GRAVITATIONAL LENSING

LECTURE 4

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CONTENTS

- Distortion and magnification (continuation)
- Second order lensing: flexion
- Time delays

EXAMPLE: FIRST ORDER DISTORTION OF A CIRCULAR SOURCE

$$\beta_1^2 + \beta_2^2 = \beta^2$$

In the reference frame where A is diagonal:

$$\begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} = \begin{pmatrix} 1-\kappa-\gamma & 0 \\ 0 & 1-\kappa+\gamma \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}$$

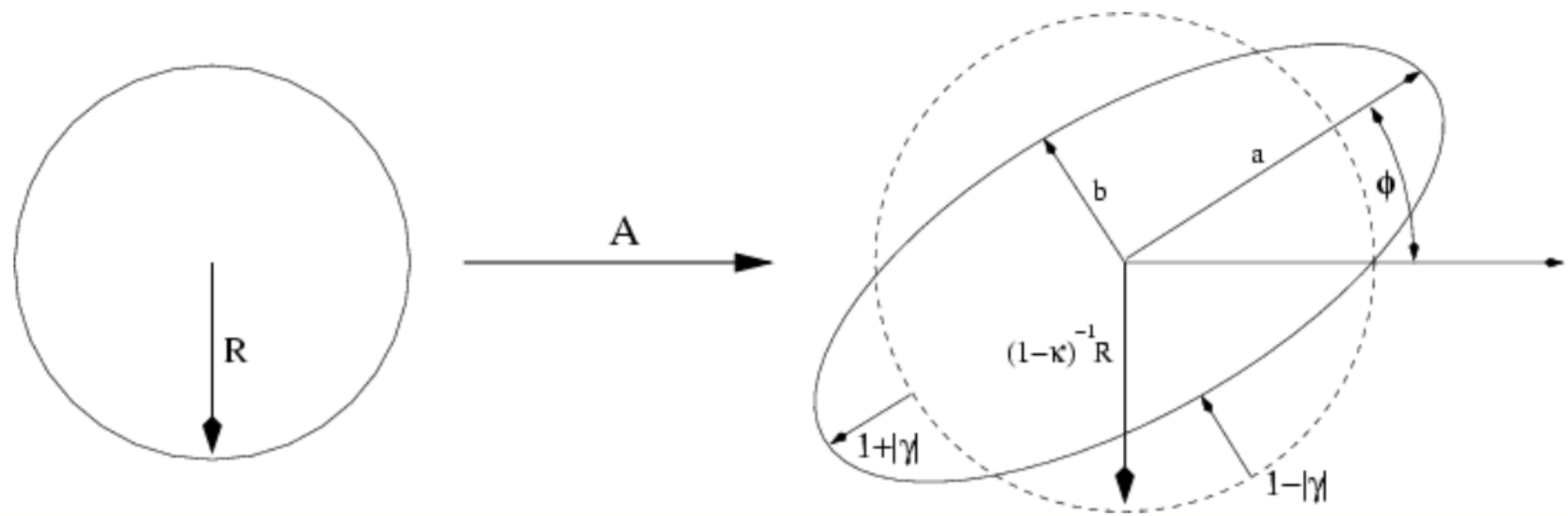
$$\begin{aligned} \beta_1 &= (1-\kappa-\gamma)\theta_1 \\ \beta_2 &= (1-\kappa+\gamma)\theta_2 \end{aligned}$$

$$\beta^2 = \beta_1^2 + \beta_2^2 = (1-\kappa-\gamma)^2\theta_1^2 + (1-\kappa+\gamma)^2\theta_2^2$$

This is the equation of an ellipse with semi-axes:

$$a = \frac{\beta}{1-\kappa-\gamma} \qquad b = \frac{\beta}{1-\kappa+\gamma}$$

EXAMPLE: FIRST ORDER DISTORTION OF A CIRCULAR SOURCE



convergence: *responsible for isotropic expansion or contraction*

shear: *responsible for anisotropic distortion*

Ellipticity: $$e = \frac{a-b}{a+b} = \frac{\gamma}{1-\kappa} = g$$

EXAMPLE: FIRST ORDER DISTORTION OF A CIRCULAR SOURCE

What is the orientation of the ellipse? Let's find the eigenvectors corresponding to the eigenvalue λ_t

$$E_{\lambda_t} = N(A - \lambda_t I) = \quad = \quad N \begin{pmatrix} \gamma - \gamma_1 & -\gamma_2 \\ -\gamma_2 & \gamma + \gamma_1 \end{pmatrix}$$

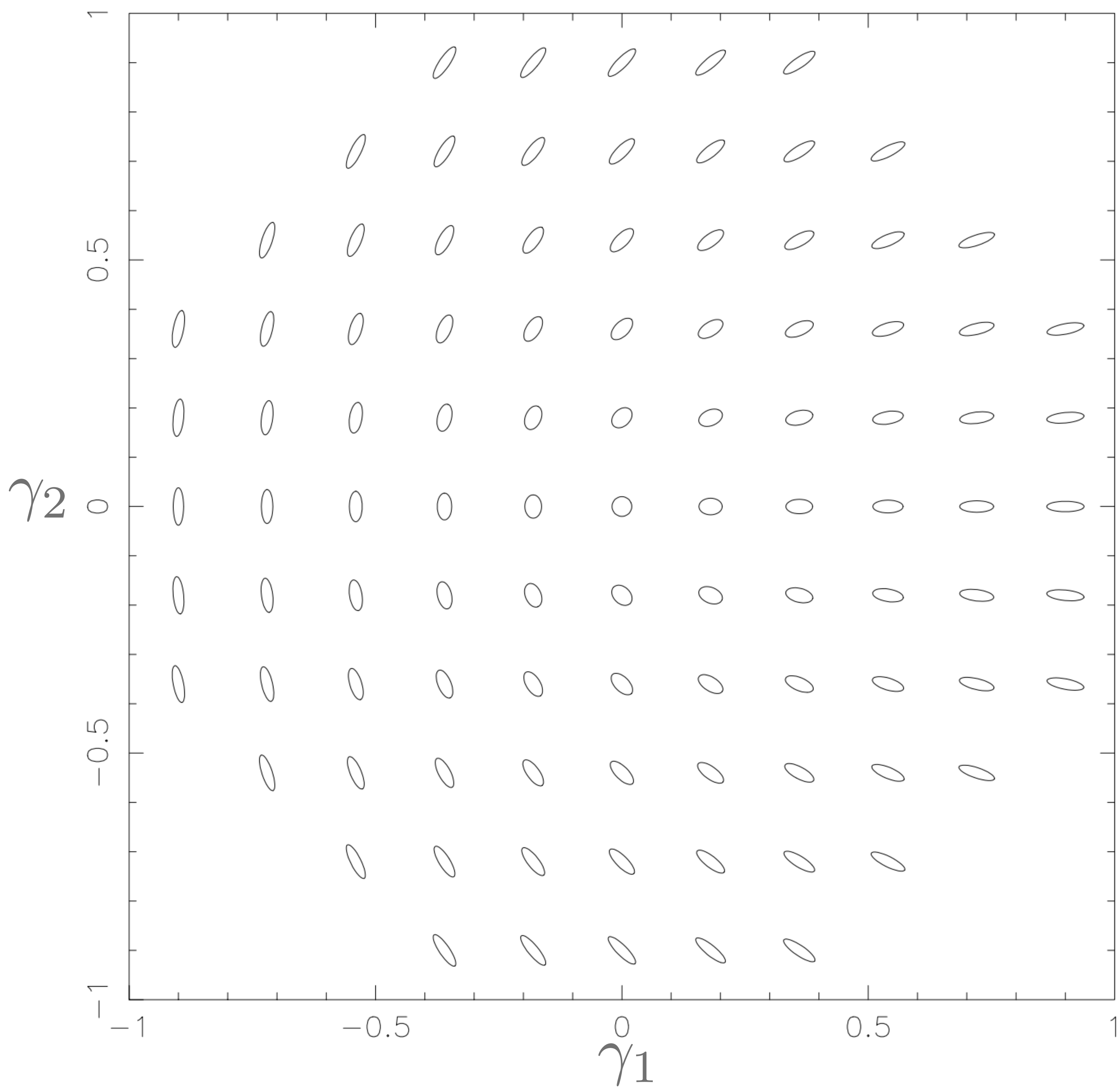
Result: $\vec{v} = (v_1, v_2) = |v|(\cos\phi', \sin\phi')$ *with* $v_1 = \frac{\gamma_2}{\gamma - \gamma_1} v_2$

After some math: $\cos\phi' = \pm\cos\phi$

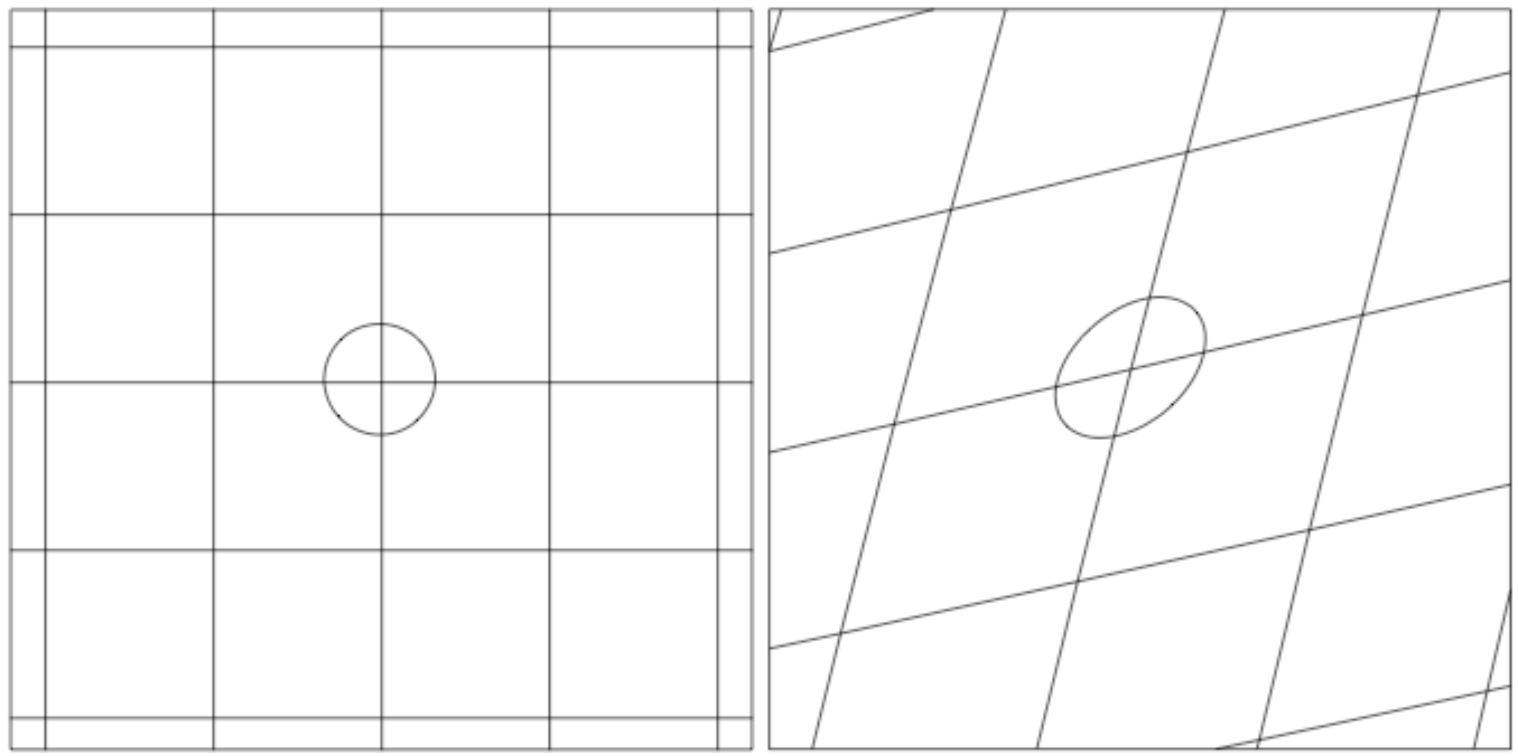
$$\phi' = \phi$$

$$\phi' = \phi + \pi$$

SHEAR DISTORTIONS

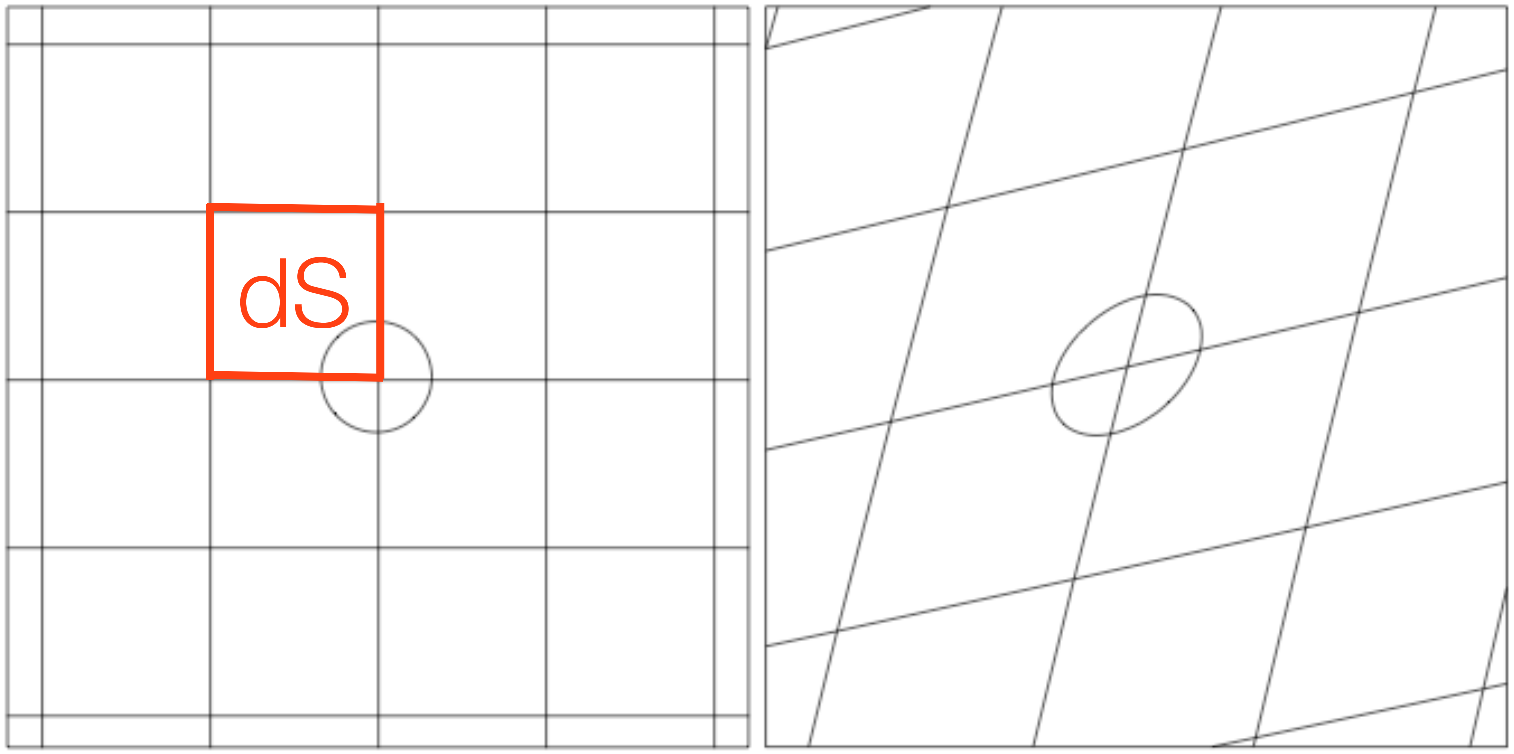


MAGNIFICATION



Kneib & Natarajan (2012)

MAGNIFICATION



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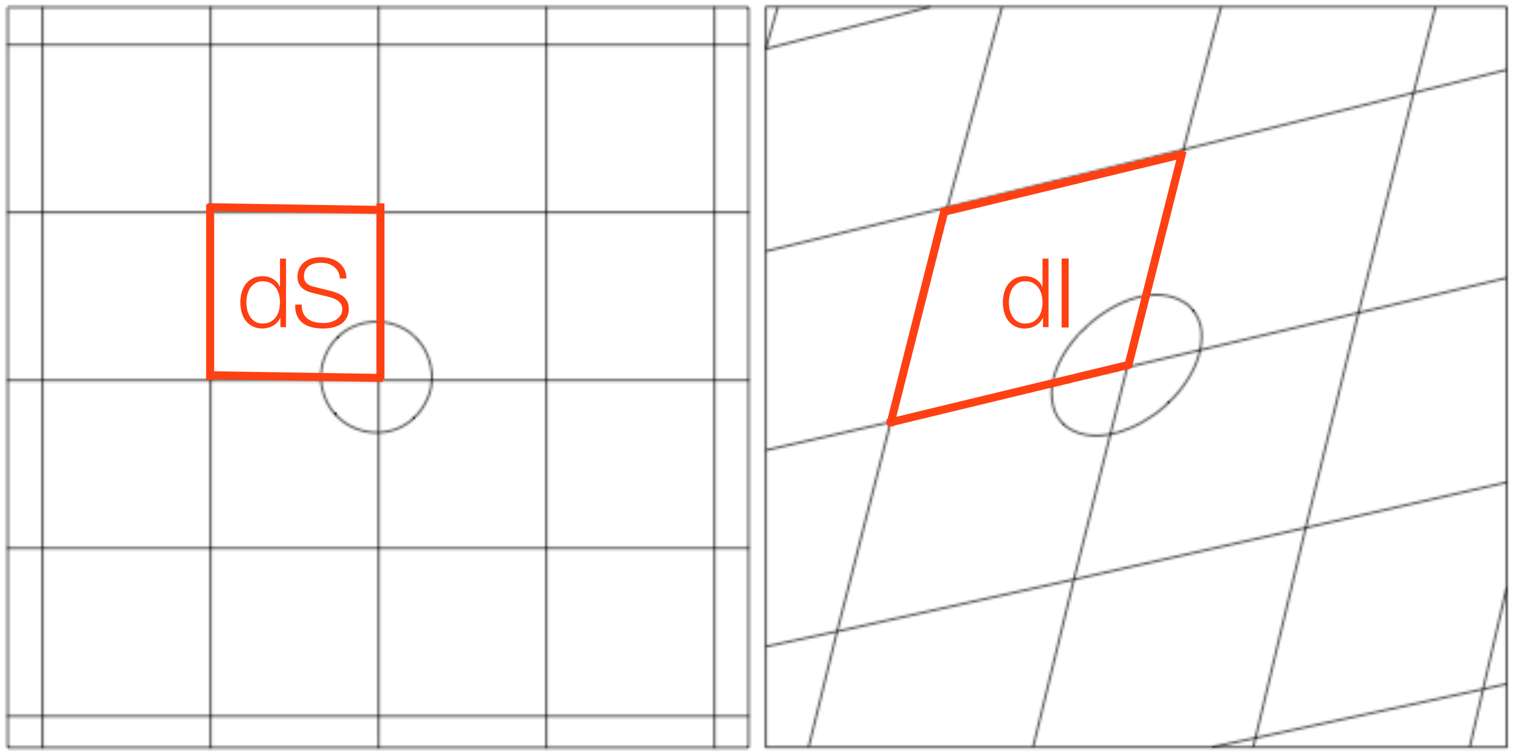

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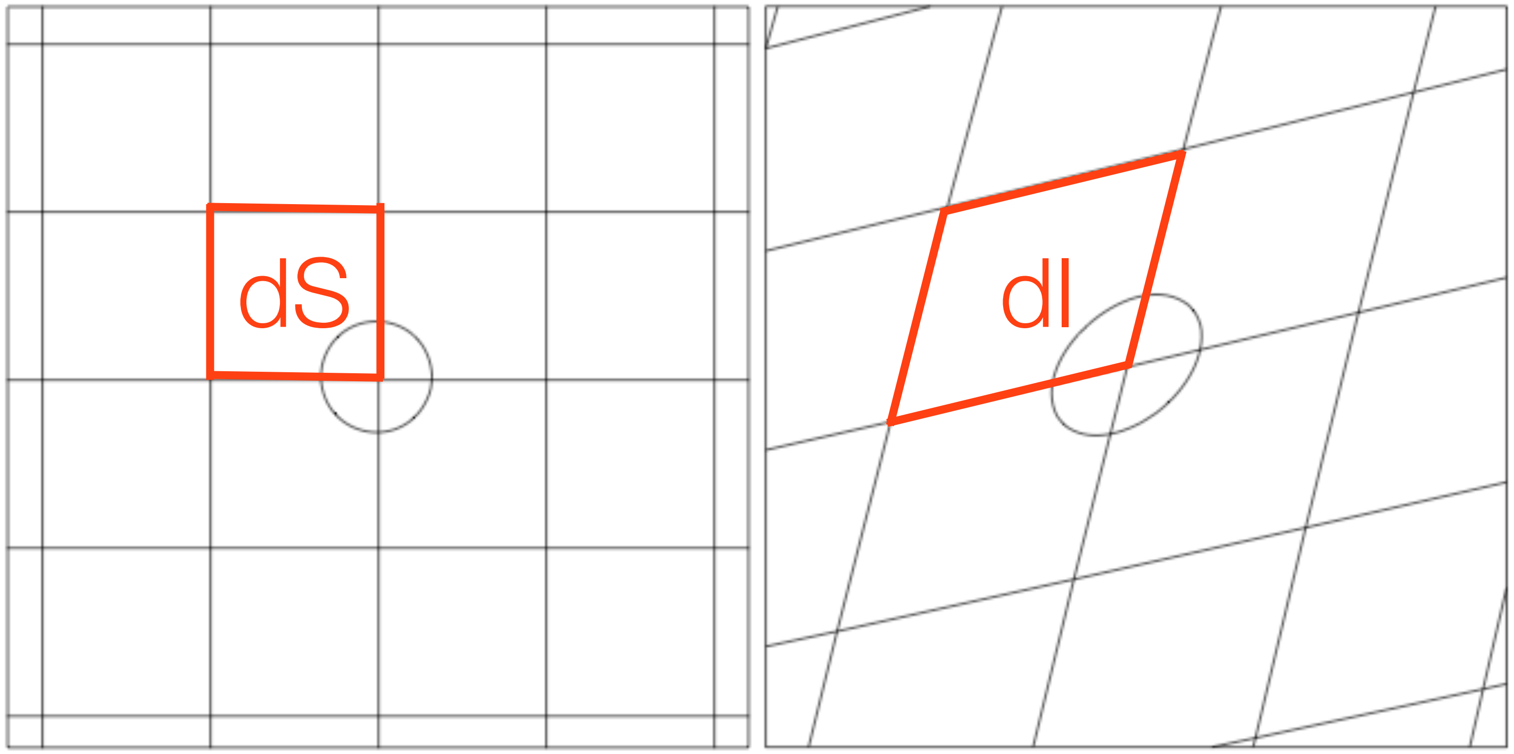

Kneib & Natarajan (2012)

$$\mu = \frac{dI}{dS} = \frac{\delta\theta^2}{\delta\beta^2} = \det A^{-1}$$

CONSERVATION OF SURFACE BRIGHTNESS

The source ***surface brightness*** *is*

$$I_\nu = \frac{dE}{dt dA d\Omega d\nu}$$

In phase space, the radiation emitted is characterized by the density

$$f(\vec{x}, \vec{p}, t) = \frac{dN}{d^3x d^3p}$$

In absence of photon creations or absorptions, f is conserved (Liouville theorem)

$$dN = \frac{dE}{h\nu} = \frac{dE}{cp}$$

$$d^3x = c dt dA$$

$$d^3\vec{p} = p^2 dp d\Omega$$

$$f(\vec{x}, \vec{p}, t) = \frac{dN}{d^3x d^3p} = \frac{dE}{hcp^3 dA dt d\nu d\Omega} = \frac{I_\nu}{hcp^3}$$

Since GL does not involve creation or absorption of photons, neither it changes the photon momenta (achromatic!), surface brightness is conserved!

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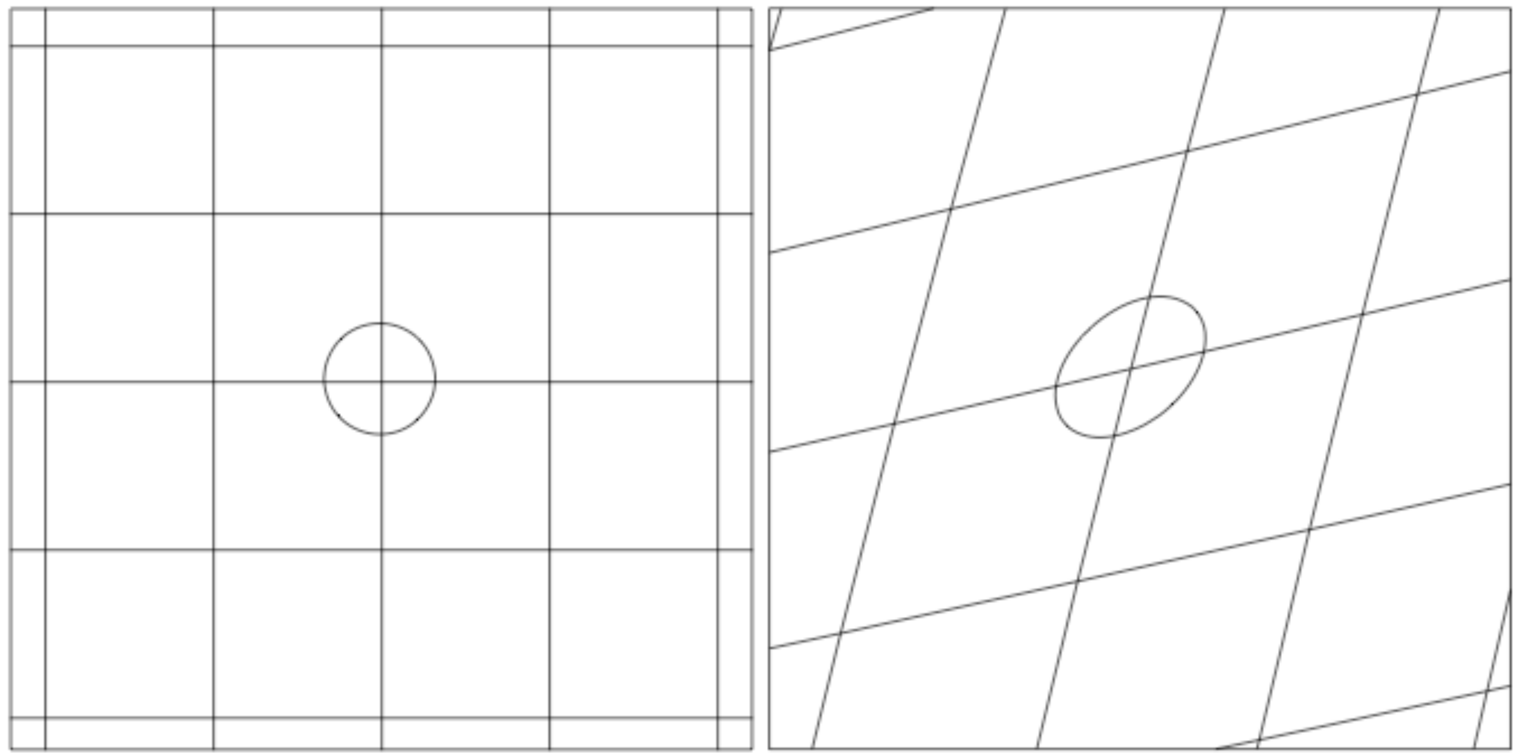

Kneib & Natarajan (2012)

$$F_\nu = \int_I I_\nu(\vec{\theta}) d^2\theta = \int_S I_\nu^S[\vec{\beta}(\vec{\theta})]\mu d^2\beta$$

Lensing changes the amount of photons (flux) we receive from the source by changing the solid angle the source subtends

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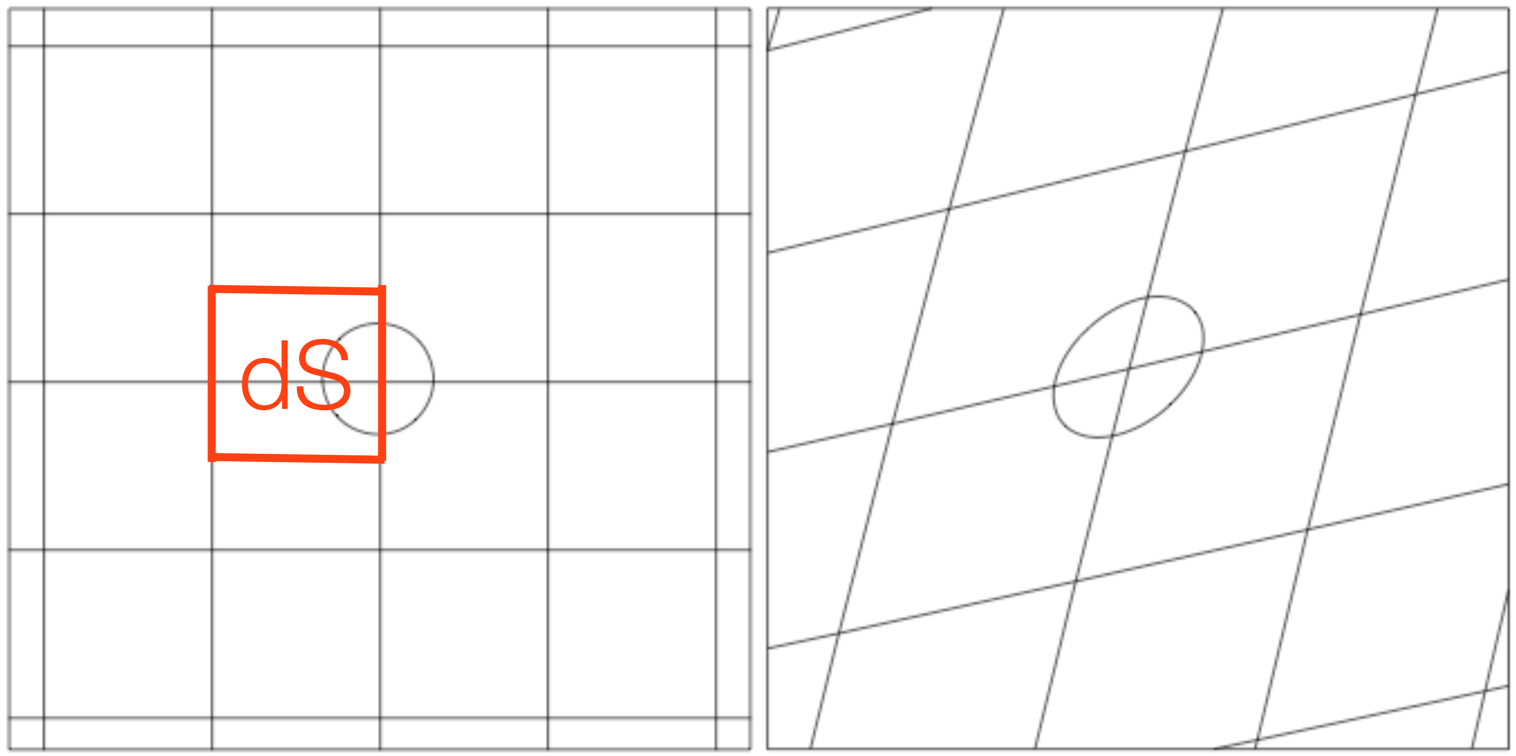

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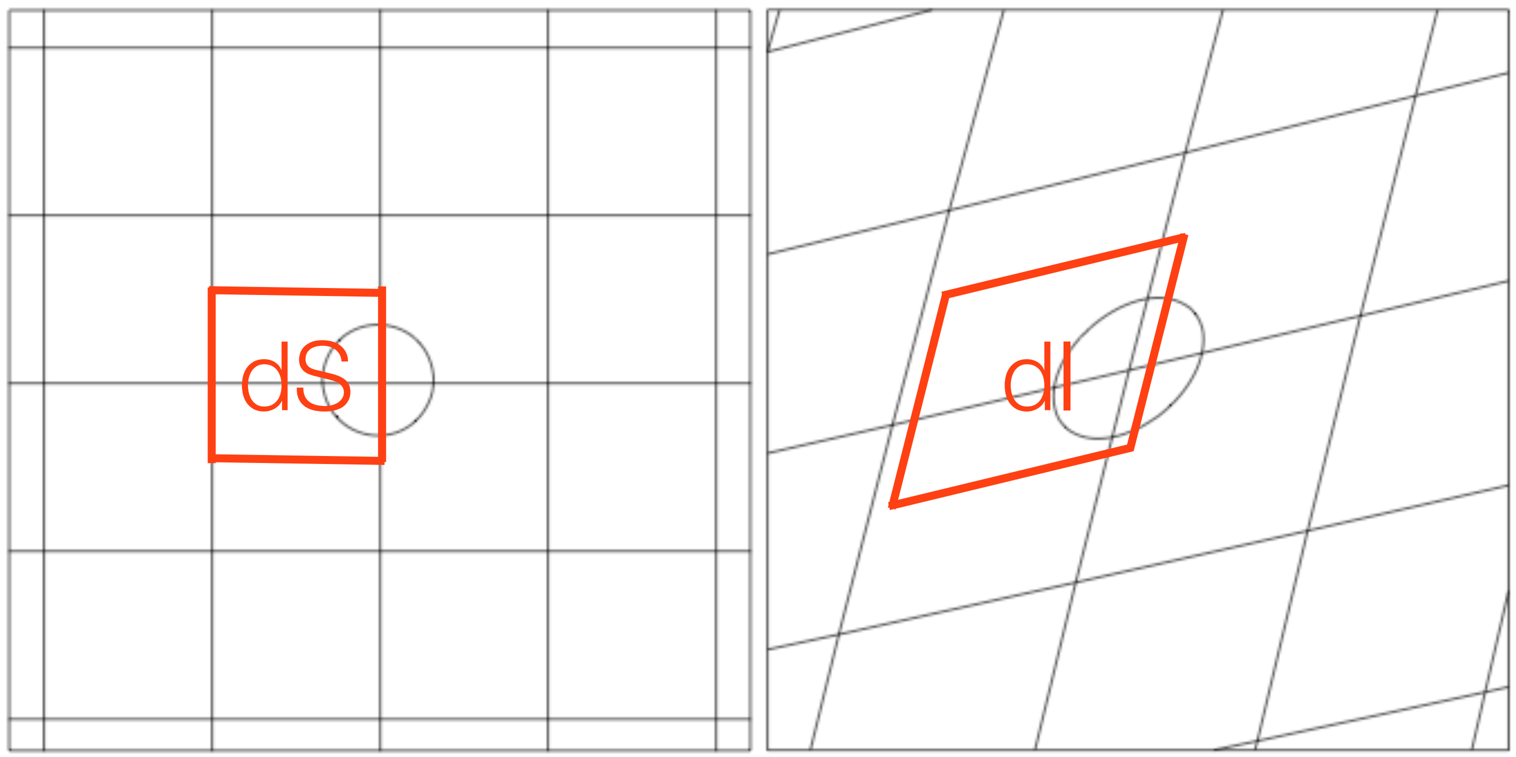

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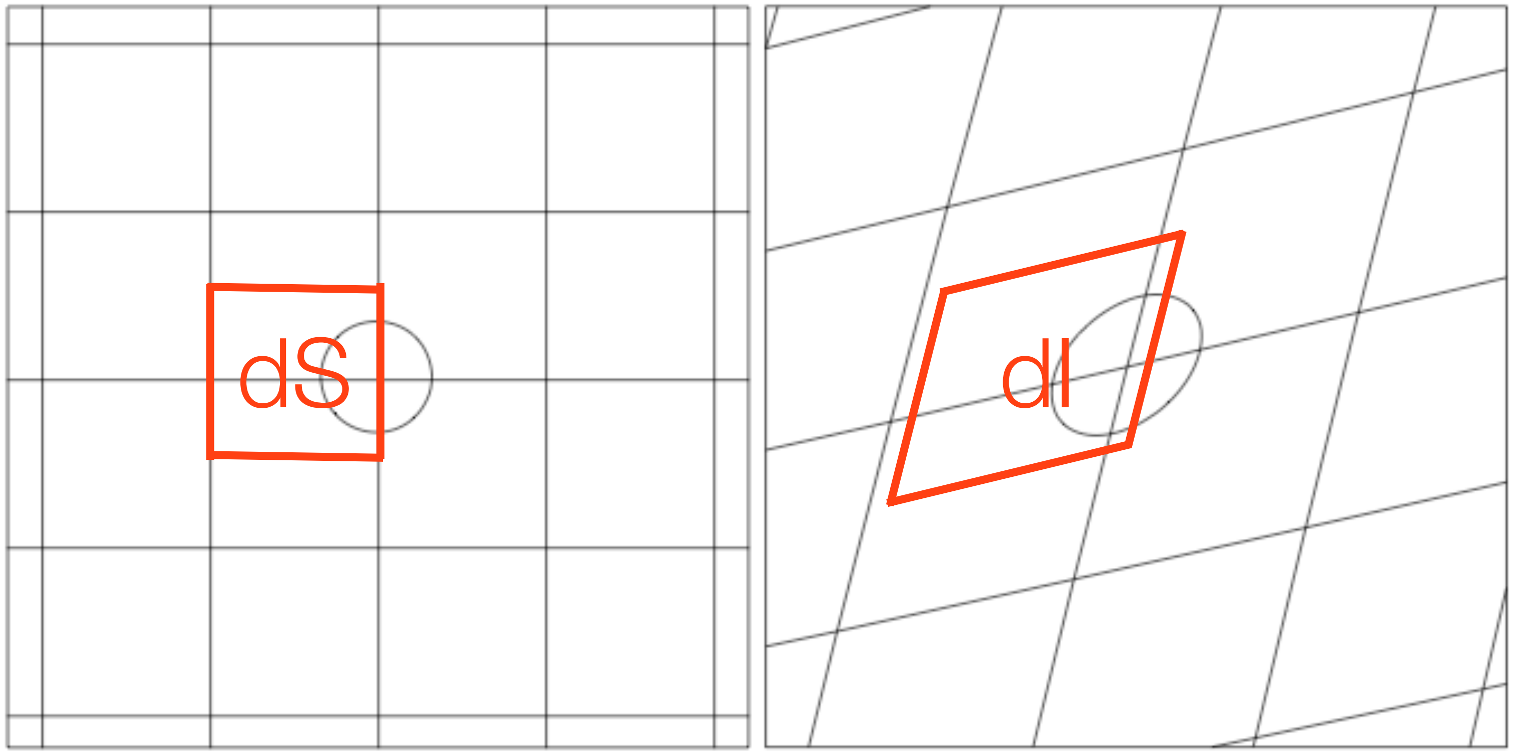

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CRITICAL LINES AND CAUSTICS

Both convergence and shear are functions of position on the lens plane:

$\kappa = \kappa(\vec{\theta})$

$\gamma = \gamma(\vec{\theta})$

The determinant of the lensing Jacobian is

$$\det A = (1 - \kappa - \gamma)(1 - \kappa + \gamma) = \mu^{-1}$$

The ***critical lines*** *are the lines where the eigenvalues of the Jacobian are zero:*

$(1 - \kappa - \gamma) = 0$ *tangential critical line*

$(1 - \kappa + \gamma) = 0$ *radial critical line*

Along these lines the magnification diverges!

Via the lens equations, they are mapped into the ***caustics****…*

SECOND ORDER LENS EQUATION

$$\beta_i \simeq \frac{\partial \beta_i}{\partial \theta_j} \theta_j$$

A_{ij}

SECOND ORDER LENS EQUATION

$$\beta_i \simeq \frac{\partial \beta_i}{\partial \theta_j}\theta_j + \frac{1}{2}\frac{\partial^2 \beta_i}{\partial \theta_j \partial \theta_k}\theta_j \theta_k$$

$$A_{ij}$$

$$\frac{\partial A_{ij}}{\partial \theta_k} = D_{ijk}$$

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$$D_{ij1} = \begin{pmatrix} -2\gamma_{1,1} - \gamma_{2,2} & -\gamma_{2,1} \\ -\gamma_{2,1} & -\gamma_{2,2} \end{pmatrix} \qquad D_{ij2} = \begin{pmatrix} -\gamma_{2,1} & -\gamma_{2,2} \\ -\gamma_{2,2} & 2\gamma_{1,2} - \gamma_{2,1} \end{pmatrix}$$