GRAVITATIONAL LENSING

LECTURE 5

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CONTENTS

- Second order lensing: flexion (continuation)
- Time delays

SECOND ORDER LENS EQUATION

$$\beta_i \simeq \frac{\partial \beta_i}{\partial \theta_j} \theta_j$$

A_{ij}

SECOND ORDER LENS EQUATION

$$\beta_i \simeq \frac{\partial \beta_i}{\partial \theta_j}\theta_j + \frac{1}{2}\frac{\partial^2 \beta_i}{\partial \theta_j \partial \theta_k}\theta_j\theta_k$$

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$$\frac{\partial A_{ij}}{\partial \theta_k} = D_{ijk}$$

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$$D_{ij1} = \begin{pmatrix} -2\gamma_{1,1} - \gamma_{2,2} & -\gamma_{2,1} \\ -\gamma_{2,1} & -\gamma_{2,2} \end{pmatrix} \qquad D_{ij2} = \begin{pmatrix} -\gamma_{2,1} & -\gamma_{2,2} \\ -\gamma_{2,2} & 2\gamma_{1,2} - \gamma_{2,1} \end{pmatrix}$$

COMPLEX NOTATION

$$v = (v_1, v_2) \longrightarrow v = v_1 + iv_2$$

therefore,

$$\alpha = \alpha_1 + i\alpha_2$$

$$\gamma = \gamma_1 + i\gamma_2.$$

we can also define complex differential operators:

$$\partial = \partial_1 + i\partial_2$$

$$\partial^\dagger = \partial_1 - i\partial_2$$

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we can also define complex differential operators:

$\partial = \partial_1 + i\partial_2$ *Spin raising operator*

$\partial^\dagger = \partial_1 - i\partial_2$ *Spin lowering operator*

COMPLEX NOTATION

$$\partial\hat{\Psi} = \partial_1\hat{\Psi} + i\partial_2\hat{\Psi} = \alpha_1 + i\alpha_2 = \alpha$$

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From spin-0 scalar field to spin-1 vector field (deflection angle)

COMPLEX NOTATION

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From spin-0 scalar field to spin-1 vector field (deflection angle)

$$\partial^\dagger\partial = \partial_1^2 + \partial_2^2 = \triangle$$

$$\partial^\dagger\partial\hat{\Psi} = \triangle\hat{\Psi} = 2\kappa$$

COMPLEX NOTATION

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From spin-1 vector field to spin-0 scalar field

COMPLEX NOTATION

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From spin-0 scalar field to spin-1 vector field (deflection angle)

$$\partial^\dagger\partial = \partial_1^2 + \partial_2^2 = \triangle$$

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From spin-1 vector field to spin-0 scalar field

$$\frac{1}{2}\partial\partial\hat{\Psi} = \frac{1}{2}\partial\alpha = \gamma$$

The shear is a spin-2 tensor field

$$\partial^{-1}\partial^\dagger\gamma = \frac{1}{2}\partial^{-1}\partial^\dagger\partial\partial\hat{\Psi} = \partial^\dagger\partial\hat{\Psi} = \kappa$$

COMPLEX NOTATION

$$
\begin{aligned}
F &= \frac{1}{2}\partial\partial^{\dagger}\partial\hat{\Psi} = \partial\kappa \\
G &= \frac{1}{2}\partial\partial\partial\hat{\Psi} = \partial\gamma
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$$F = F_1 + iF_2 = (\gamma_{1,1} + \gamma_{2,2}) + i(\gamma_{2,1} - \gamma_{1,2})$$

$$G = G_1 + iG_2 = (\gamma_{1,1} - \gamma_{2,2}) + i(\gamma_{2,1} + \gamma_{1,2})$$

$$D_{ij1} = \begin{pmatrix} -2\gamma_{1,1} - \gamma_{2,2} & -\gamma_{2,1} \\ -\gamma_{2,1} & -\gamma_{2,2} \end{pmatrix} \qquad D_{ij2} = \begin{pmatrix} -\gamma_{2,1} & -\gamma_{2,2} \\ -\gamma_{2,2} & 2\gamma_{1,2} - \gamma_{2,1} \end{pmatrix}$$

COMPLEX NOTATION

$$F = \frac{1}{2}\partial\partial^\dagger\partial\hat{\Psi} = \partial\kappa$$ *Spin-1*

$$G = \frac{1}{2}\partial\partial\partial\hat{\Psi} = \partial\gamma$$ *Spin-3*

$$F = F_1 + iF_2 = (\gamma_{1,1} + \gamma_{2,2}) + i(\gamma_{2,1} - \gamma_{1,2})$$

$$G = G_1 + iG_2 = (\gamma_{1,1} - \gamma_{2,2}) + i(\gamma_{2,1} + \gamma_{1,2})$$

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GRAVITATIONAL TIME DELAY

In Lecture 1: $n = 1 - \frac{2\Phi}{c^2}$

$$t_{grav} = \int \frac{dz}{c'} - \int \frac{dz}{c} = \frac{1}{c}\int (n-1)dz = -\frac{2}{c^3}\int \Phi dz$$

Remember that $\hat{\Psi}(\vec{\theta}) = \frac{D_{\mathsf{LS}}}{D_{\mathsf{L}} D_{\mathsf{S}}} \frac{2}{c^2} \int \Phi(D_{\mathsf{L}}\vec{\theta}, z)\mathrm{d}z$

Therefore, $t_{\mathrm{grav}} = -\frac{D_{\mathsf{L}} D_{\mathsf{LS}}}{D_{\mathsf{S}}} \frac{1}{c} \hat{\Psi}$

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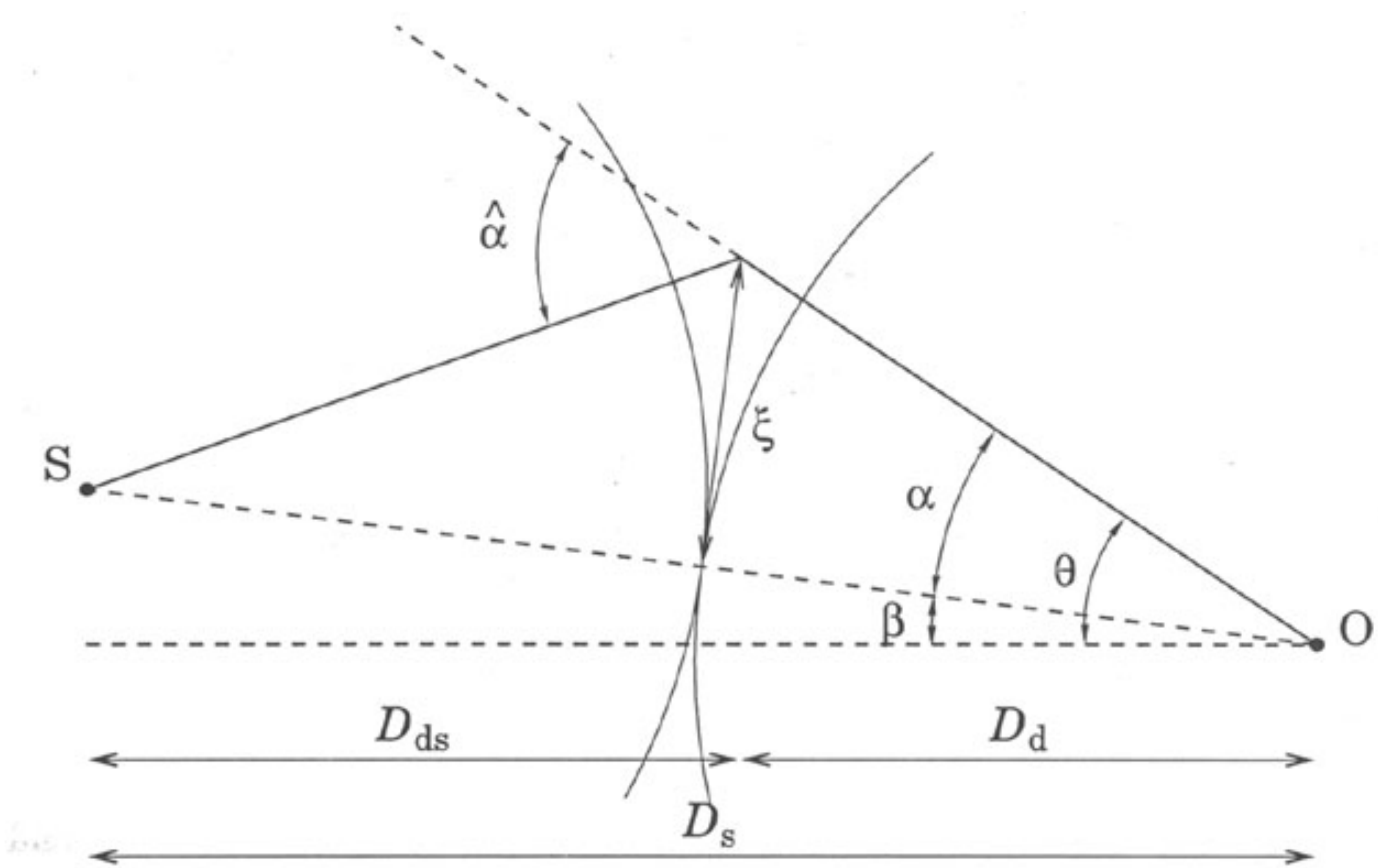
Therefore, $t_{\mathrm{grav}} = -\frac{D_{\mathsf{L}} D_{\mathsf{LS}}}{D_{\mathsf{S}}} \frac{1}{c} \hat{\Psi}$

Integrating along the line of sight!

GEOMETRICAL TIME DELAY



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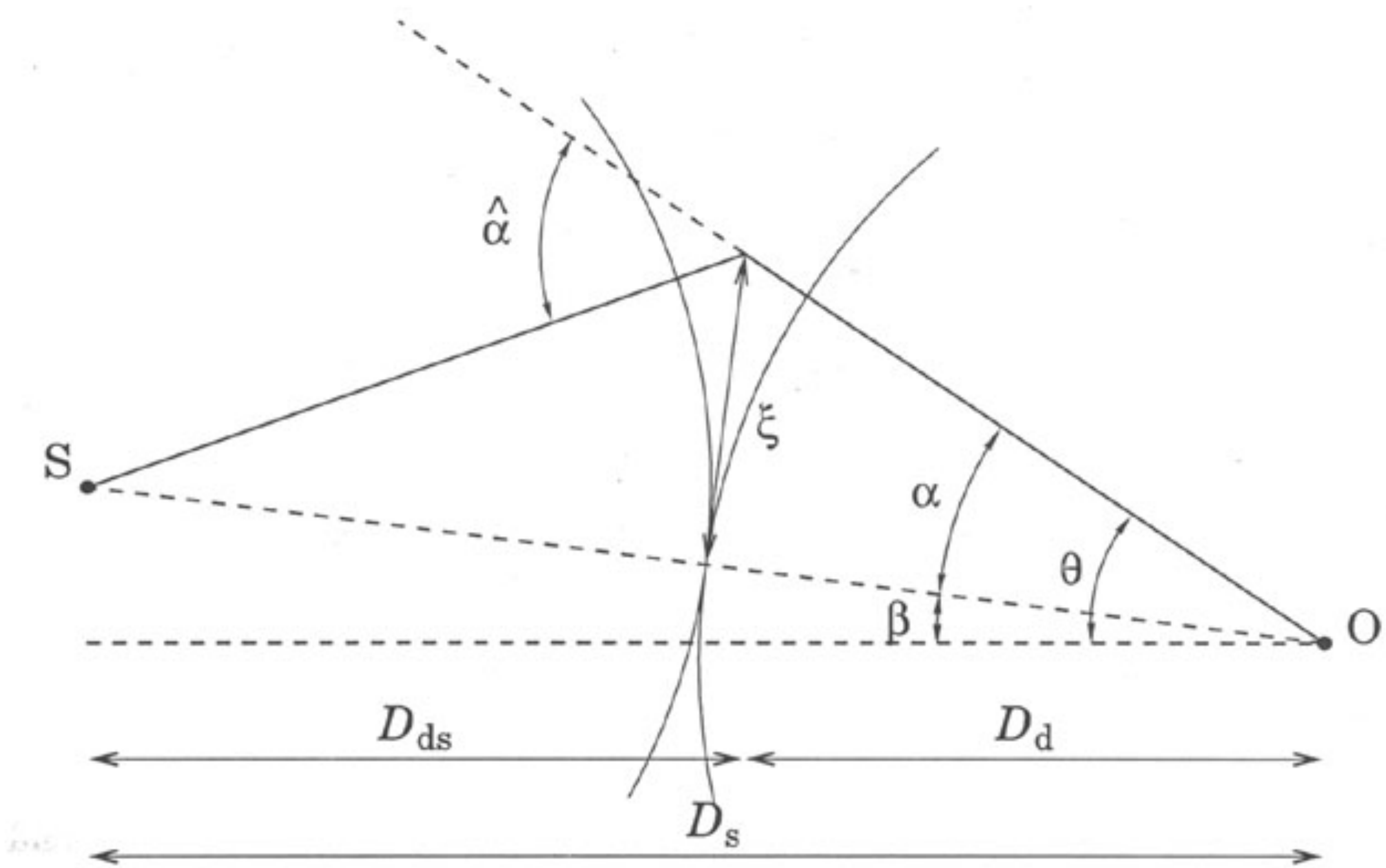

$$\Delta l = \xi \frac{\hat{\vec{\alpha}}}{2} = (\vec{\theta} - \vec{\beta}) \frac{D_{\mathsf{L}} D_{\mathsf{LS}}}{D_{\mathsf{S}}} \frac{\vec{\alpha}}{2} = (\vec{\theta} - \vec{\beta})^2 \frac{D_{\mathsf{L}} D_{\mathsf{LS}}}{2 D_{\mathsf{S}}}$$

TIME DELAY SURFACE

$$t(\vec{\theta}) = t_{geom} + t_{grav} \propto \left(\frac{1}{2}(\vec{\theta} - \vec{\beta})^2 - \hat{\Psi} \right)$$

$$\vec{\nabla} t(\vec{\theta}) \propto \left(\vec{\theta} - \vec{\beta} - \vec{\nabla}\hat{\Psi} \right)$$

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Lens equation!

TIME DELAY SURFACE

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Images form at the stationary points of t!

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Images form at the stationary points of t!

$$T_{ij} = \frac{\partial^2 t(\vec{\theta})}{\partial\theta_i \partial\theta_j} \propto (\delta_{ij} - \Psi_{ij})$$