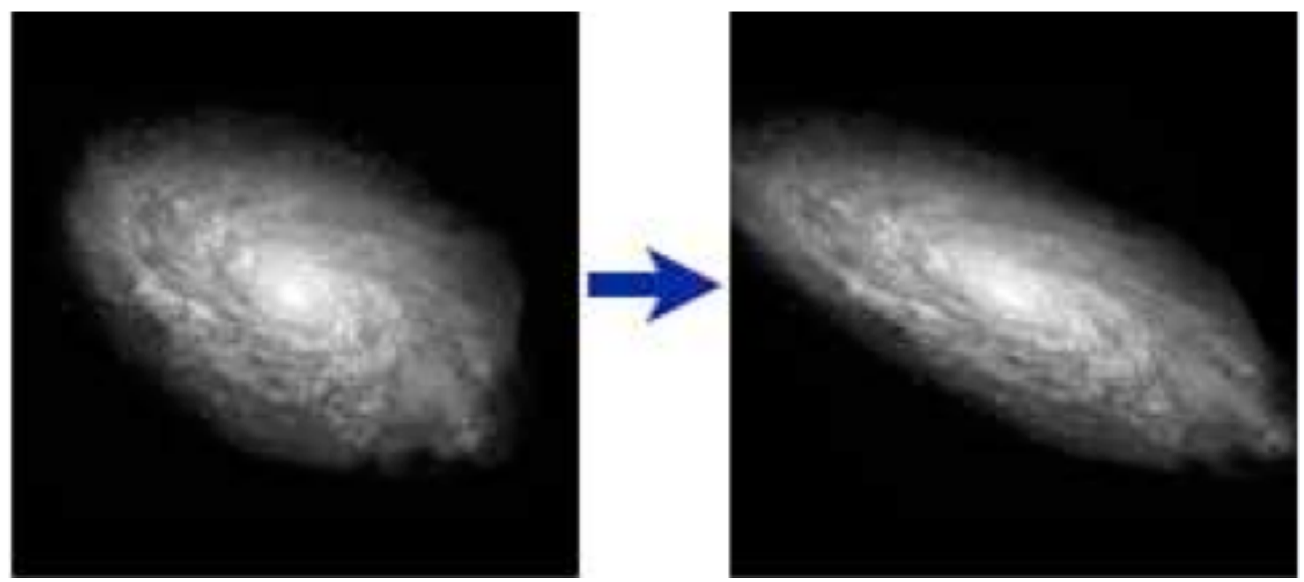

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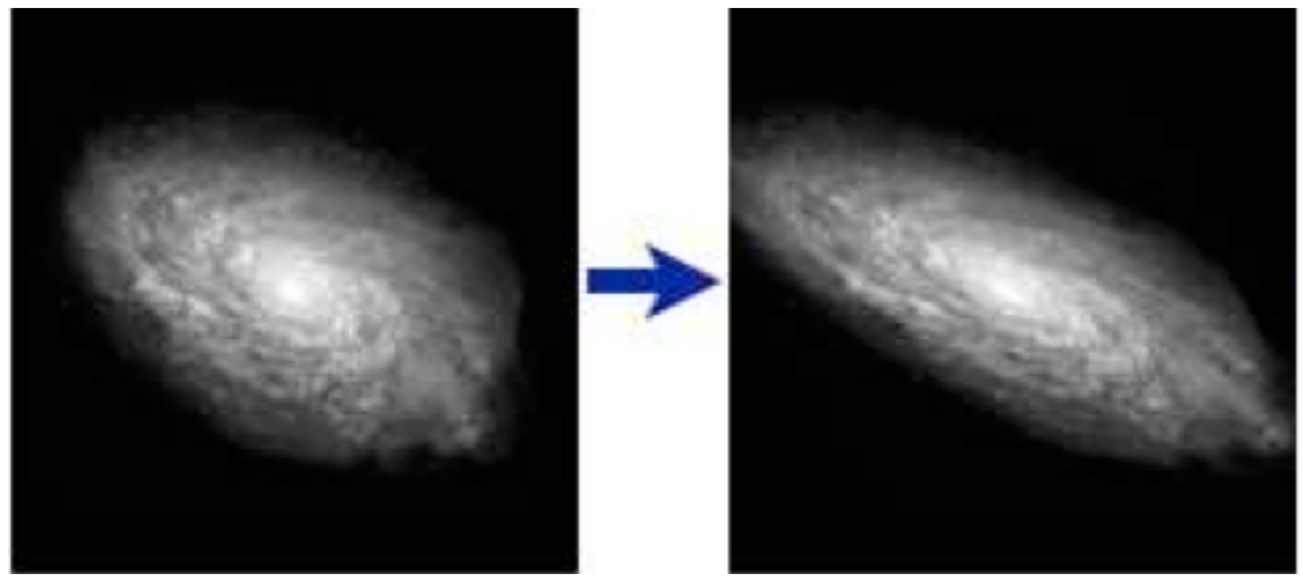
$$I(\boldsymbol{\theta}) = I^{(\mathrm{s})}[\boldsymbol{\beta}_0 + \mathcal{A}(\boldsymbol{\theta}_0) \cdot (\boldsymbol{\theta} - \boldsymbol{\theta}_0)]$$

FROM SOURCE TO IMAGE ELLIPTICITY



$$Q_{ij}^{(s)} = \frac{\int d^2\beta \, I^{(s)}(\boldsymbol{\theta}) \, q_I[I^{(s)}(\boldsymbol{\beta})] \, (\beta_i - \bar{\beta}_i) \, (\beta_j - \bar{\beta}_j)}{\int d^2\beta \, I^{(s)}(\boldsymbol{\theta}) \, q_I[I^{(s)}(\boldsymbol{\beta})]} \qquad Q_{ij} = \frac{\int d^2\theta \, I(\boldsymbol{\theta}) \, q_I[I(\boldsymbol{\theta})] \, (\theta_i - \bar{\theta}_i) \, (\theta_j - \bar{\theta}_j)}{\int d^2\theta \, I(\boldsymbol{\theta}) \, q_I[I(\boldsymbol{\theta})]} , \quad i, j \in \{1, 2\}$$

Given that $\beta - \bar{\beta} = \mathcal{A} \left(\boldsymbol{\theta} - \bar{\boldsymbol{\theta}} \right)$ $\qquad d^2\beta = \det \mathcal{A} \, d^2\theta,$

We find that $Q^{(s)} = \mathcal{A} \, Q \, \mathcal{A}^T = \mathcal{A} \, Q \, \mathcal{A}$

which gives:

$$\epsilon^{(s)} = \begin{cases} \dfrac{\epsilon - g}{1 - g^* \epsilon} & \text{if } |g| \le 1 \, ; \\ \\ \dfrac{1 - g \epsilon^*}{\epsilon^* - g^*} & \text{if } |g| > 1 \, . \end{cases}$$

The inverse transformations are found by changing the source and the image ellipticities and g with -g

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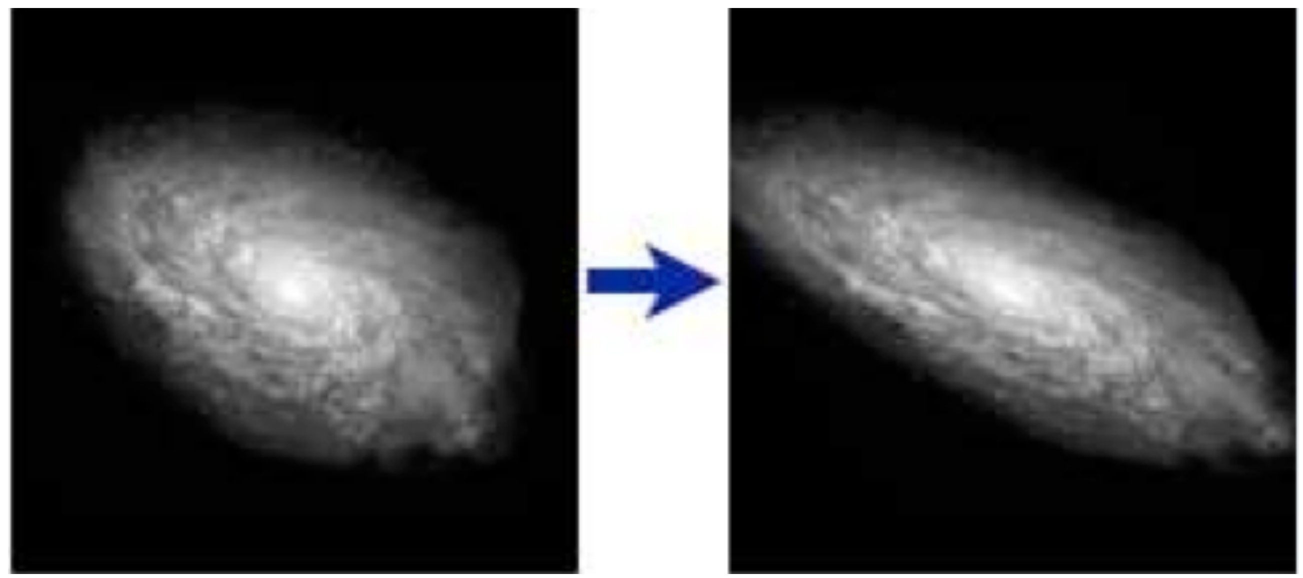

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EXTRACTING THE SIGNAL

Still, we have the problem that we don't know the intrinsic ellipticity: without it, we cannot determine g!

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Suppose that sources have intrinsically random phases.

In this case, averaging over a number of sources, the expectation value of the ellipticity is...

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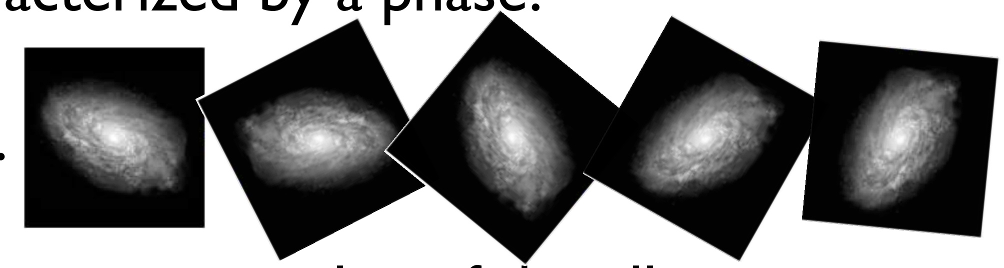

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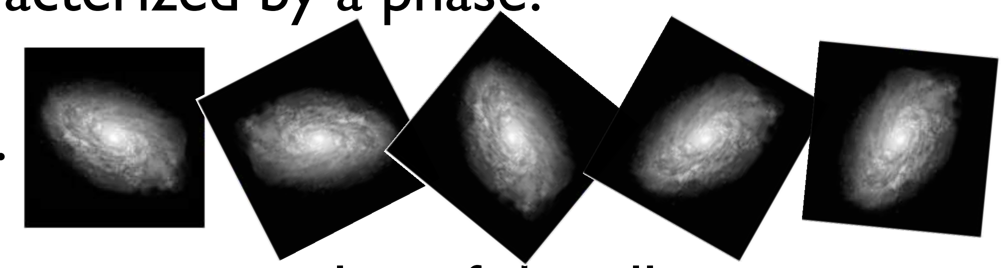


In this case, averaging over a number of sources, the expectation value of the ellipticity is...

$$E(\epsilon^{(s)}) = 0$$

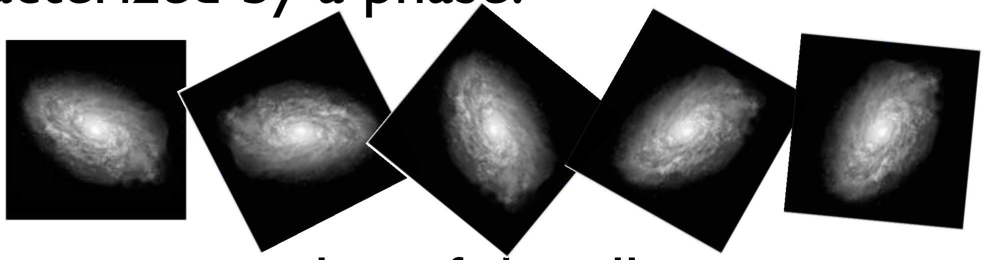
EXTRACTING THE SIGNAL

Still, we have the problem that we don't know the intrinsic ellipticity: without it, we cannot determine g!

$$\epsilon^{(\mathrm{s})} = \begin{cases} \dfrac{\epsilon - g}{1 - g^*\epsilon} & \text{if } |g| \leq 1 \; ; \\ \dfrac{1 - g\epsilon^*}{\epsilon^* - g^*} & \text{if } |g| > 1 \; . \end{cases}$$

Remember that ellipticities are complex numbers characterized by a phase.

Suppose that sources have intrinsically random phases.

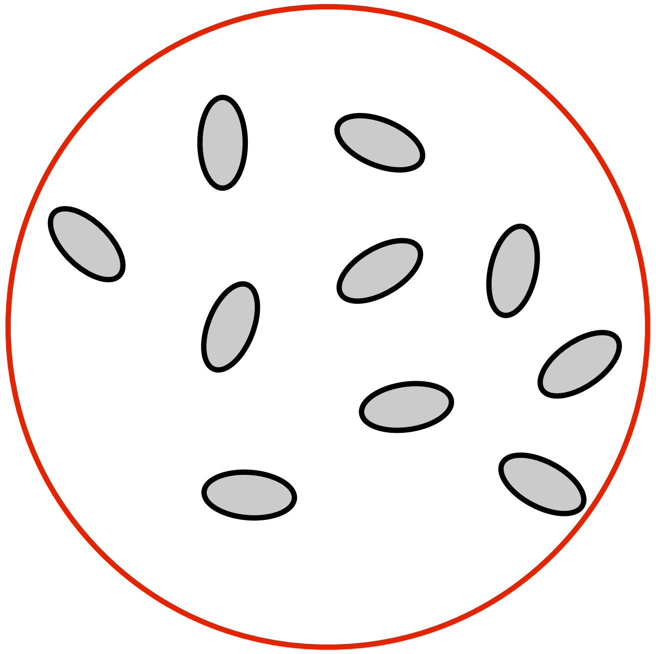


In this case, averaging over a number of sources, the expectation value of the ellipticity is...

$$E(\epsilon^{(s)}) = 0$$

Thus, from the above equations:

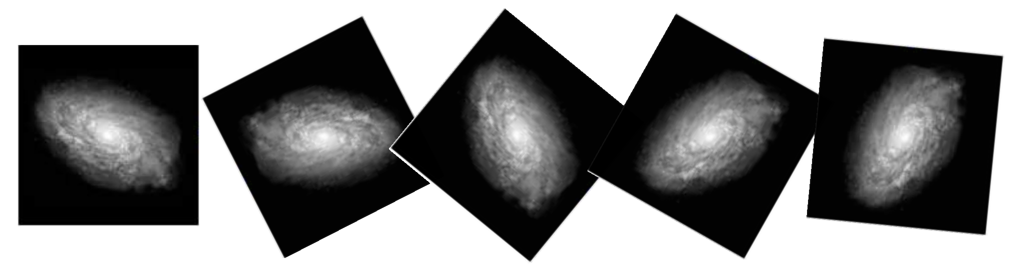
$$\mathrm{E}(\epsilon) = \begin{cases} g & \text{if } |g| \leq 1 \\ 1/g^* & \text{if } |g| > 1 \end{cases}$$



Which means that we can estimate the reduced shear by averaging over a number of sources:

$$g \approx \langle \epsilon \rangle$$

NOISE



The noise is given by the dispersion in the intrinsic ellipticity distribution

$$\sigma_\epsilon = \sqrt{\left\langle \epsilon^{(s)} \epsilon^{(s)*} \right\rangle}$$

Averaging over N galaxies, the 1-σ deviation from the mean ellipticity is

$$\sigma_\epsilon / \sqrt{N}$$

Thus, we can beat the noise by averaging over many galaxies!

- ☐ select a number of galaxies in a region and assume that the shear is constant within the region
- ☐ if the region is too large, the shear is smoothed
- ☐ increase the number density of galaxies

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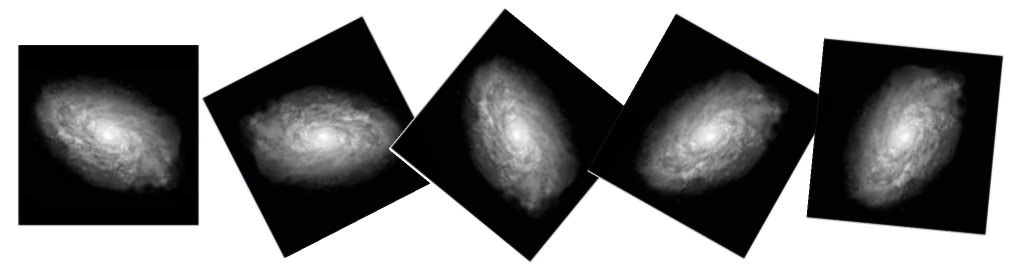

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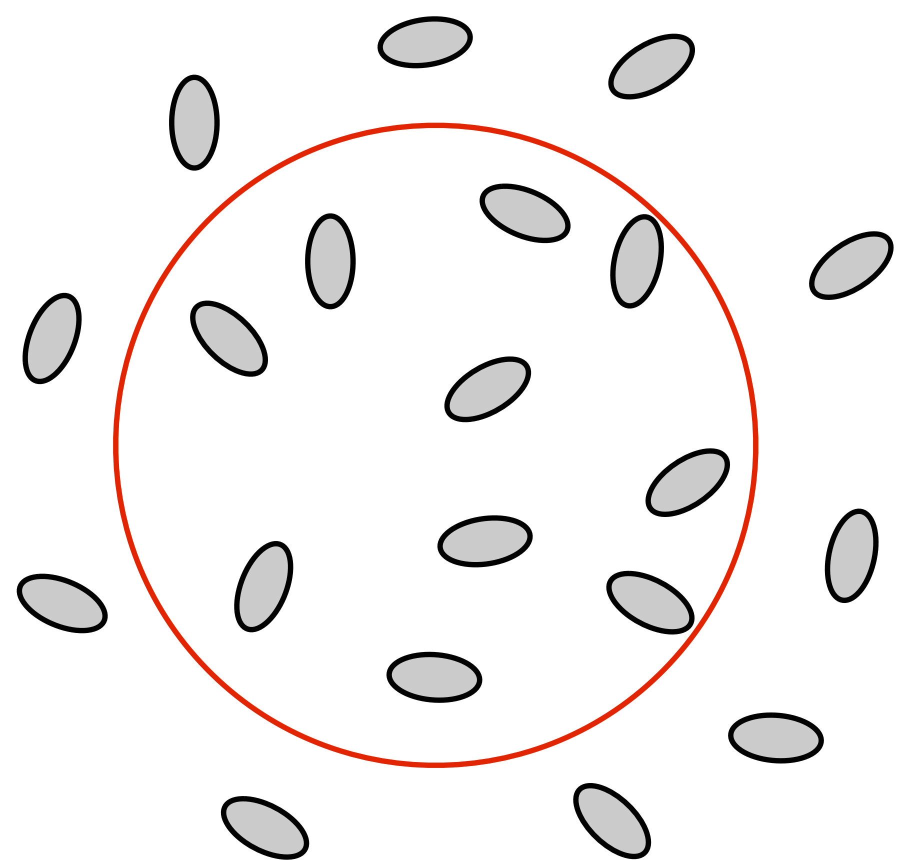

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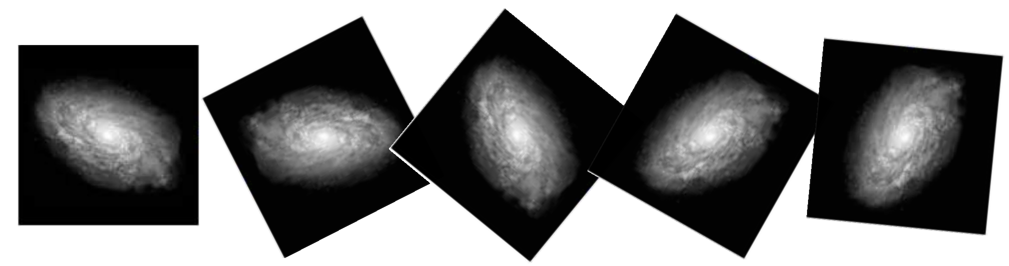

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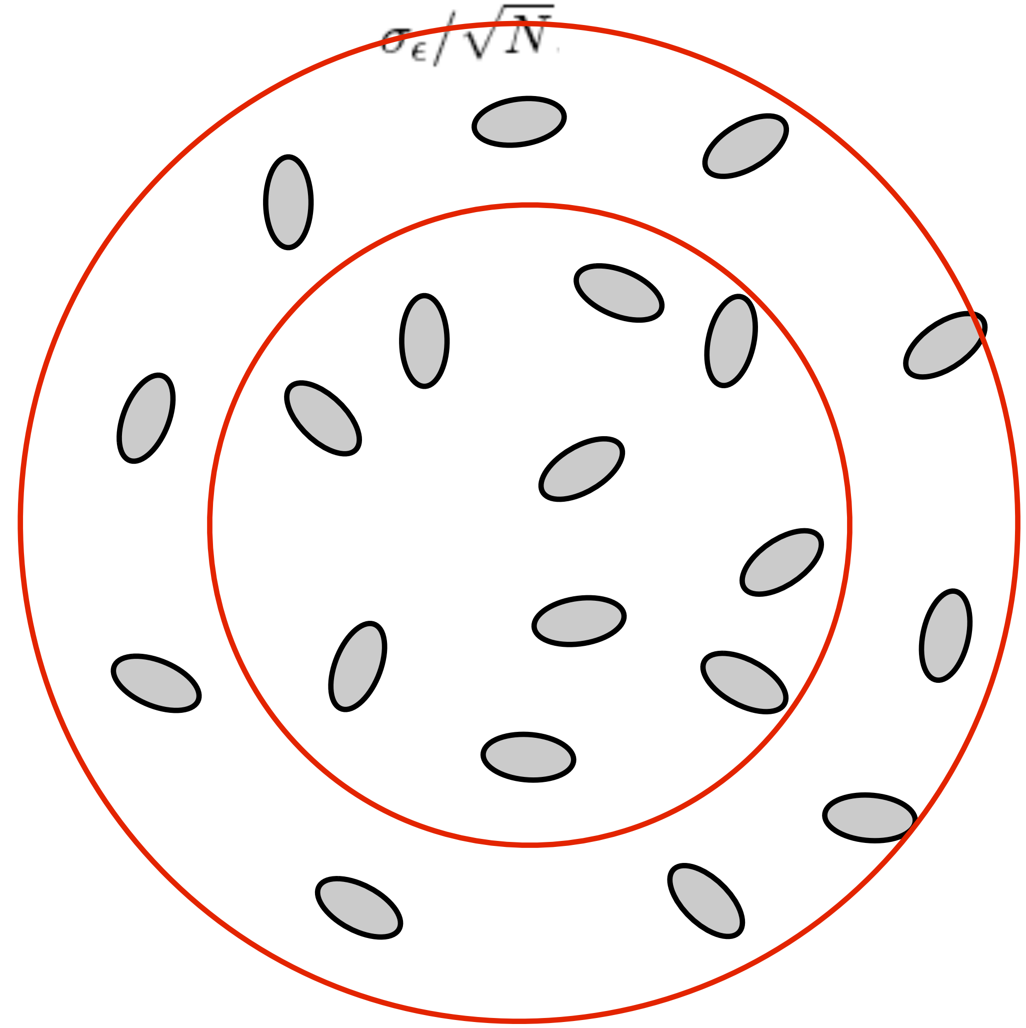

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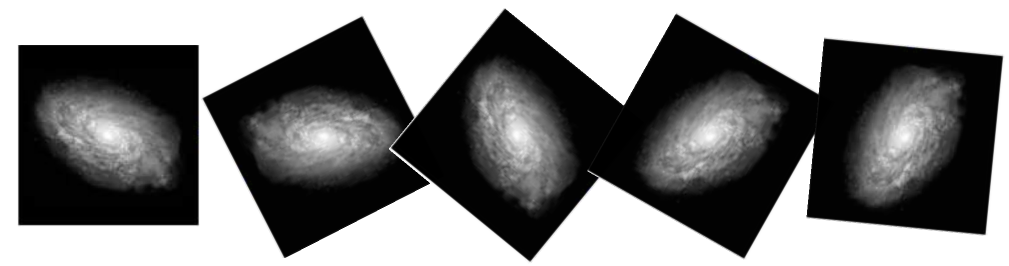

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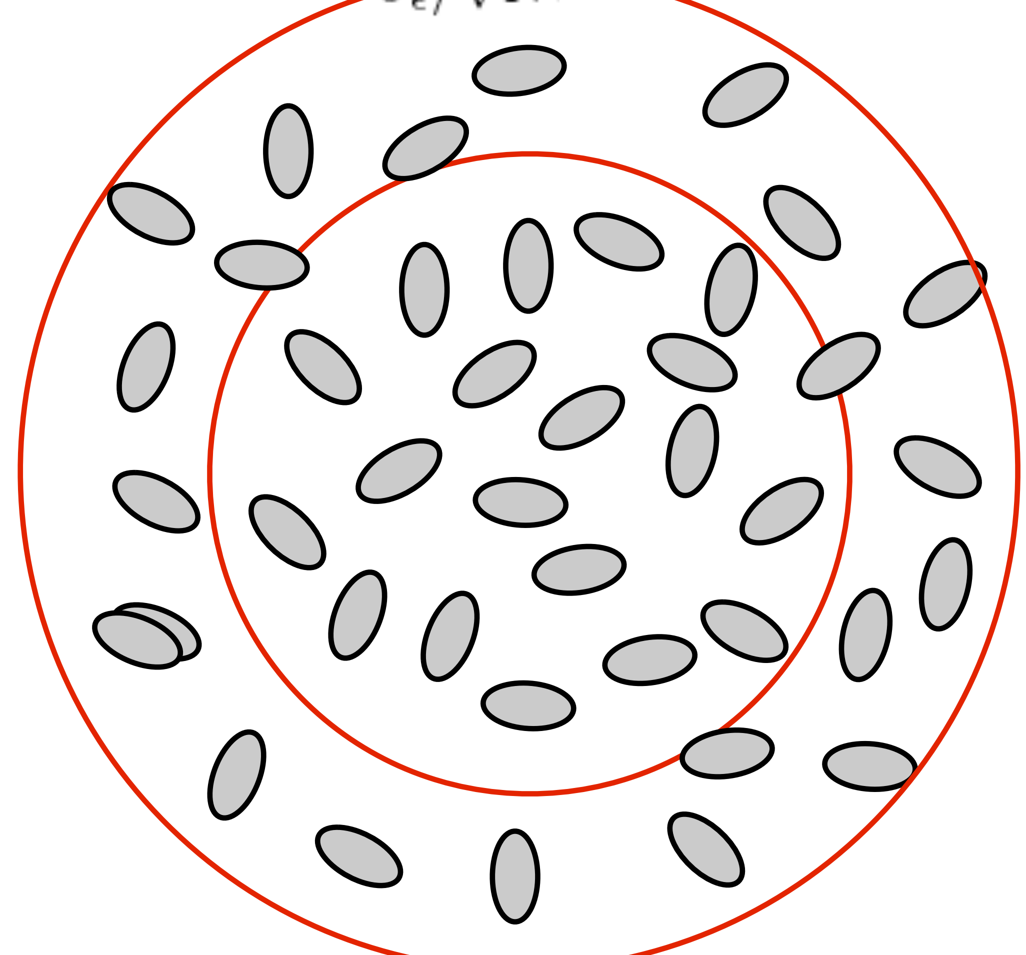

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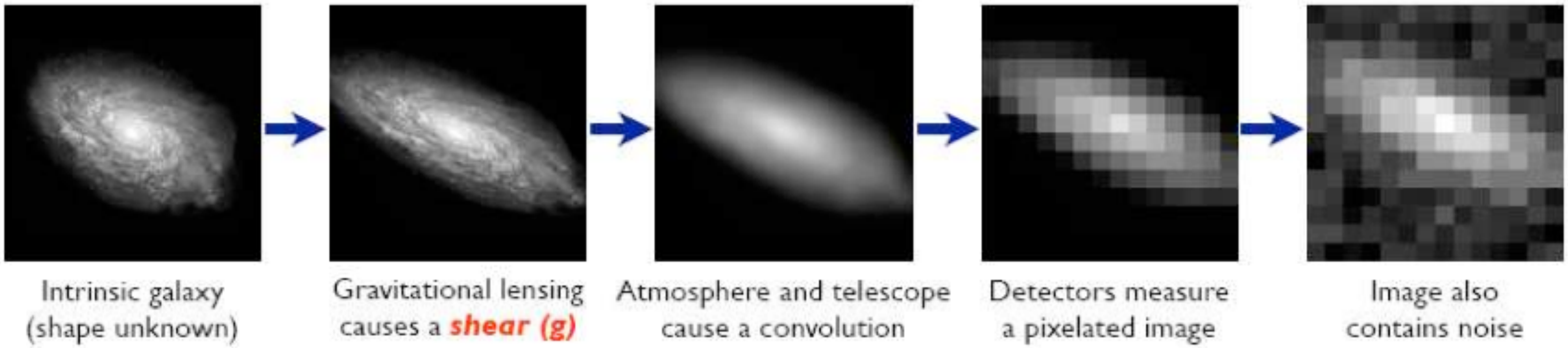
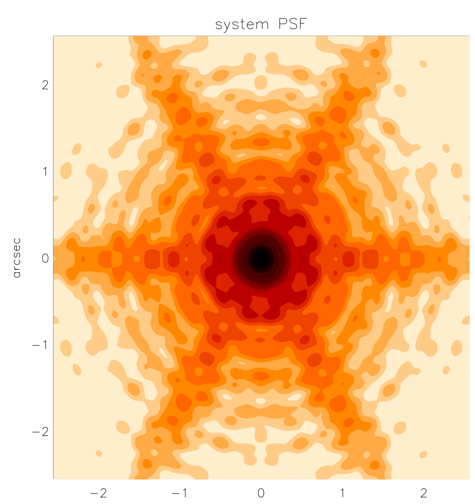
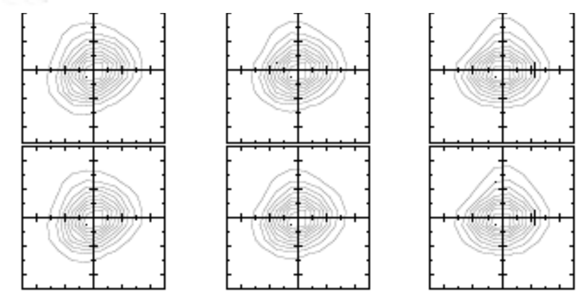
POINT SPREAD FUNCTION



$$I^{\mathrm{obs}}(\theta) = \int \mathrm{d}^2\vartheta \; I(\vartheta)\, P(\theta - \vartheta)$$

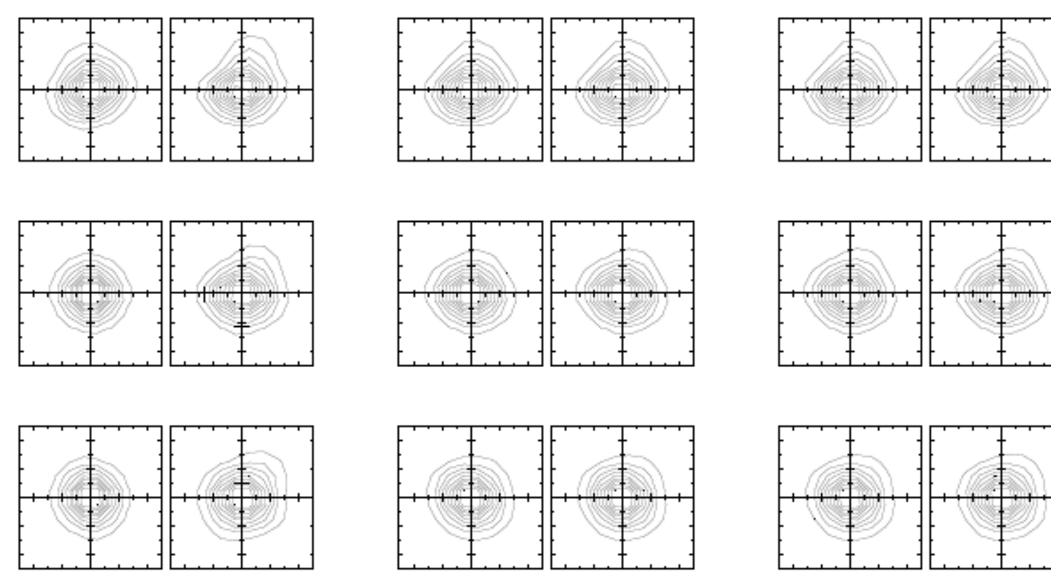




PSF has several contributors: telescope (airy disk), atmosphere, AOCS,...

PSF can have weird shapes (anisotropy caused by coma, jitter, defocus, astigmatism, ecc.) and change across the field!

LBT →



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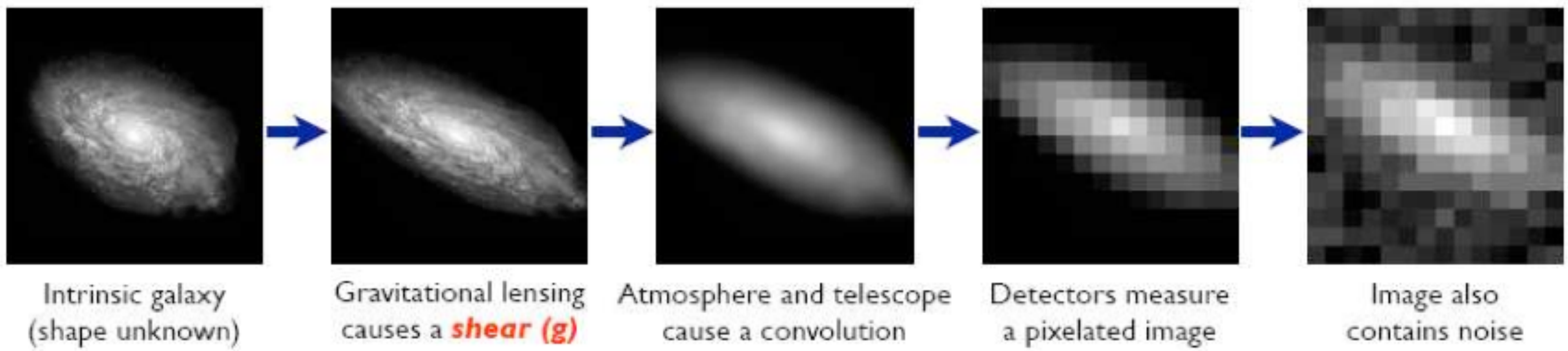

$$I^{\rm obs}(\boldsymbol{\theta}) = \int \mathrm{d}^2\vartheta \; I(\boldsymbol{\vartheta})\, P(\boldsymbol{\theta} - \boldsymbol{\vartheta})$$

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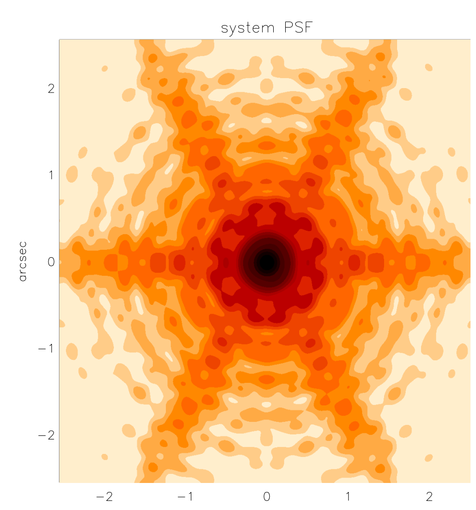

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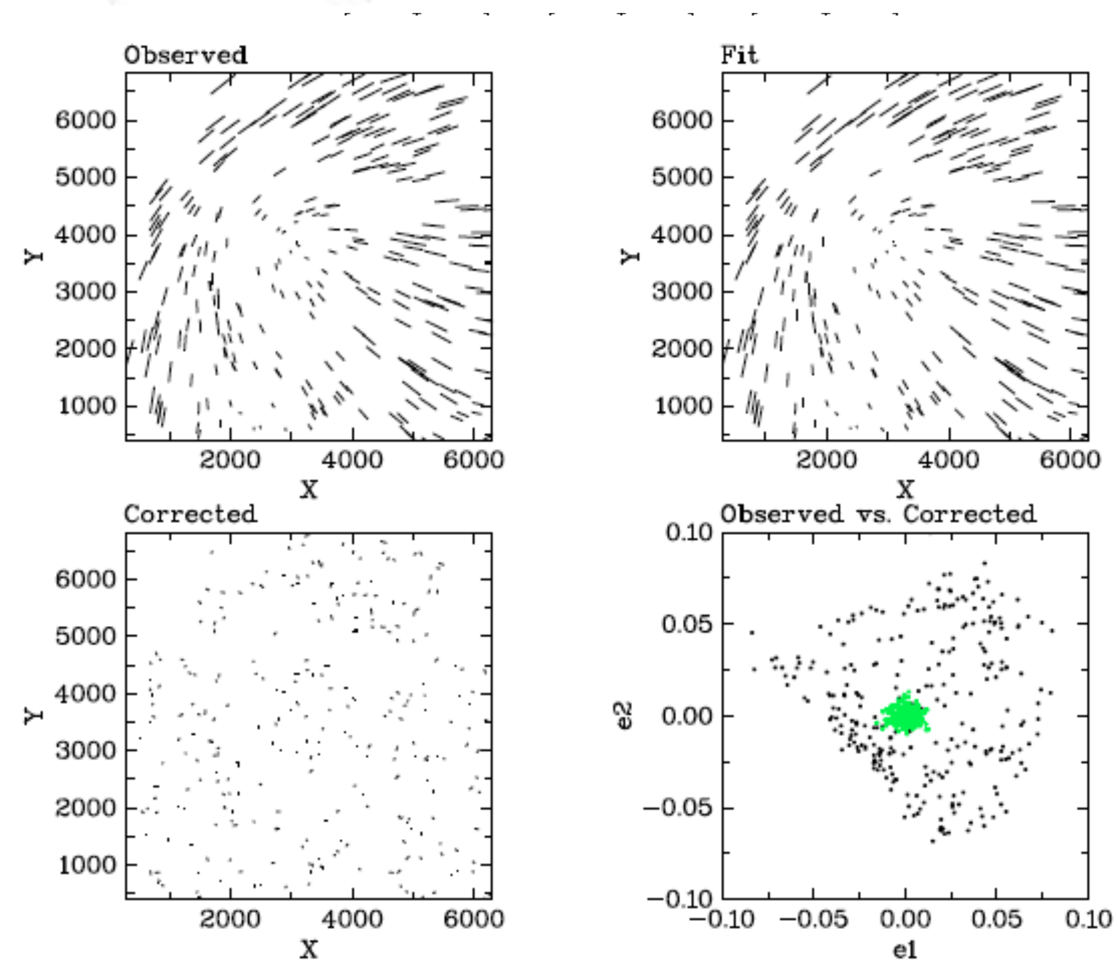

PRACTICAL MEASUREMENT OF SHEAR: THE KSB METHOD

$$\epsilon_\alpha^{obs} = \epsilon_\alpha^s + P_{\alpha\beta}^{sm} p_\beta + P_{\alpha\beta}^g g_\beta$$

Smear polarisability tensor: describes how the image ellipticity responds to the presence of a PSF anisotropy

Shear polarisability tensor: describes the response of the image ellipticity to the shear in presence of smearing

For a star:

$$p_\mu = (P^{\text{sm}*})^{-1}_{\mu\alpha}\, \varepsilon_\alpha^{*\text{obs}}$$

the anisotropy of the PSF is estimated from the stars near the galaxy

The estimated shear is

$$\epsilon = (P^g)^{-1}(\epsilon^{obs} - P^{sm} p)$$

ONCE MEASURED THE SHEAR, WHAT DO WE DO?

Several ways to convert the shear measurement into a mass estimate:

- some methods are parametric
- other methods are free-form

THE KAISER & SQUIRES INVERSION ALGORITHM

Fourier transform: $\hat{f}(k) = \int_{-\infty}^{+\infty} f(x) e^{-ikx} dx \qquad f(x) = \int_{-\infty}^{+\infty} \hat{f}(k) e^{ikx} dk$

$$\hat{f}(\vec{k}) = \int_{-\infty}^{+\infty} f(\vec{x}) e^{-2i\vec{k}\vec{x}} d^2k \quad f(\vec{x}) = \int_{-\infty}^{+\infty} \hat{f}(\vec{k}) e^{2i\vec{k}\vec{x}} d^2x$$

Shear and convergence:

$$\kappa = \frac{1}{2}(\psi_{11} + \psi_{22}) \quad \Rightarrow \quad \hat{\kappa} = -\frac{1}{2}(k_1^2 + k_2^2)\hat{\psi}$$

$$\gamma_1 = \frac{1}{2}(\psi_{11} - \psi_{22}) \quad \Rightarrow \quad \hat{\gamma}_1 = -\frac{1}{2}(k_1^2 - k_2^2)\hat{\psi}$$

$$\gamma_2 = \psi_{12} \quad \Rightarrow \quad \hat{\gamma}_2 = -k_1 k_2 \hat{\psi} \ ,$$

Real space Fourier space

THE KAISER & SQUIRES INVERSION ALGORITHM

From:

$$
\begin{aligned}
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$$\begin{pmatrix} \hat{\gamma}_1 \\ \hat{\gamma}_2 \end{pmatrix} = k^{-2} \begin{pmatrix} k_1^2 - k_2^2 \\ 2k_1 k_2 \end{pmatrix} \hat{\kappa}$$

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using:

$$\left[k^{-2} \begin{pmatrix} k_1^2 - k_2^2 \\ 2k_1k_2 \end{pmatrix}\right] [k^{-2} (\ k_1^2 - k_2^2 \quad 2k_1k_2 \ )] = 1$$

$$\hat{\kappa} = k^{-2} (\ k_1^2 - k_2^2 \quad 2k_1k_2 \ ) \begin{pmatrix} \hat{\gamma}_1 \\ \hat{\gamma}_2 \end{pmatrix} = k^{-2}[(k_1^2 - k_2^2)\hat{\gamma}_1 + 2k_1k_2\hat{\gamma}_2]$$

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$$(\widehat{f * g}) = \hat{f}\hat{g}$$

$$\hat{\kappa} = k^{-2} \begin{pmatrix} k_1^2 - k_2^2 & 2k_1 k_2 \end{pmatrix} \begin{pmatrix} \hat{\gamma}_1 \\ \hat{\gamma}_2 \end{pmatrix} = k^{-2}[(k_1^2 - k_2^2)\hat{\gamma}_1 + 2k_1 k_2 \hat{\gamma}_2]$$

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$$\kappa(\vec{\theta}) = \frac{1}{\pi} \int \mathrm{d}^2\theta' [D_1(\vec{\theta} - \vec{\theta}')\gamma_1 + D_2(\vec{\theta} - \vec{\theta}')\gamma_2]$$

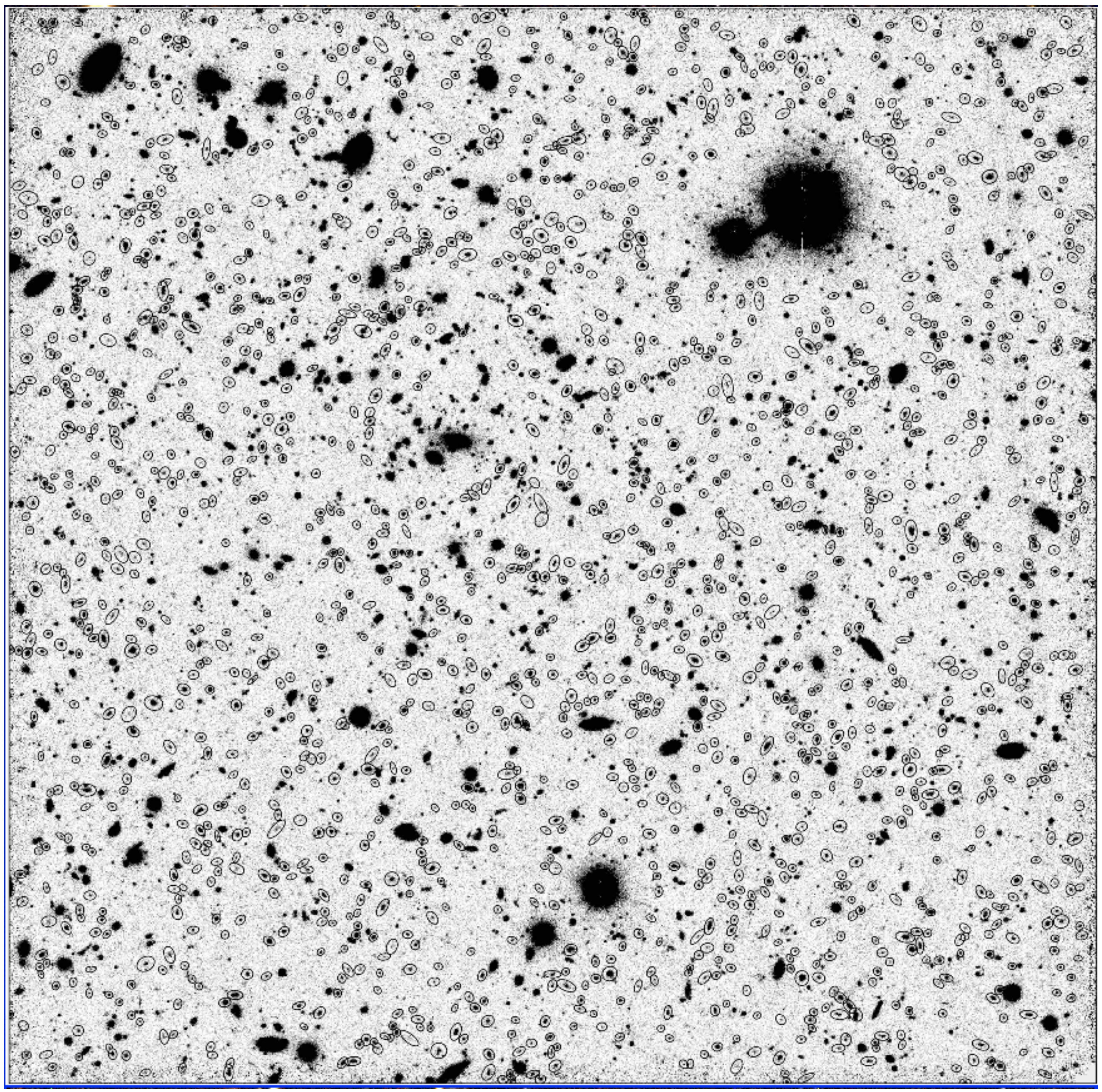
$$(f \hat{*} g) = \hat{f}\hat{g}$$

$$\hat{\kappa} = k^{-2} (\ k_1^2 - k_2^2 \quad 2k_1 k_2 \ ) \begin{pmatrix} \hat{\gamma}_1 \\ \hat{\gamma}_2 \end{pmatrix} = k^{-2}[(k_1^2 - k_2^2)\hat{\gamma}_1 + 2k_1 k_2 \hat{\gamma}_2]$$

THE KAISER & SQUIRES INVERSION ALGORITHM

CL1232-1250
(Clowe et al.)

THE KAISER & SQUIRES INVERSION ALGORITHM



CL1232-1250
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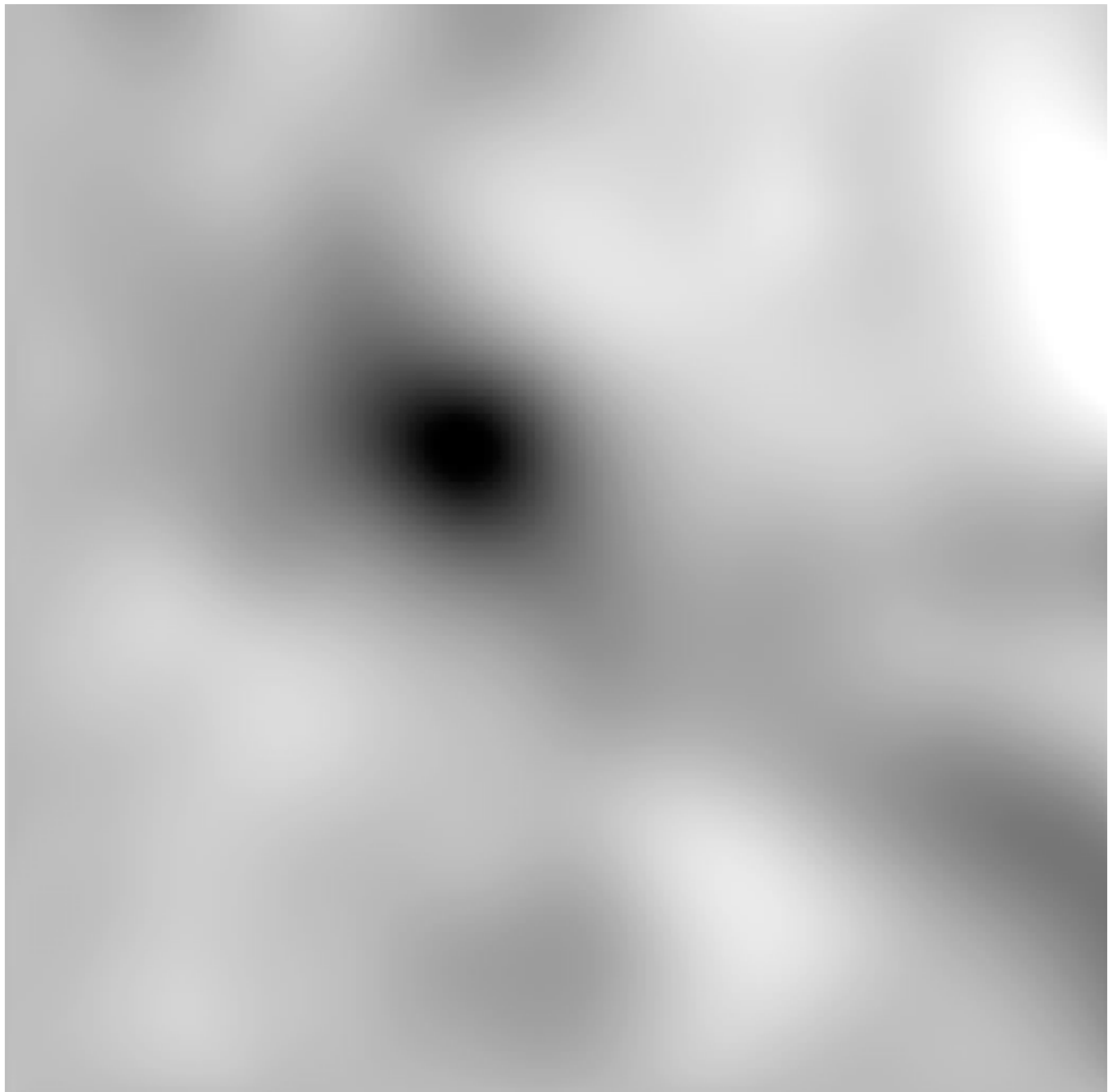


CL1232-1250
(Clowe et al.)

THE KAISER & SQUIRES INVERSION ALGORITHM

- infinite fields would be required: wide field + boundary conditions.
- ellipticity measures the reduced shear, not the shear:

$$\kappa(\vec{\theta}) = \frac{1}{\pi} \int \mathrm{d}^2\theta' [D_1(\vec{\theta} - \vec{\theta}')g_1(1-\kappa) + D_2(\vec{\theta} - \vec{\theta}')g_2(1-\kappa)]$$

This equation can be solved iteratively starting from κ=0

• mass sheet degeneracy...

MASS SHEET DEGENERACY

The mass sheet transformation on shear and convergence is:

$$\kappa \to \kappa' = (1-\lambda) + \lambda\kappa$$
$$\gamma \to \gamma' = \lambda\gamma$$

The ellipticity is then: $$\epsilon' = g' = \frac{\lambda\gamma}{1-(1-\lambda)-\lambda\kappa} = \frac{\lambda\gamma}{\lambda(1-\kappa)} = g = \epsilon$$

Thus, weak lensing is also variant under mass-sheet transformations!

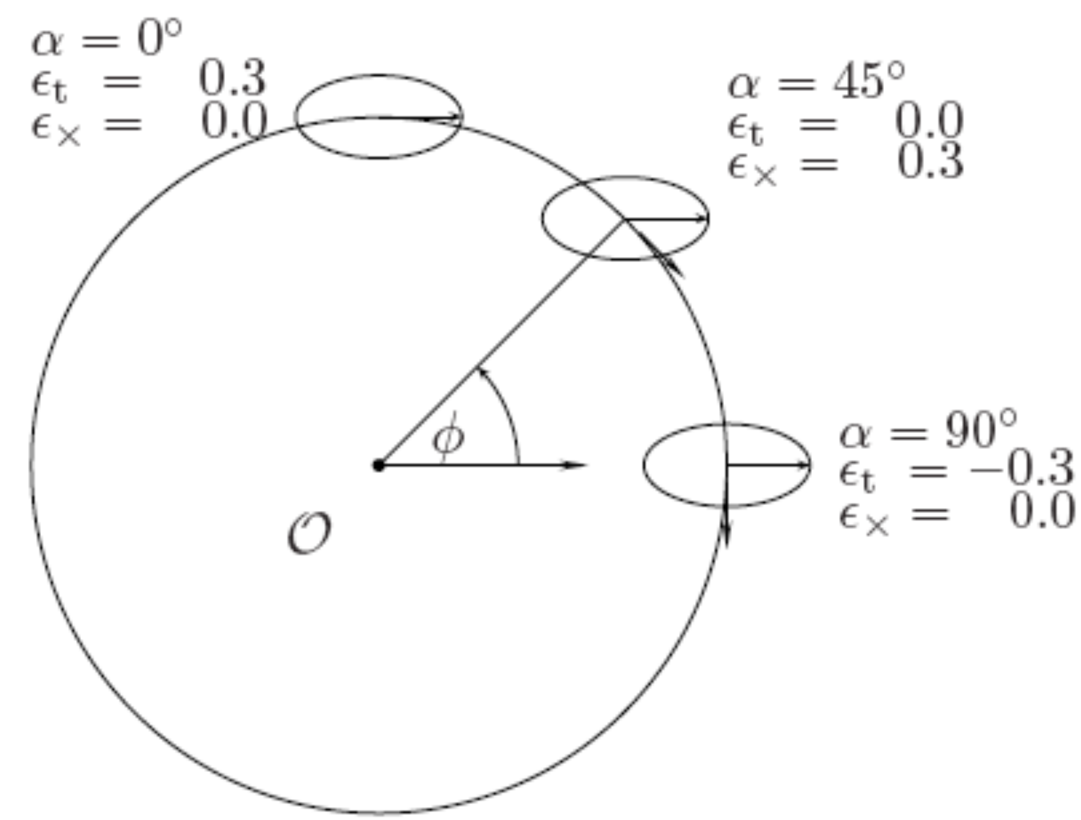
TANGENTIAL AND CROSS COMPONENT OF THE SHEAR

Given a direction ϕ we can define a tangential and a cross component of the ellipticity/shear relative to this direction.

$$\gamma_{\rm t} = -\mathcal{R}{\rm e}\left[\gamma\,{\rm e}^{-2{\rm i}\phi}\right] \quad , \quad \gamma_{\times} = -\mathcal{I}{\rm m}\left[\gamma\,{\rm e}^{-2{\rm i}\phi}\right]$$

Note that, under this convention, "tangential" means both tangentially and radially oriented ellipticities

With this we want to emphasize that lensing, being caused by a scalar potential is curl-free



The signs are chosen such that the tangential component is positive for tangentially distorted images, and it is negative for radially distorted images.

FIT OF THE TANGENTIAL SHEAR PROFILE

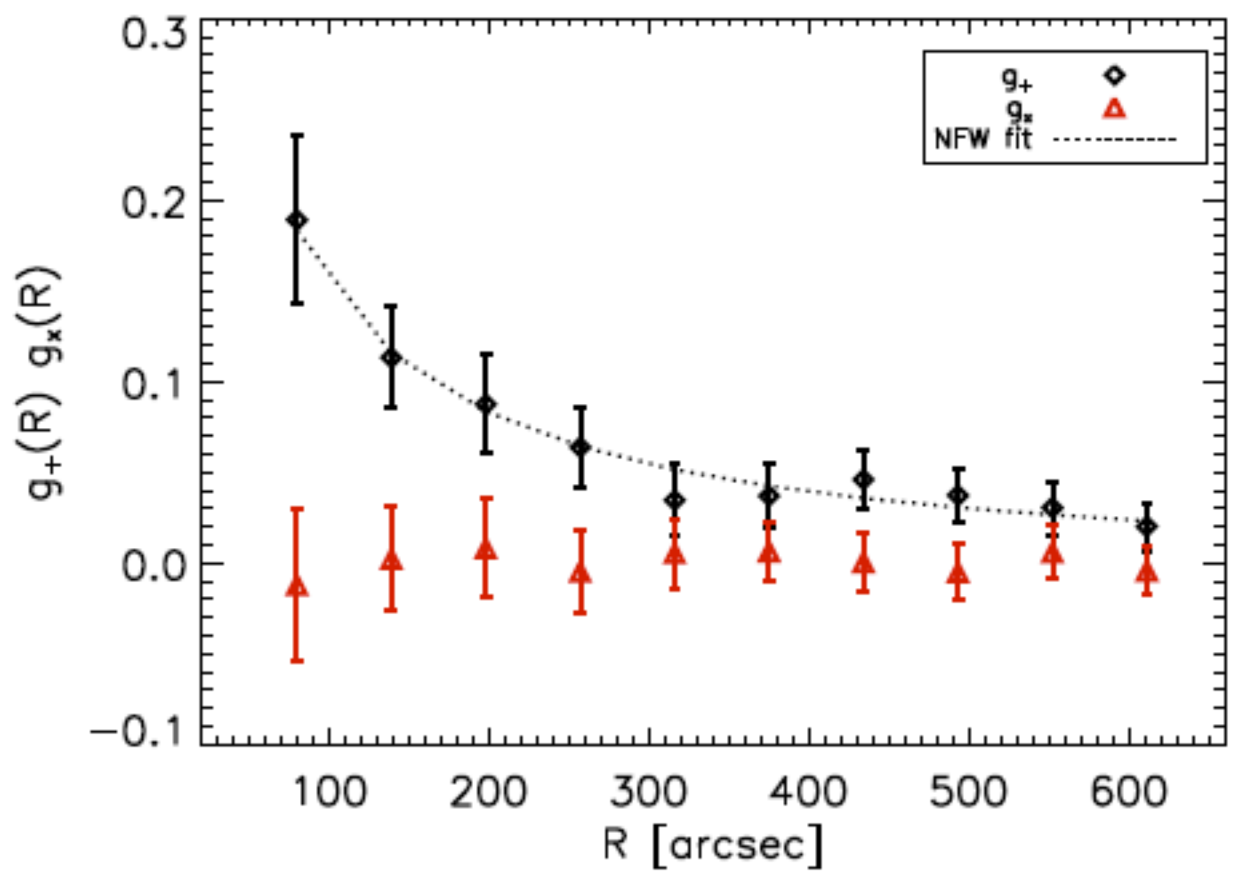
Having measured the tangential shear profile, we can fit it with some parametric model

SIS $\gamma(x) = (\gamma_1^2 + \gamma_2^2)^{1/2} = \frac{1}{2x} = \kappa(x)$

NFW $\kappa(x) = \frac{\Sigma(\xi_0 x)}{\Sigma_{cr}} = 2\kappa_s \frac{f(x)}{x^2 - 1}$

$$f(x) = \begin{cases} 1 - \frac{2}{\sqrt{x^2-1}} \text{arctan}\sqrt{\frac{x-1}{x+1}} & (x > 1) \\ 1 - \frac{2}{\sqrt{1-x^2}} \text{arctanh}\sqrt{\frac{1-x}{1+x}} & (x < 1) \\ 0 & (x = 1) \end{cases}$$

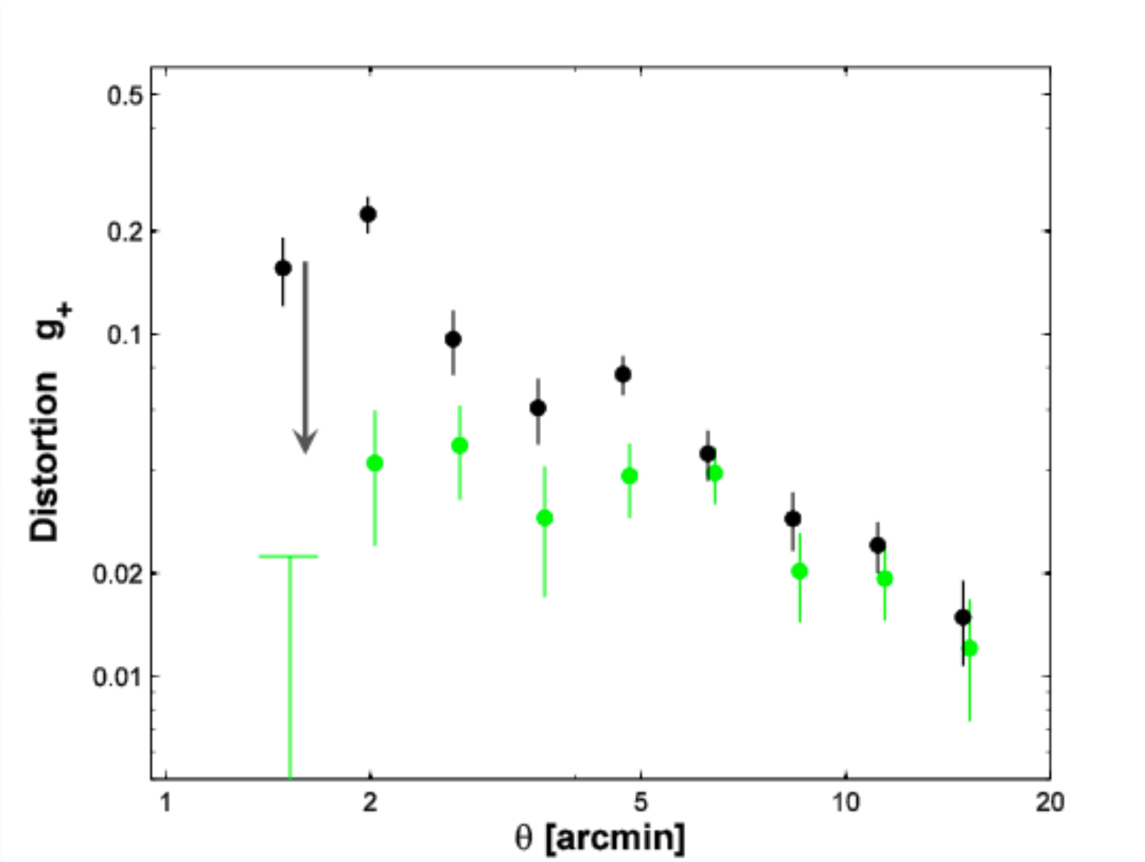
$$\gamma(x) = \overline{\kappa}(x) - \kappa(x)$$

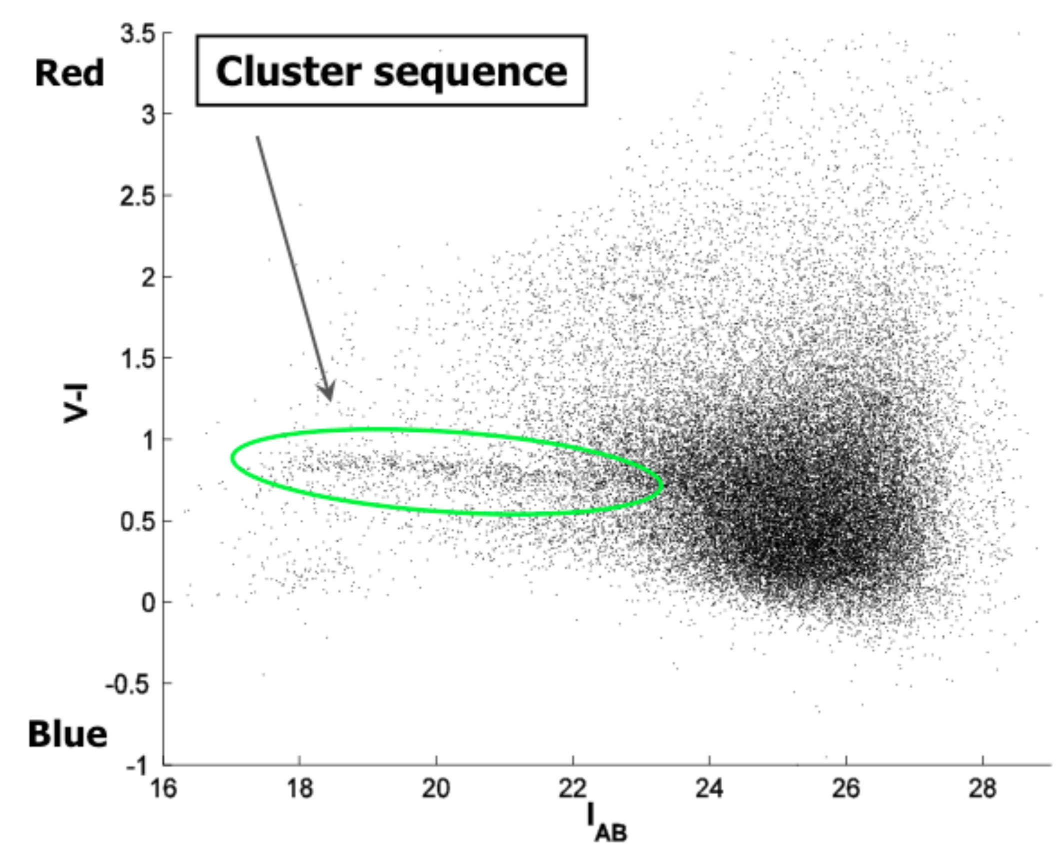


$$l_\gamma = \sum_{i=1}^{N_\gamma} \left[\frac{|\epsilon_i - g(\theta_i)|^2}{\sigma^2[g(\theta_i)]} + 2 \ln \sigma[g(\theta_i)] \right]$$

LIMITATIONS AND BIASES

- redshift distribution of the sources (assume median redshift for a given exposure time)
- contaminations by cluster members



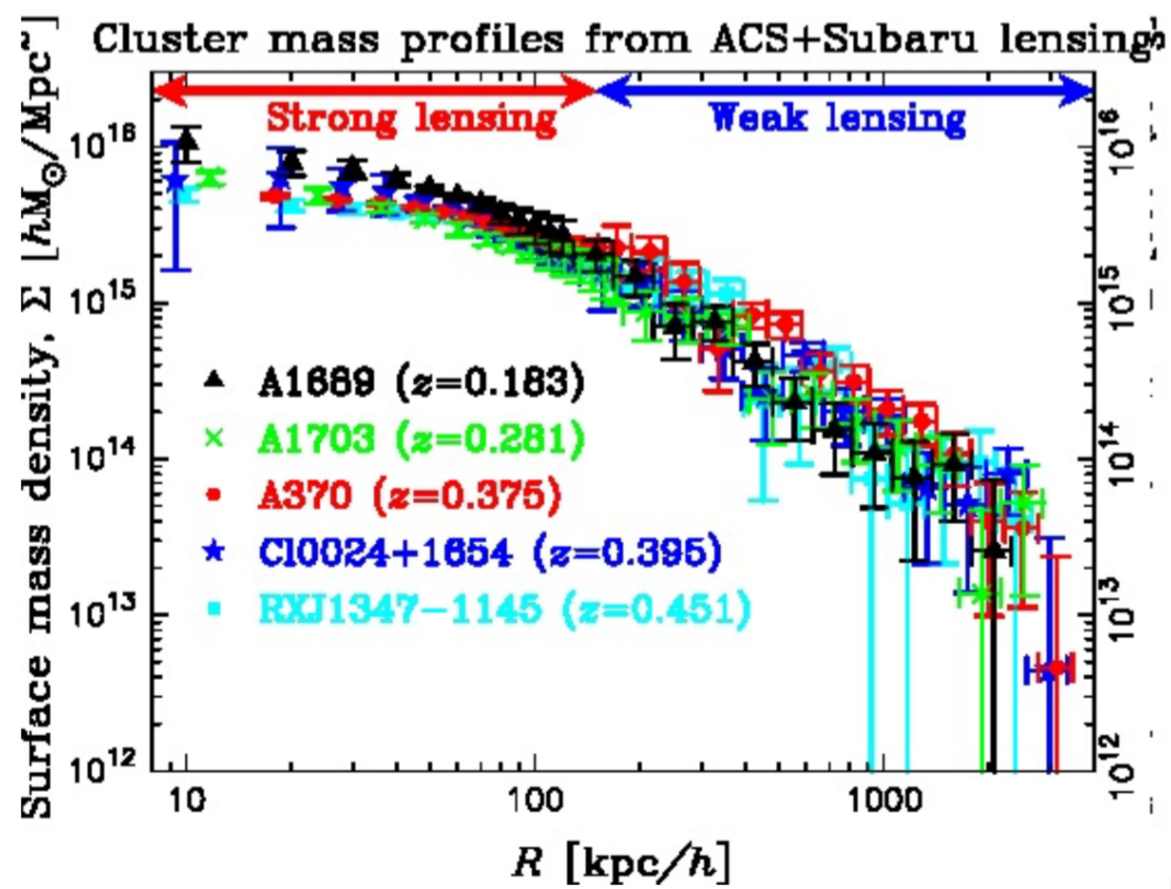


APPLICATIONS

- Mass profiles
- The nature of dark matter
- Cosmology
- …

MASS PROFILES

- SL in clusters probes scales ~10-50”. Assuming z~0.5, this corresponds to scales ~60-300 kpc
- typical scale radii for cluster-sized halos are >200 kpc
- thus, SL alone cannot constrain well the scale radius, neither the virial radius, i.e. the concentration
- weak lensing probes the mass distribution outside the SL region
- the combination of SL and WL is a powerful method to measure the total mass profile



Umetsu et al. 2011

CLASH observations

HST 524 orbits: 25 clusters, each imaged in 16 passbands. (0.23 – 1.6 μm) ~20 orbits per cluster.

CLASH observations are 80% done - 20 clusters completed, all HST data by July.

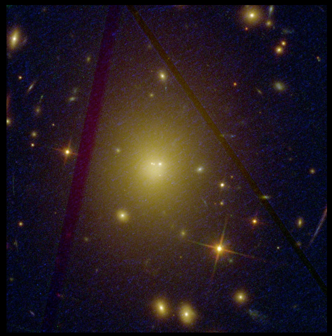
Chandra x-ray Observatory archival data (0.5 – 7 keV)

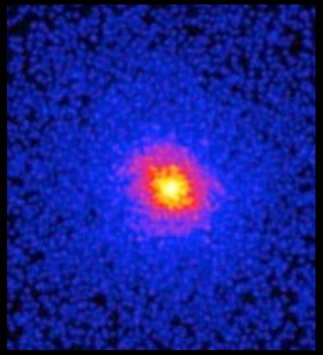
Spitzer Space Telescope archival and new cycle 8 data (3.6, 4.5 μm)

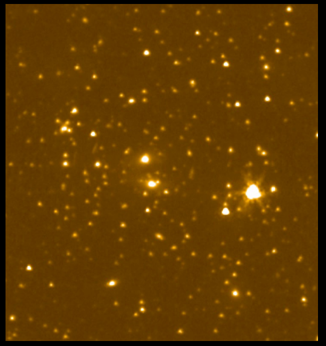
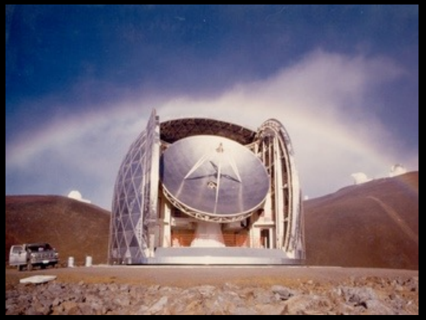
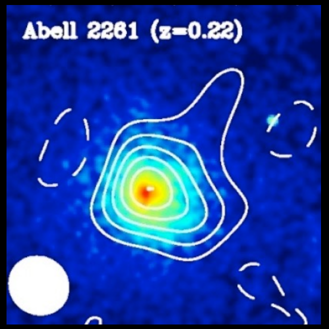
SZE observations (Bolocam, Mustang) to augment existing data (sub-mm)

Subaru wide-field imaging (0.4 – 0.9 μm)

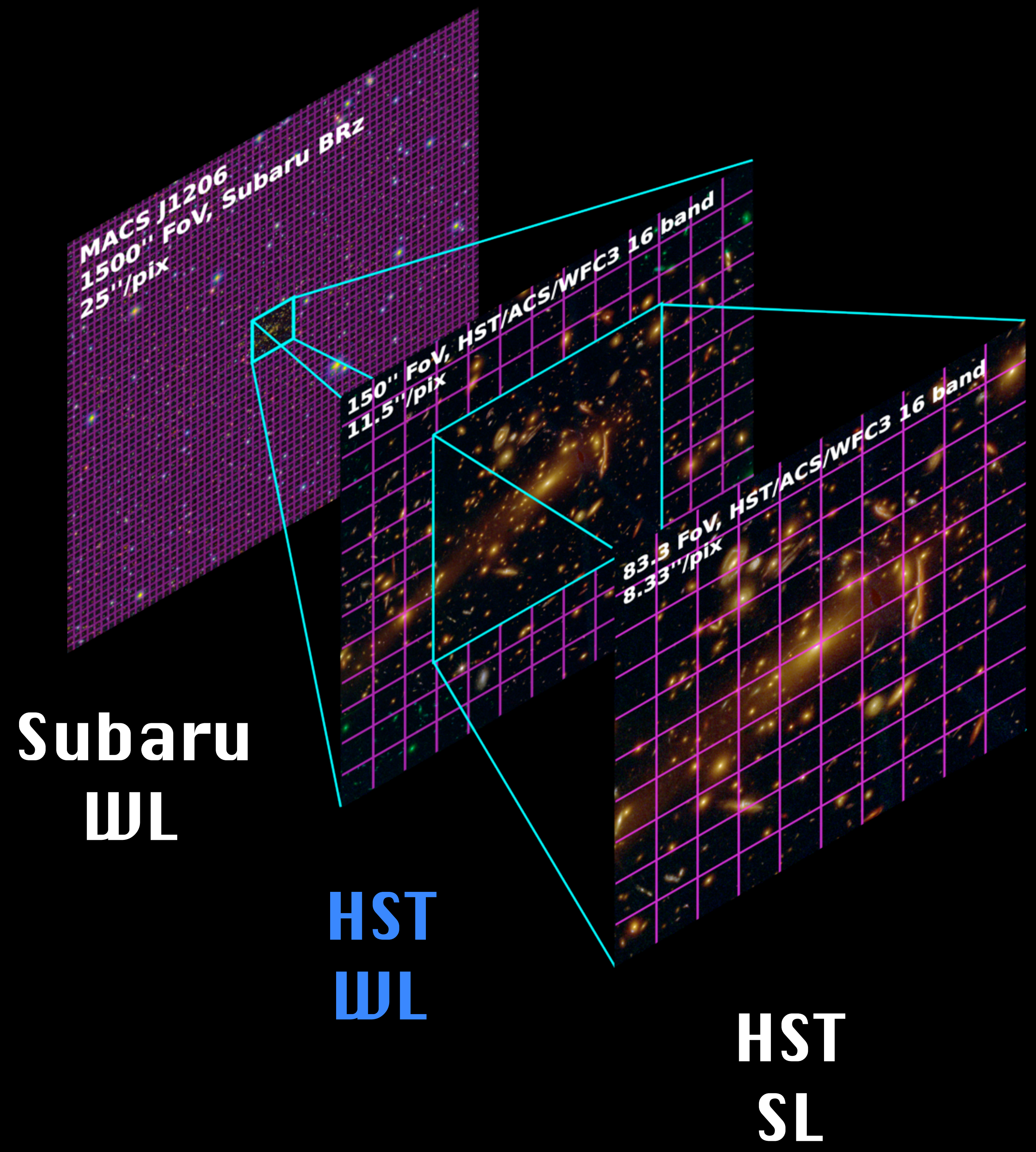
VLT, LBT, Magellan, MMT, Palomar Spectroscopy




CLASH reconstructions



SaWLens

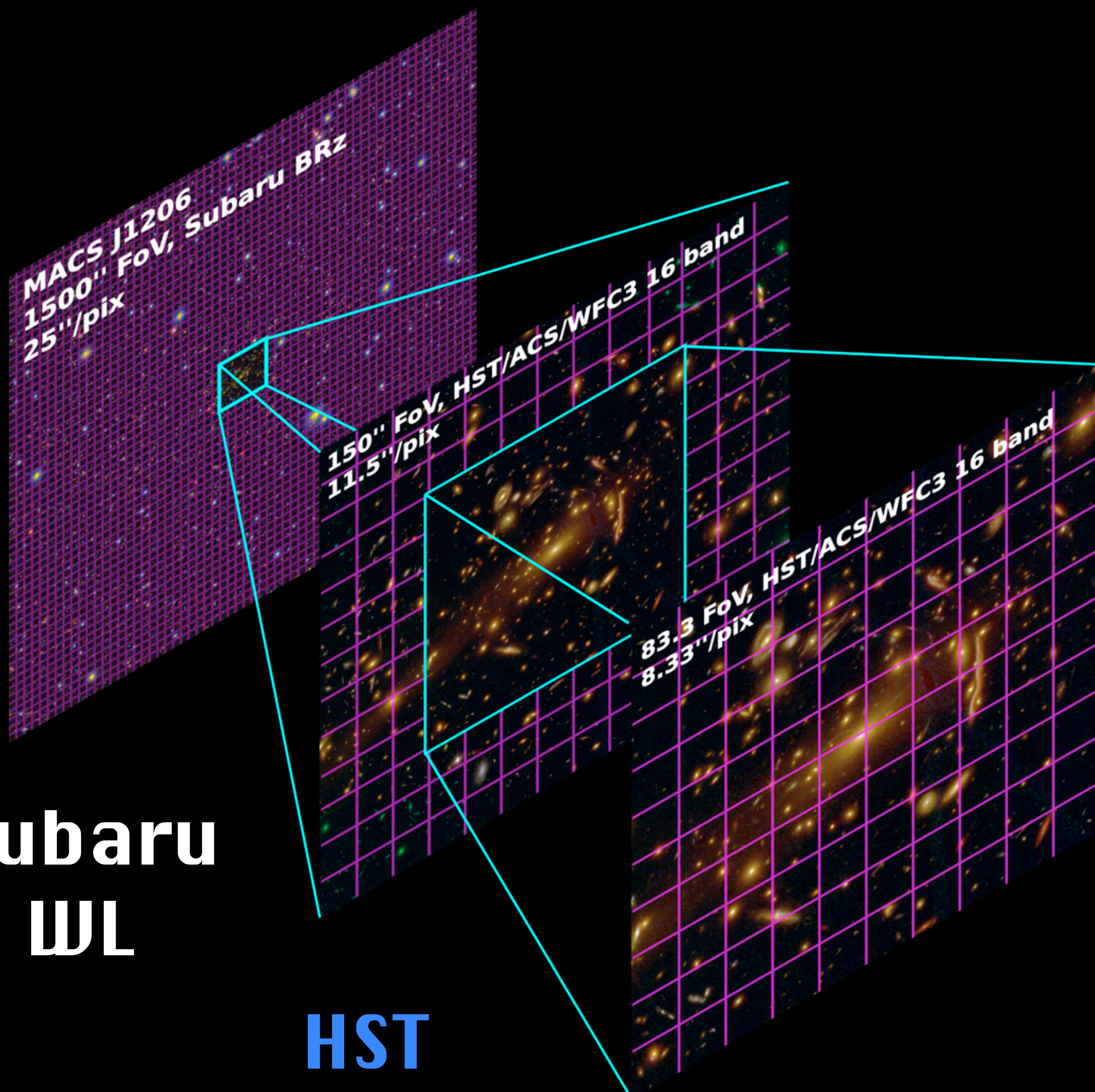
implements WL, SL and (Flexion)

operates on adaptively refined grids (AMR)

non-parametric method, this means that we make no assumptions on the lens' mass profile

JM et al. 2009

CLASH reconstructions



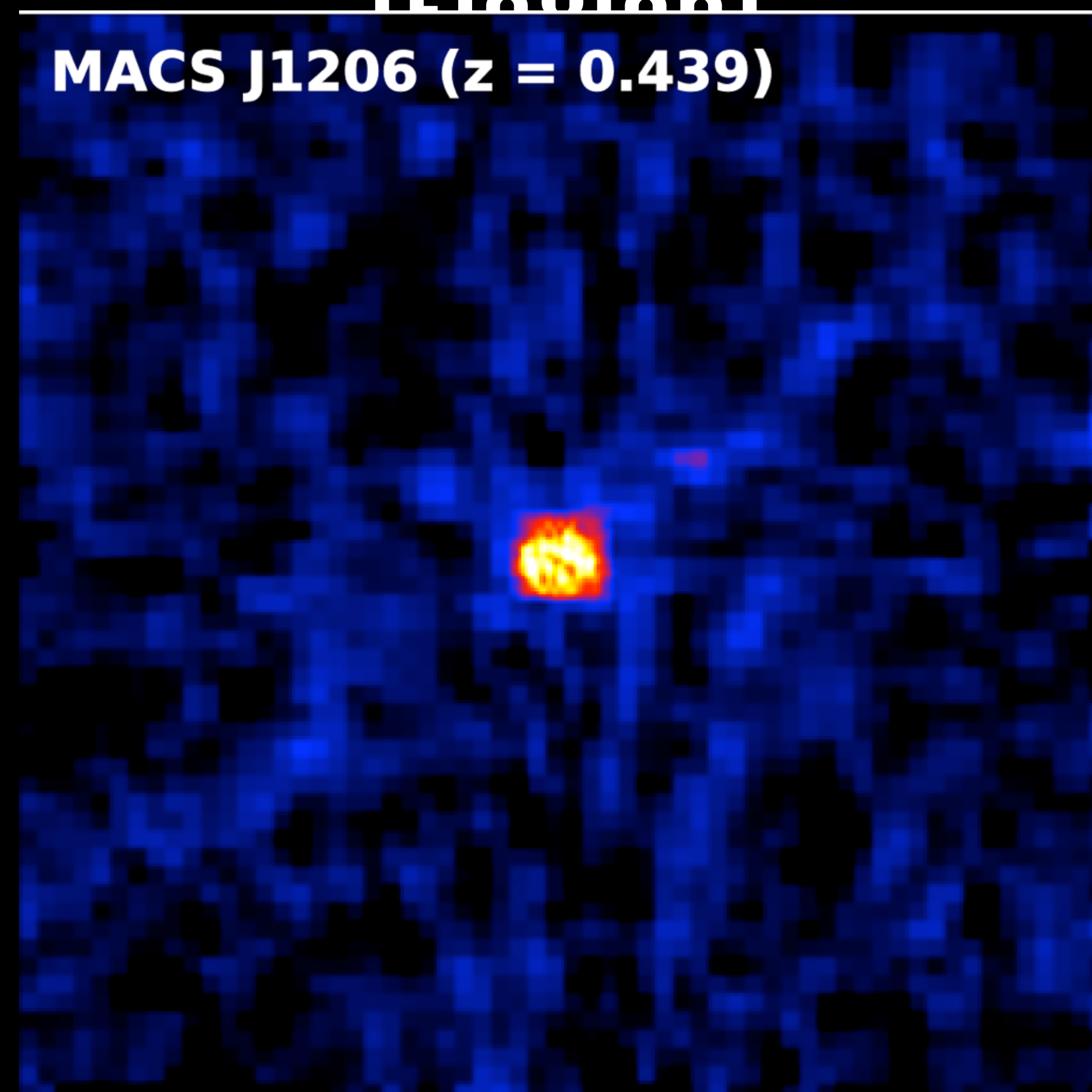
Subaru WL

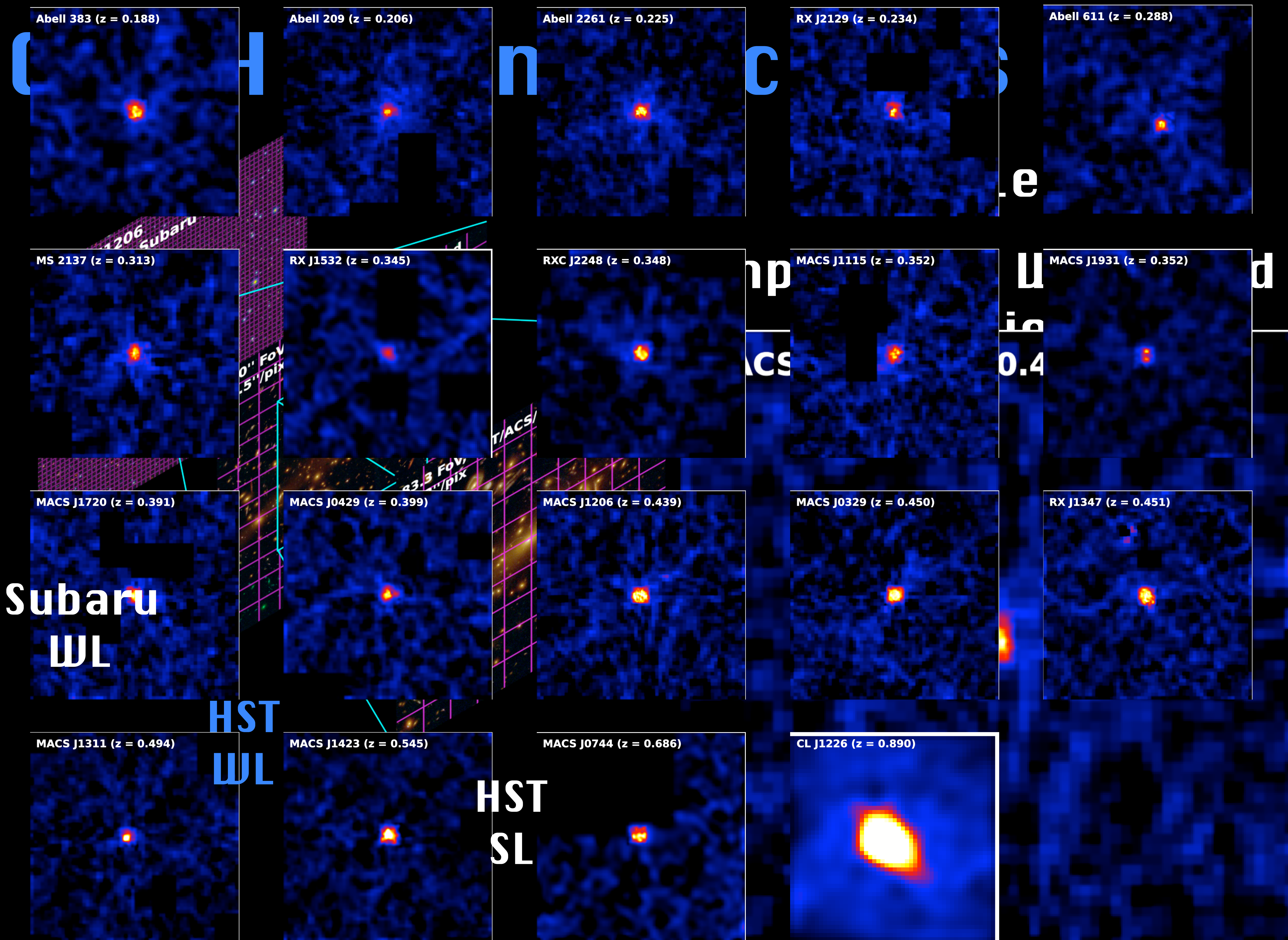
HST WL

HST SL

SaWLens

implements WL, SL and (Flexion)

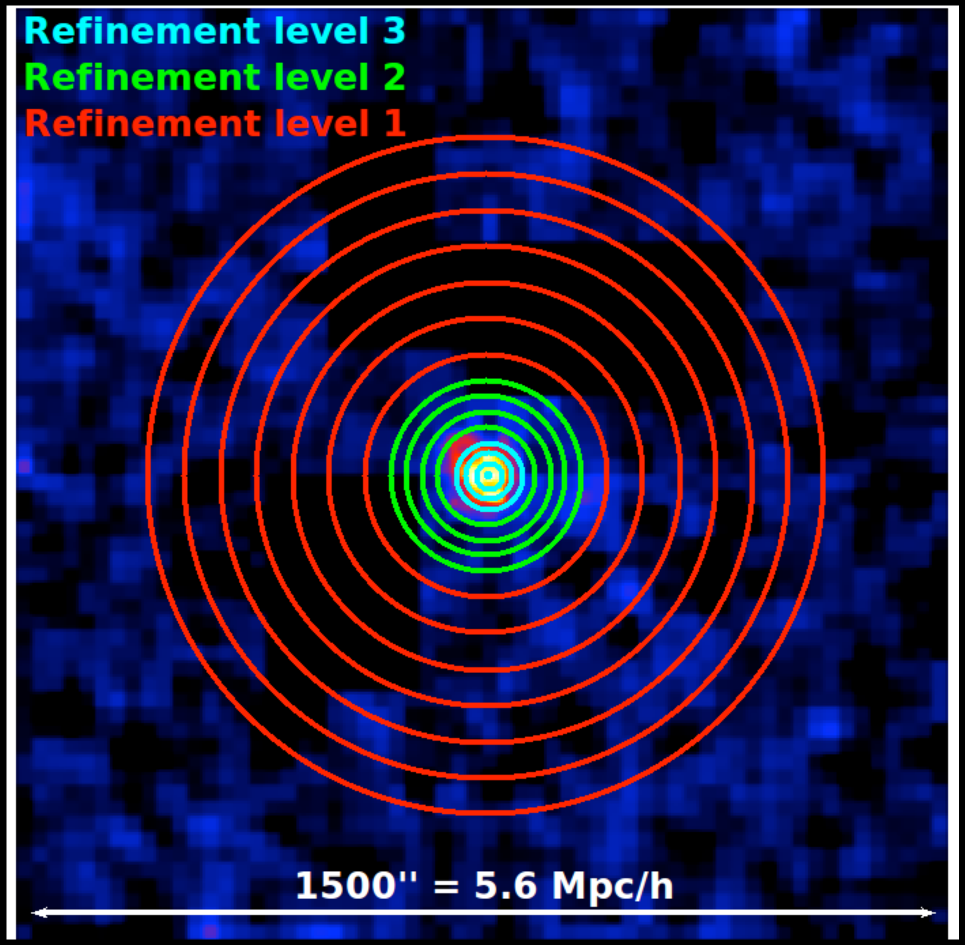


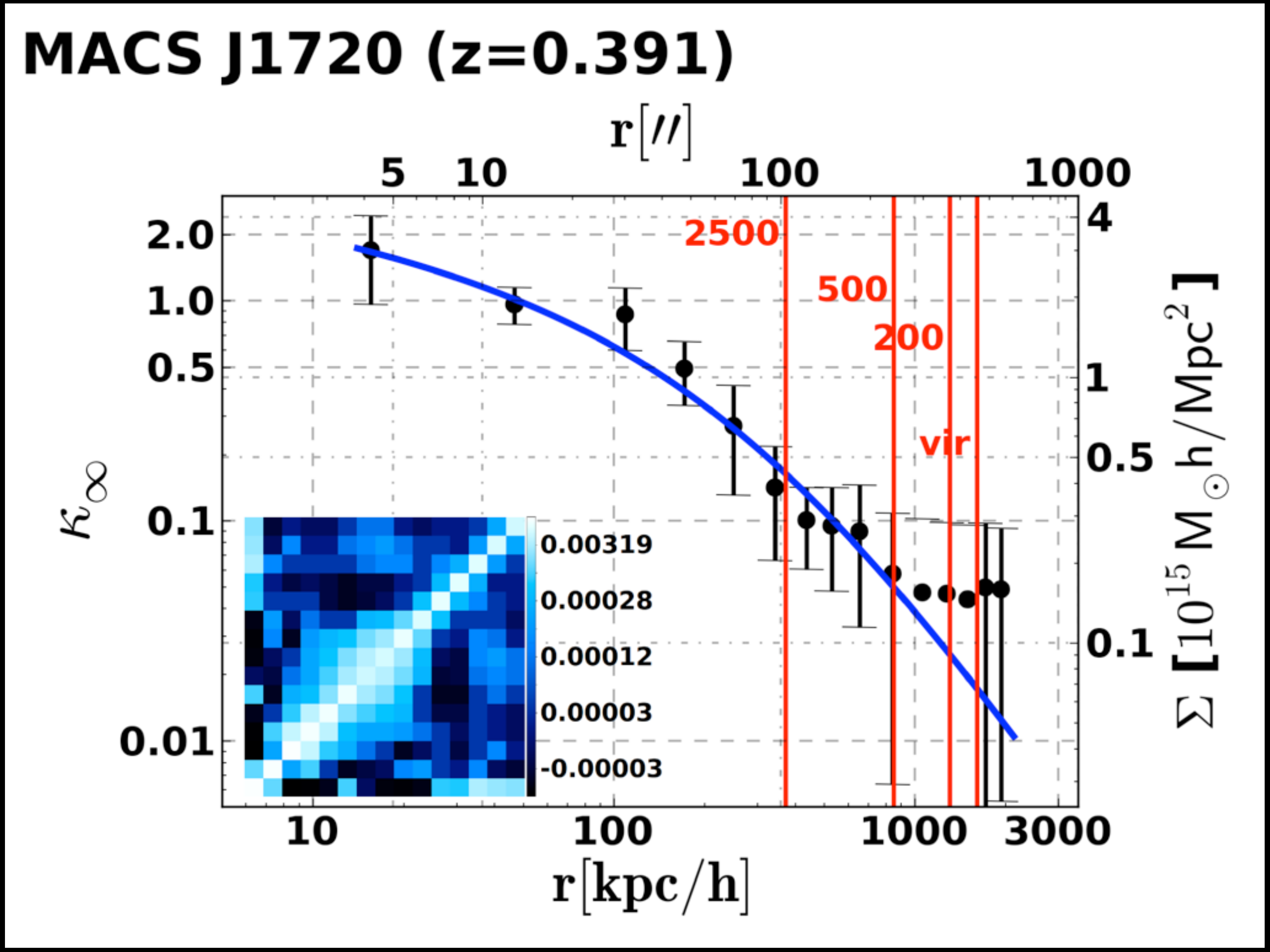


Abell 383 (z = 0.188)
Abell 209 (z = 0.206)
Abell 2261 (z = 0.225)
RX J2129 (z = 0.234)
Abell 611 (z = 0.288)
MS 2137 (z = 0.313)
RX J1532 (z = 0.345)
RXC J2248 (z = 0.348)
MACS J1115 (z = 0.352)
MACS J1931 (z = 0.352)
MACS J1720 (z = 0.391)
MACS J0429 (z = 0.399)
MACS J1206 (z = 0.439)
MACS J0329 (z = 0.450)
RX J1347 (z = 0.451)
MACS J1311 (z = 0.494)
MACS J1423 (z = 0.545)
MACS J0744 (z = 0.686)
CL J1226 (z = 0.890)
Subaru WL
HST WL
HST SL

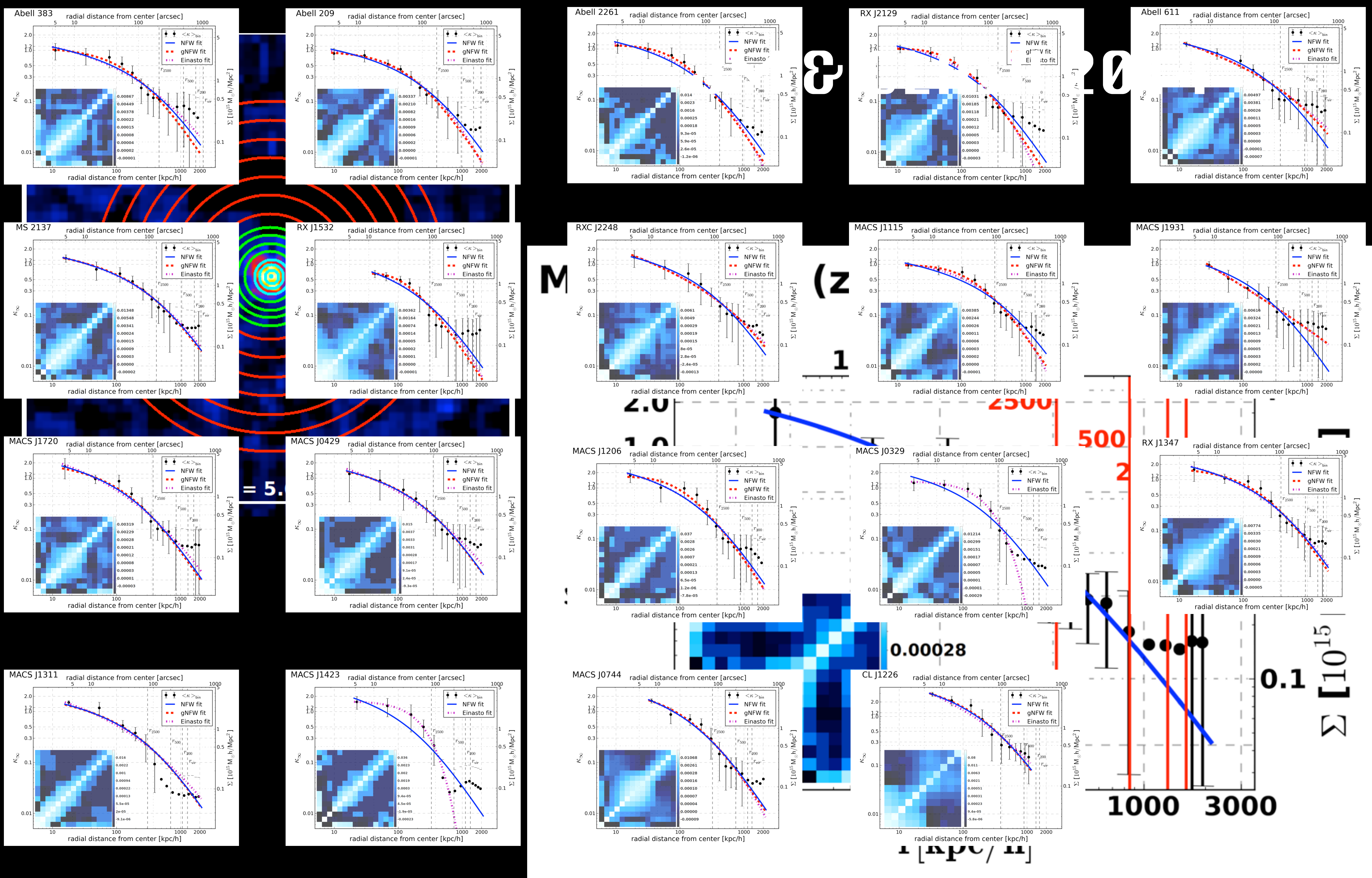
CLASH profiles

JM & CLASH 2014





CLASH profiles

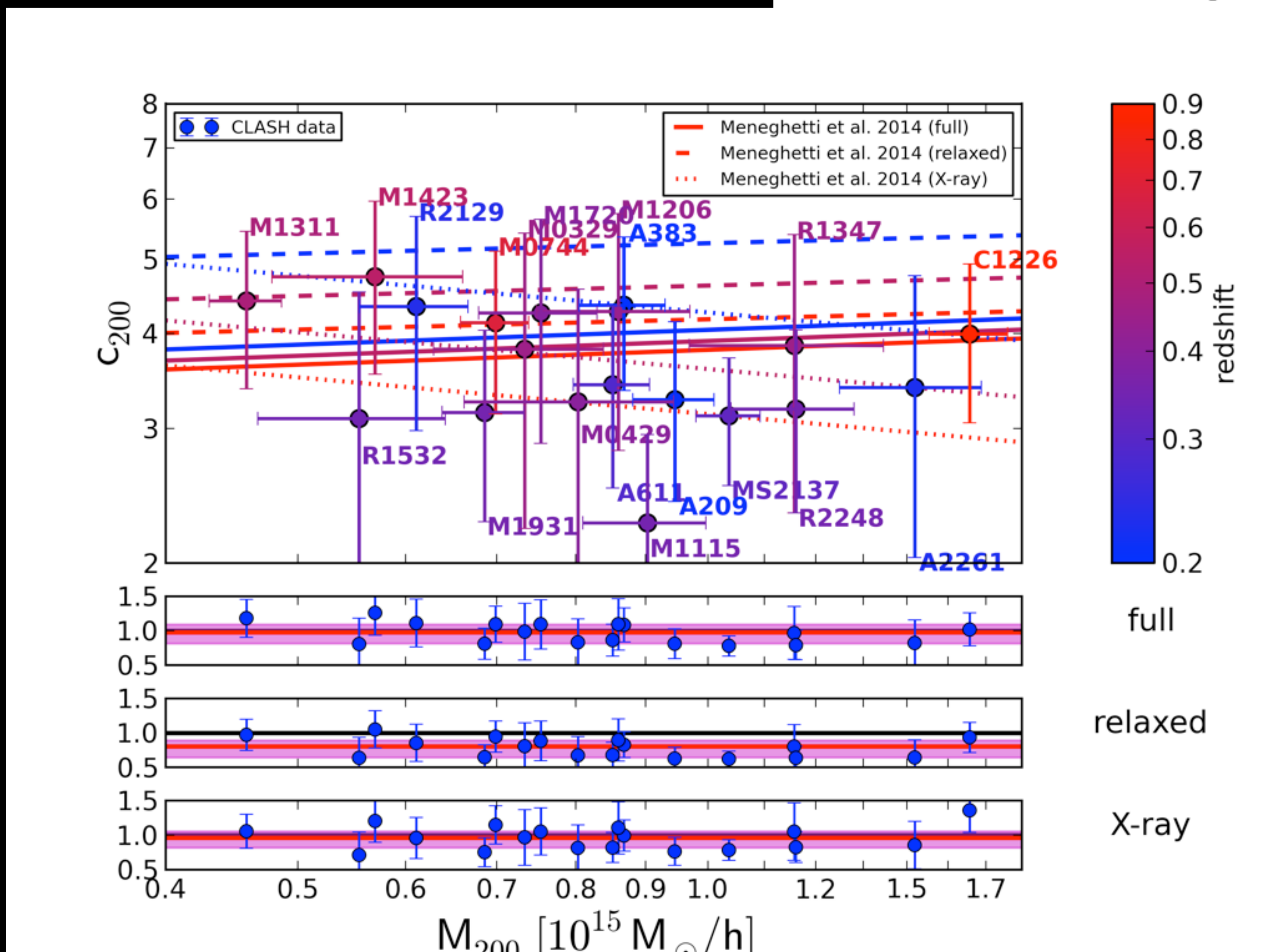


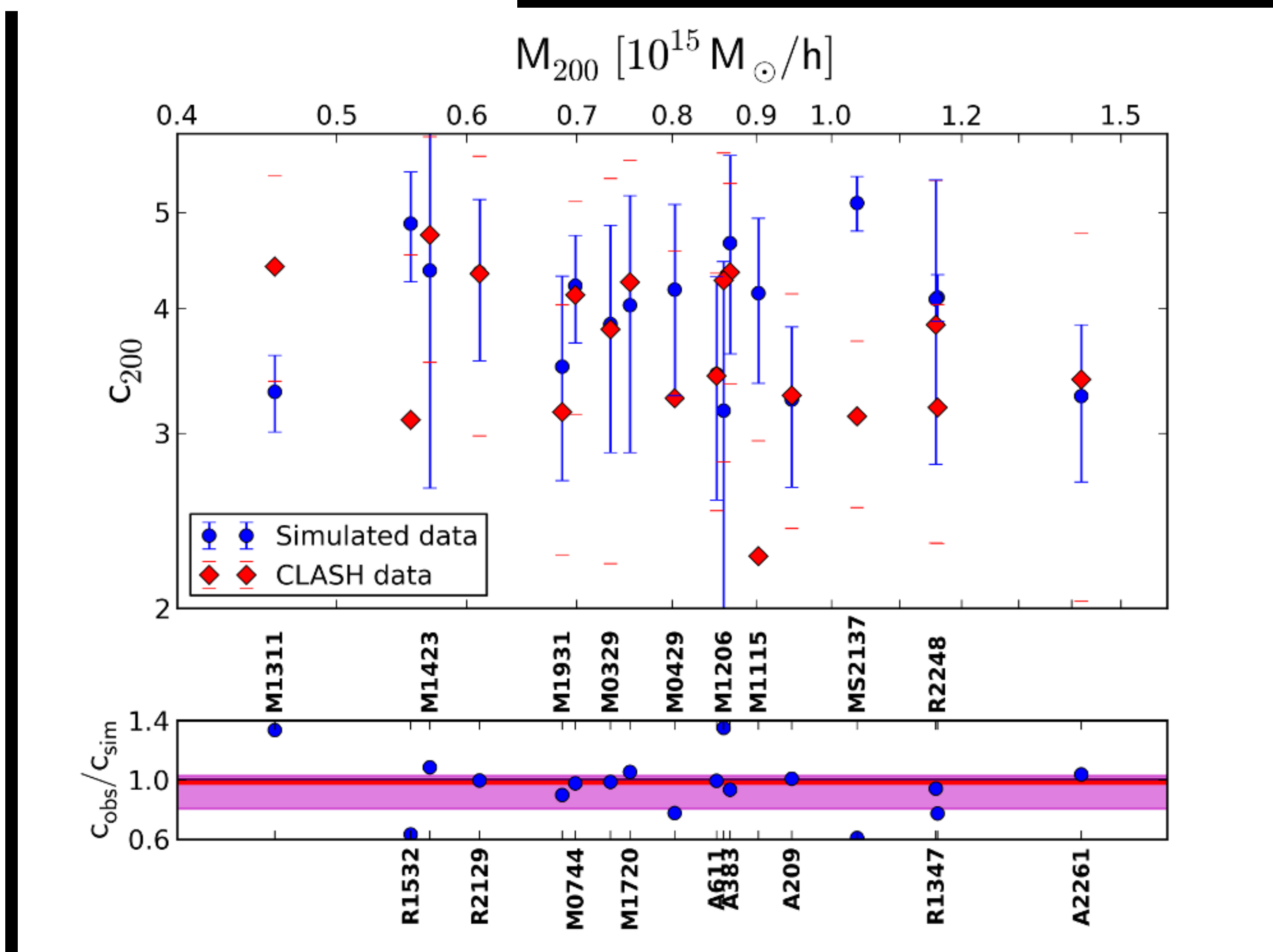
Tailored comparisons

Table 9
Goodness-of-fit: Meneghetti et al. 2014

Sample	$\langle c_{\rm obs}/c_{\rm sim}\rangle$	Q_2	Q_1	Q_3	χ^2	p-value
3D full	1.05 ± 0.16	1.09	0.90	1.20	5.7	0.97
3D relaxed	0.85 ± 0.15	0.88	0.71	0.97	18.7	0.18
2D full	0.97 ± 0.15	0.98	0.82	1.09	7.3	0.92
2D relaxed	0.79 ± 0.14	0.81	0.64	0.89	33.7	0.00
2D rel.+SL	0.82 ± 0.18	0.81	0.69	0.94	28.6	0.01
2D X-ray	0.96 ± 0.18	0.96	0.82	1.06	9.5	0.80

Note. — The column explanations are identical to Tab. 8.



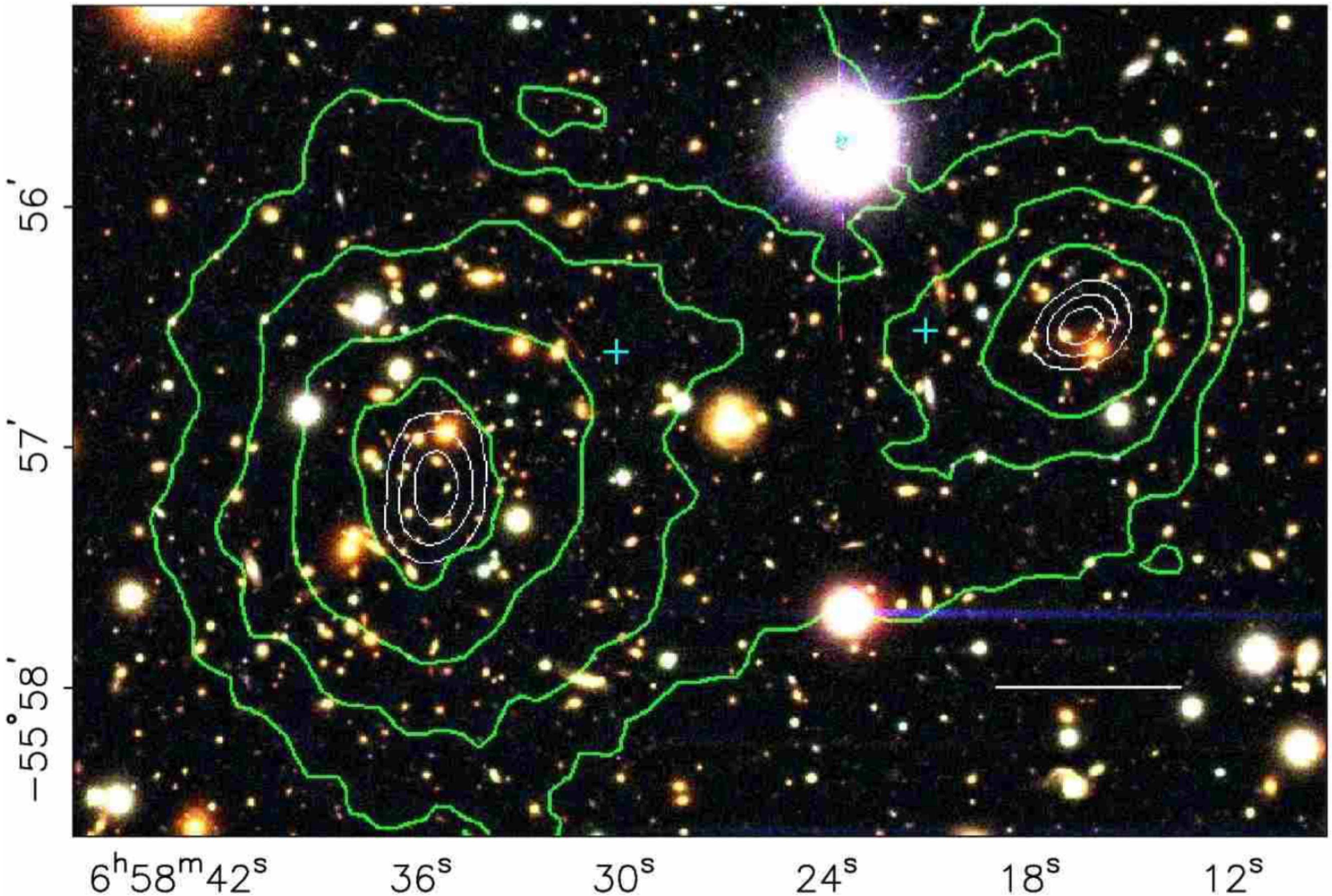


JM & CLASH 2014

THE NATURE OF DM FROM WEAK LENSING

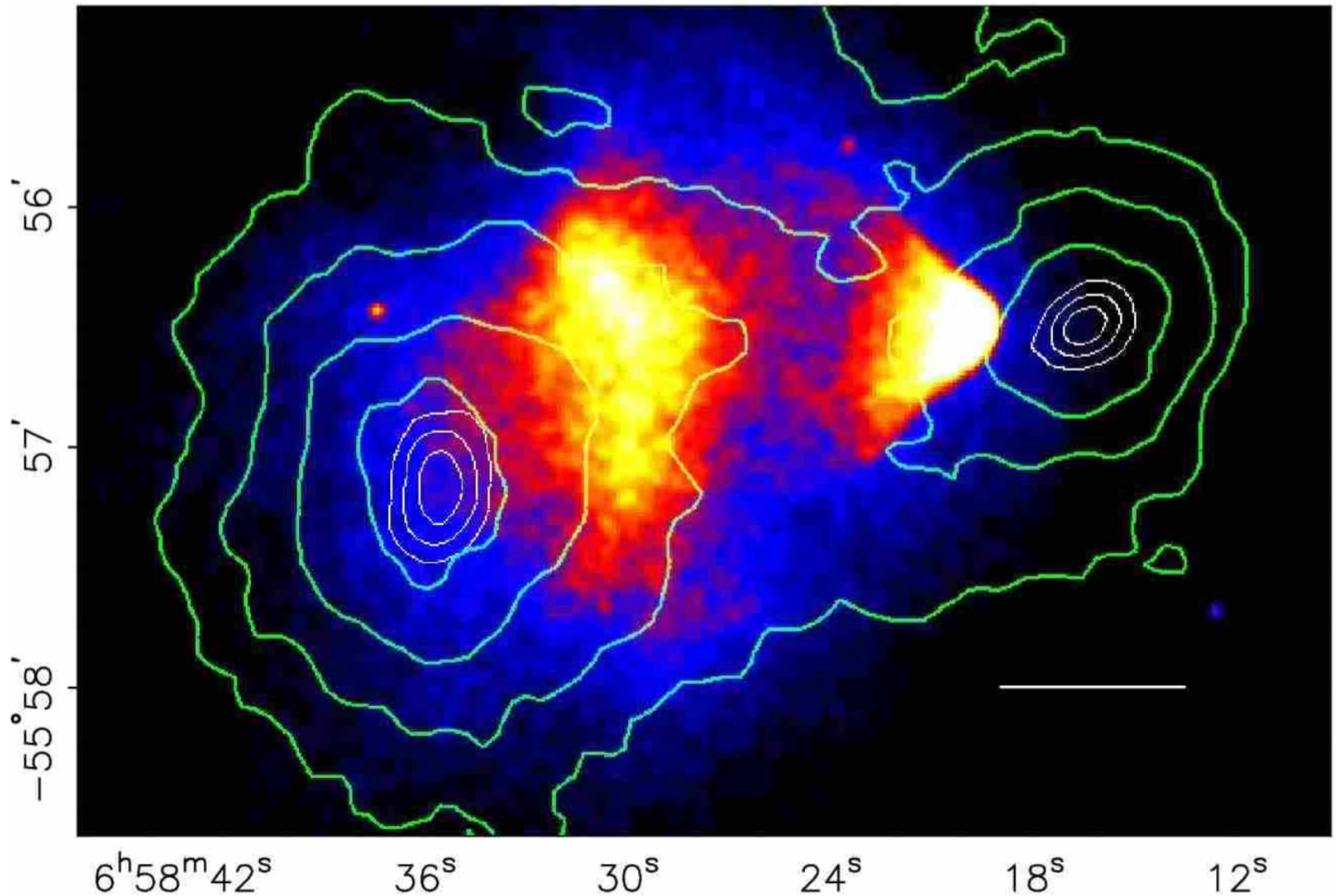
Clusters are dominated by collisionless matter:

The Bullet Cluster is a pair of colliding galaxy clusters (Clowe et al. 2006)



THE NATURE OF DM FROM WEAK LENSING

X-ray emission from the bullet cluster

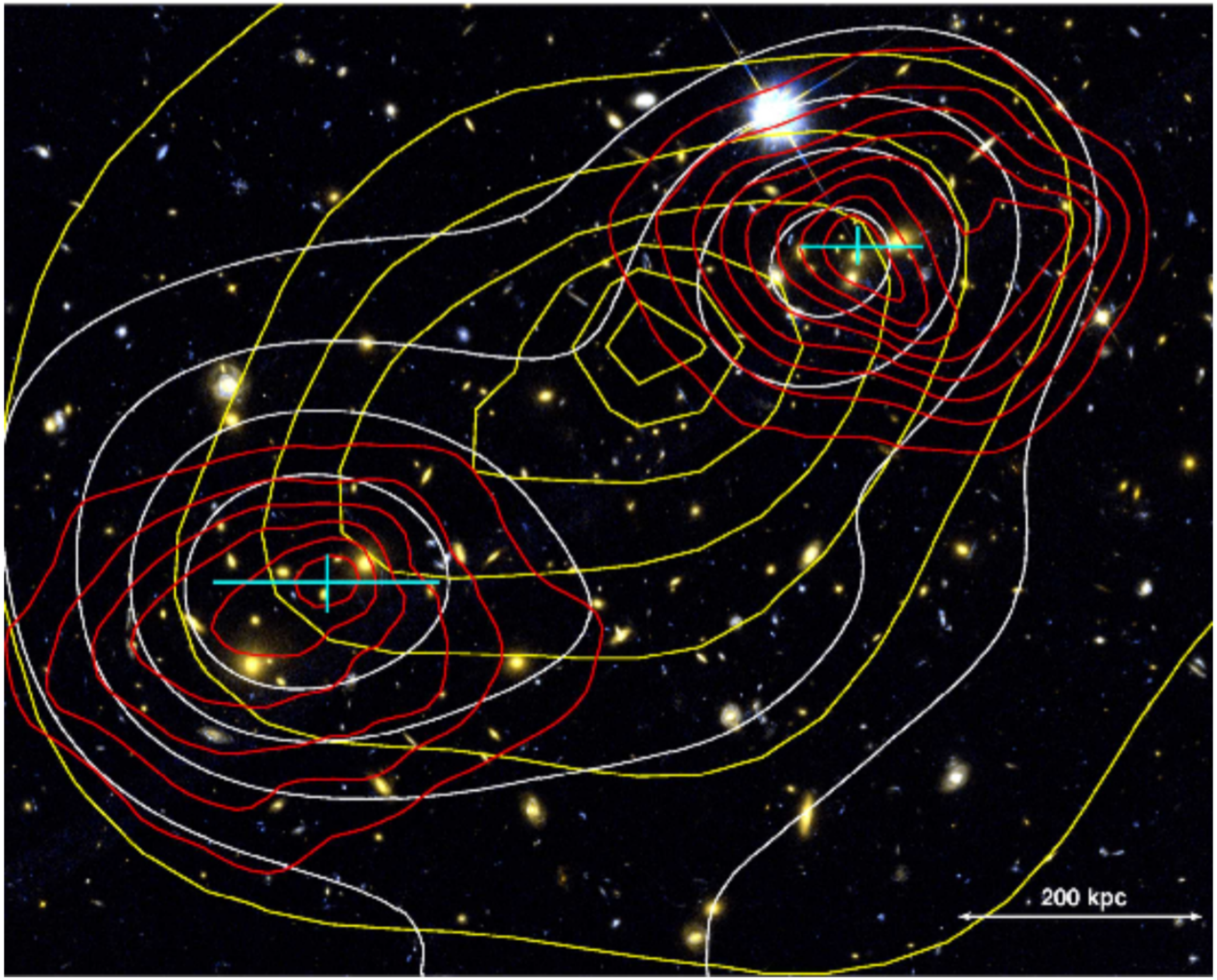


THE NATURE OF DM FROM WEAK LENSING

- Lensing shows that most of the mass is located near the galaxies,
- and not centered on the gas, which is displaced by the collision.
- $\Rightarrow$ Most of the mass in this cluster pair must behave collisionless, like galaxies.
- Most of the mass is dark matter – the bullet cluster can not be explained by changing the law of gravity without invoking collisionless dark matter.
- The bullet cluster is not the only case where this clear distinction can be made...

THE NATURE OF DM FROM WEAK LENSING

The cluster MACS J0025.4−1222 ($z = 0.59$)



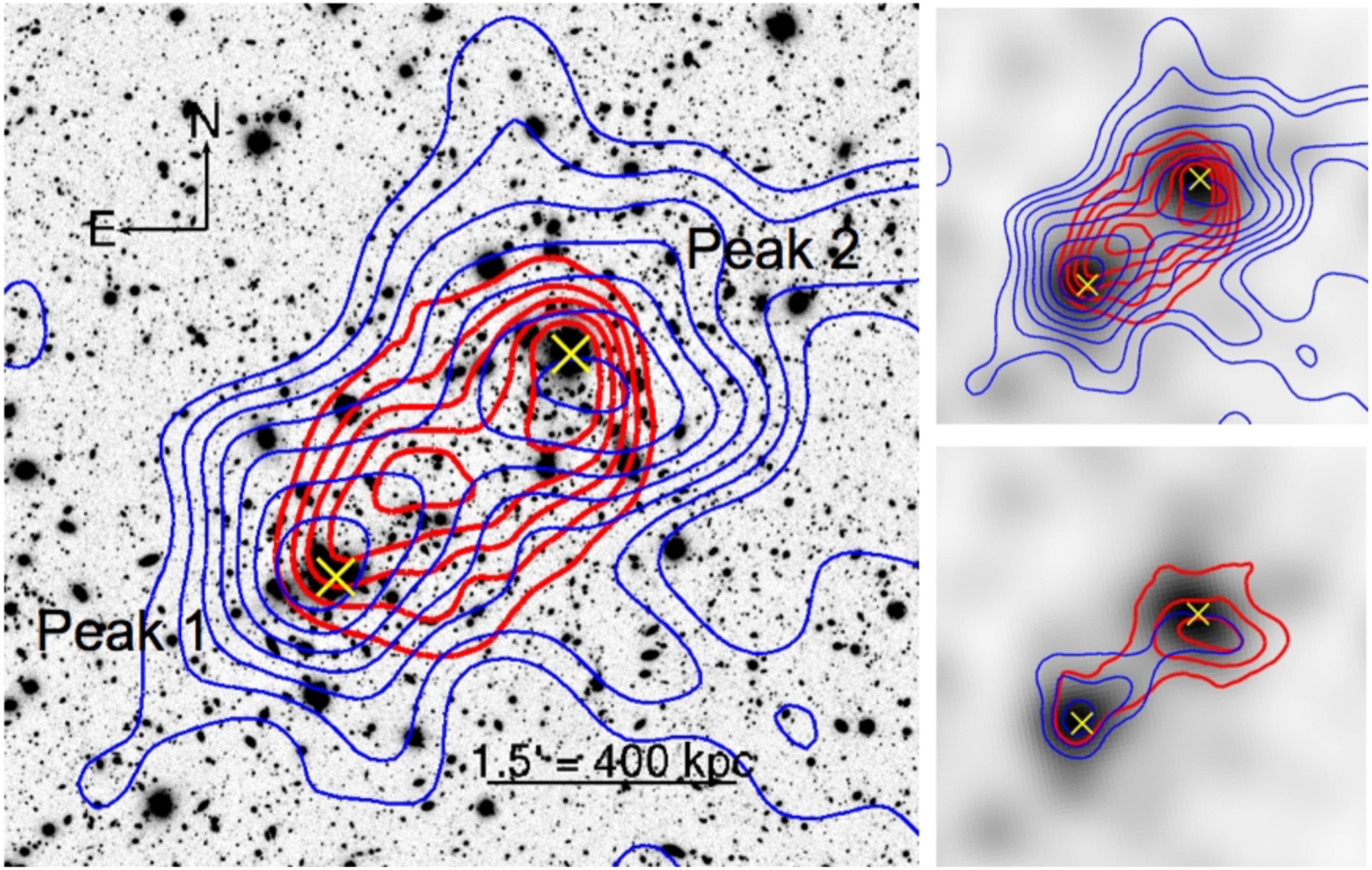
(Bradac et al. 2008)

red:
surface mass density;

yellow:
X-ray emission;

white:
smoothed optical light.

THE NATURE OF DM FROM WEAK LENSING



A1758N (Ragozzine & Clowe 2011)
Blue: mass reconstruction; red: X-ray emission